SHARP ENERGY ESTIMATES FOR FRACTIONAL DIFFUSION EQUATIONS

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In this talk, I will mainly describe recent results in collaboration with Eleonora Cinti (Barcelona and Bologna) concerning elliptic nonlinear equations with fractional diffusion.

In a work in collaboration with Y. Sire (Marseille), we studied the equation $(-\Delta)^\alpha u = f(u)$ in $\mathbb{R}^n$ with $\alpha \in (0,1)$. Crucial to our analysis is a result of Caffarelli-Silvestre which allows to realize this nonlocal equation as a degenerate elliptic equation in $\mathbb{R}^{n+1}_+$ together with a nonlinear Neumann boundary condition on $\mathbb{R}^n = \partial\mathbb{R}^{n+1}_+$. We characterized the nonlinearities f for which there exists a layer solution —meaning, essentially, a solution increasing in one direction. We also established several properties of these solutions, such as their uniqueness and decay at infinity in $\mathbb{R}$, minimality in $\mathbb{R}^n$, and 1D symmetry in $\mathbb{R}^2$.

In a more recent work in collaboration with E. Cinti, we establish the optimal energy estimates for minimizers, layer solutions, and also other solutions such as saddle-shaped solutions. These estimates allow to prove in $\mathbb{R}^3$ the 1D symmetry result of De Giorgi type for the nonlocal equation, previously only known in $\mathbb{R}^2$.

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