In search for diffeomorphism symmetry

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[B.D., J.Ryan 0807.2806 [gr-qc]; B.D. 0810.3594 [gr-qc]; B.D., B. Bahr, to appear; w.i.p.]

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Set-up

- lattice models of gravity, i.e. triangulations
- covariant: Regge calculus, conclusions hold also for discretized Palatini, Plebanski, spin foam models
- canonical: Hamiltonian versions of Regge calculus, "LQG on a fixed lattice" (vs LQG as continuum theory) ...
- problem so far: constraint algebra not closed, many ambiguities (even classically)
- Are covariant lattice models better?

Overview

- Exact or broken diffeomorphism symmetry in discretized actions?

bad news

- Repercussions for canonical descriptions?

  Can we derive a Hamiltonian from spin foam models?

bad & good news

- Do
  discrete actions with exact diffeomorphism symmetry /
  first class diffeo constraint algebras on lattice exist?
  If not, why?

???

Why do we love diffeomorphism symmetry?

- restricting ambiguities, uniqueness results, avoiding divergencies
  quantiz. of 3d with cosm. const.,   F-LOST, ...
- control over lattice effects → Lorentz symmetry breaking, singularity resolution
- master constraint, uniform discretization   vs
  trying to keep control over symmetry
- sum over triangulations:
  diffeo invariant (effective) action could make that precise
  (triangulation independence), otherwise problems with measure

Exact or broken diffeomorphism symmetry in discretized actions?

What is a gauge symmetry?

[B.D. 08]

- property of solutions determined by action:
  n-parameter set of solutions with action of symmetry group

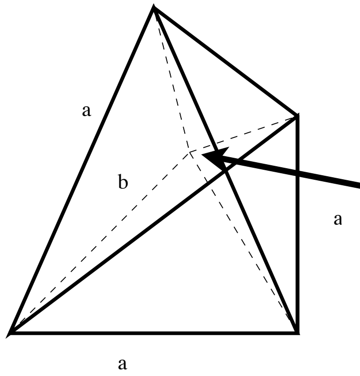
  - a) different solutions can have different gauge orbit size
    $\rightarrow$complicates definition of observables (n-point fct.s)

  - b) need to solve equations of motion

- necessary condition:
  Hessian of the action evaluated on solution has zero eigenvalues

- not sufficient:
  (configuration dependent) transformations that leave the action invariant $\rightarrow$ typically trivial action on solutions

Example: 3d grav with/ without cosm. constant

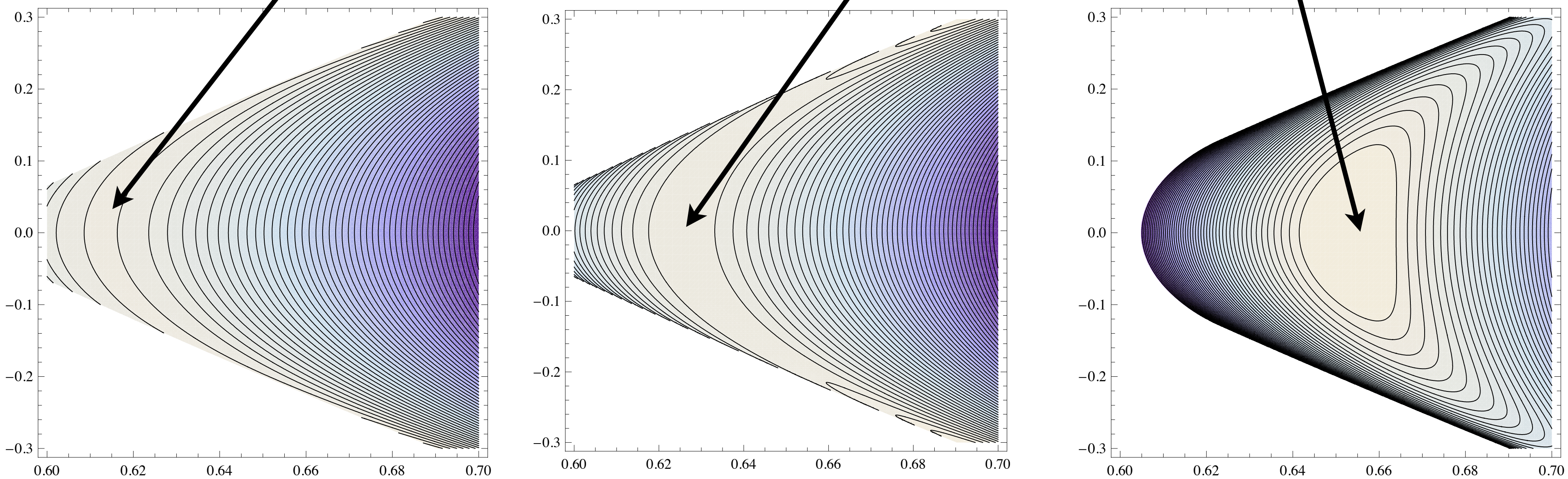


contour plot of the action as function of internal edge length

translation symmetry of inner vertex?

null eigenvalues of Hessian at solutions

non-vanishing eigenvalues of Hessian at solution



Lambda →

4d gravity (Regge calculus)?

- flat vertices (curvature vanishes at adjacent triangles):display diffeo/translation symmetry.
  There are many curved solutions with flat vertices!
- in canonical picture corresponding to 3d triangulations only admitting flat 4d geometries: first class constraint algebra explicitly known! (topological sector)
  [B.D. , Ryan 08]

4d gravity - opinions

Yes!

- [Hamber, Williams 97]
  Hessian has null eigenvalues in 2d examples, on flat background

- [Freidel, Louapre 03]
  in analogy with 3d case, argue with Bianchi identity

No?

- [Rocek & Williams, Morse, Miller ...]
  Bianchi identity leads to (configuration independent) transformations leaving action approximately invariant
  (but this applies also in 3d)

But so far no explicit test for 4d without cosmological constant!
(as numerical algorithmen for Regge use a gauge fixing)

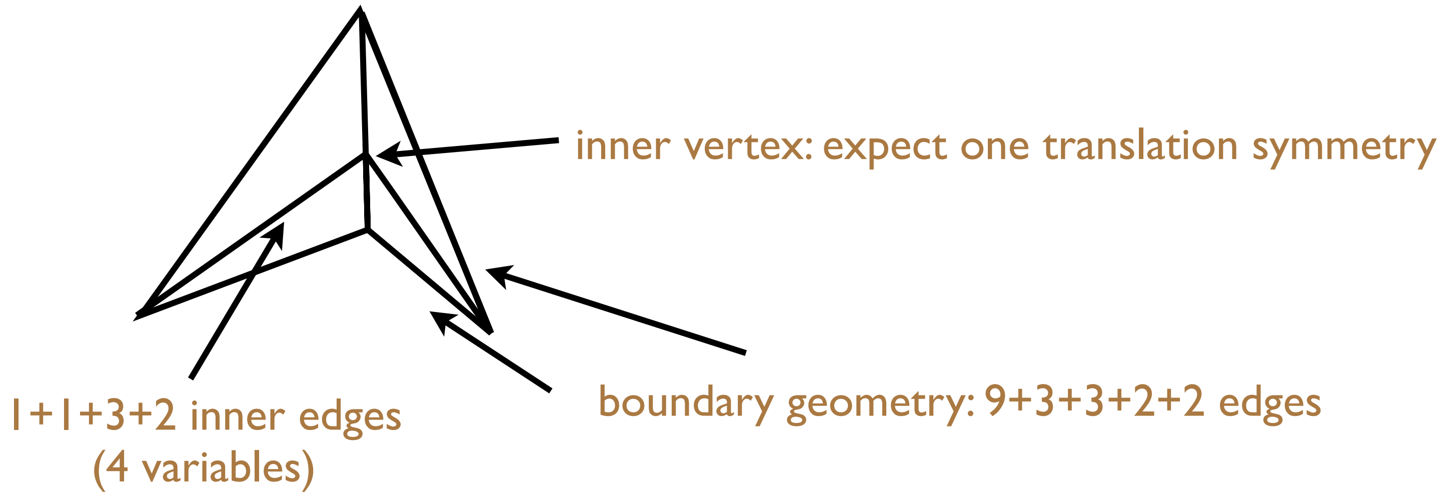
4d gravity with non-trivial vertices?

No!

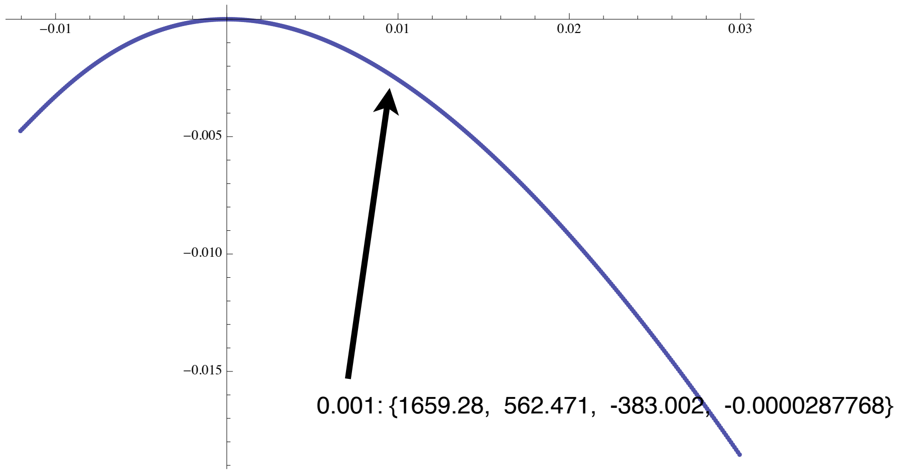
- diffeo symmetry is broken: small but non-vanishing eigenvalues of Hessian [B.D. , Bahr: to appear]
- but action is almost constant in some directions
  → path integral almost diverging
  → enforces "Gaussian" vs delta function on constraints

Example: a small 4d Regge solution with an inner vertex

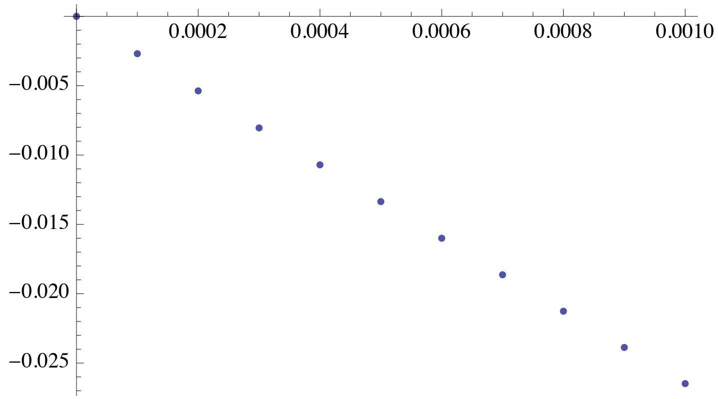
2d abstraction of 4d triang. (with 12 four-simplices, "homogeneous" configuration)



Example: a small 4d Regge solution with an inner vertex



lowest eigenvalues of Hessian as function of deviation parameter from flat configuration



curvature at one triangle as function of deviation parameter from flat configuration

Exact or broken diffeomorphism symmetry in discretized actions?

Exact or broken diffeomorphism symmetry in discretized actions?

Yes and No - but.

1) flat vertices are translation invariant

2) invariance (slightly) broken at non-trivial vertices for the Regge action

But (broken) symmetries depend on choice of action!

Example:

- 3d with cc,
  Regge action
  [Regge]
- broken translation symmetries at vertices
- constraints depend on lapse and shift (these get fixed)

- 3d with cc,
  modified Regge action
  [B.D. , Bahr: to appear]
- exact translation symmetries at vertices
- first class constraints!

Repercussions for canonical descriptions?

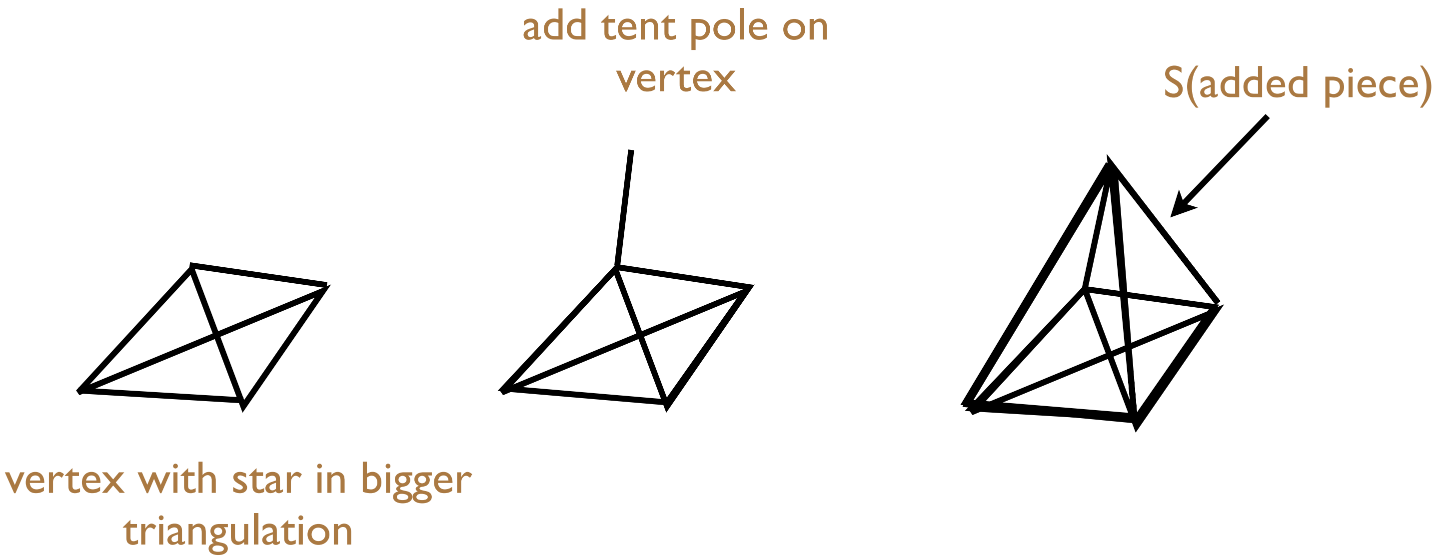
a) Canonical formalism reproducing solutions and (non-)symmetries of discretized action?

b) Constraints? Constraint algebra?

Evolving spatial triangulations with tent moves

[Sorkin 75, Barrett et al 97]

- act local, involving only star of a vertex
- do not change spatial triangulation/ number of variables

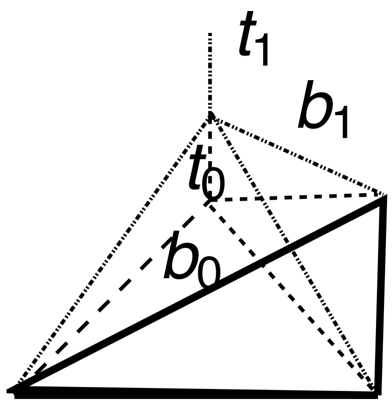


Example: 3d grav with/ without cosm. constant

[B.D. 08]

-rewrite equations of motions into canonical form (consistent discretizations)

- use S(added piece) as generating function for momenta (Hamilton-Jacobi)



$$
\begin{aligned}
0 &= p_{t_0} &&= -\frac{\partial S(b_0,b_1,t_0)}{\partial t_0} \\
0 &= p^-_{b_1} - p^+_{b_1} &&= \frac{\partial S(b_0,b_1,t_0)}{\partial b_1} + \frac{\partial S(b_1,b_2,t_1)}{\partial b_1}
\end{aligned}
$$

Example: 3d grav with/ without cosm. constant

$$0 = -\frac{\partial S(b_0, b_1, t_0)}{\partial t_0} \quad \rightarrow \quad t_0 = T_0(b_0, b1)$$

$$p^-_{b_1} = \frac{\partial S(b_0, b_1, t_0)}{\partial b_0}$$

$$\begin{aligned} p^-_{b_1} &= \frac{\partial S(b_0, b_1, t_0)}{\partial b_0}_{|t_0 \rightarrow T_0} \\ &= C_0(b_1) + \lambda C_1(b_1) + \lambda^2 C_2(b_1, b_0) + \ldots \end{aligned}$$

$\Rightarrow$ constraint hypersurface of finite width

constraint

lapse/shift dependence

Example: 3d grav with/ without cosm. constant

- obtain constraints for vanishing cc
  (lapse, shift arbitrary)
- for non-vanishing cc constraints fix lapse and shift
  (there is a unique solution!)

Canonical formalism displays exactly symmetries of covariant equations!

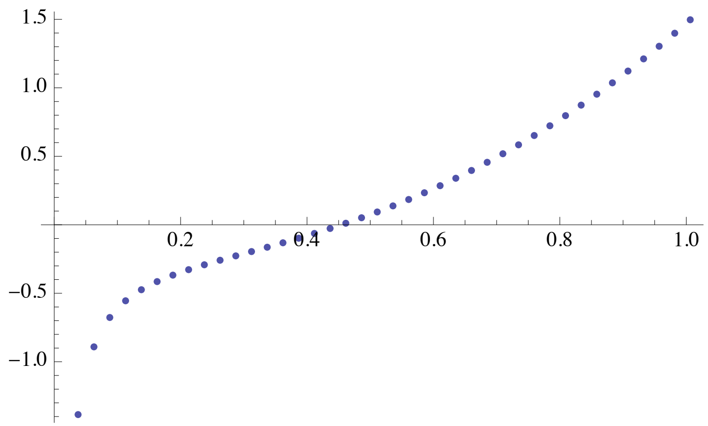
Continuum limit in time?

$$p_{b_1} \quad = \quad C_0(b_1) + \lambda C_1(b_1) + \lambda^2 C_2(b_1, b_0) + \ldots$$

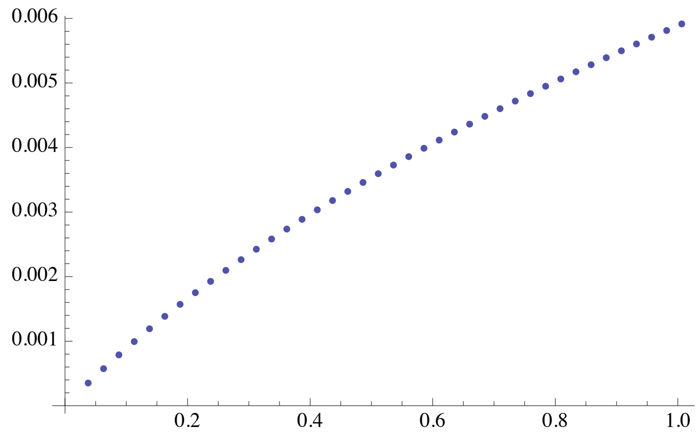
limit of small time steps: $\quad b_0 \longrightarrow b_1?$

Does not work (for Regge action)!

Continuum limit in time?



eigenvalues of the Hessian
corresponding to spatial diffeos
as function of time like length

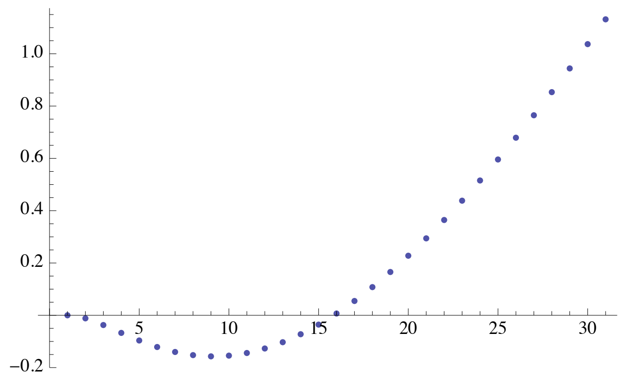


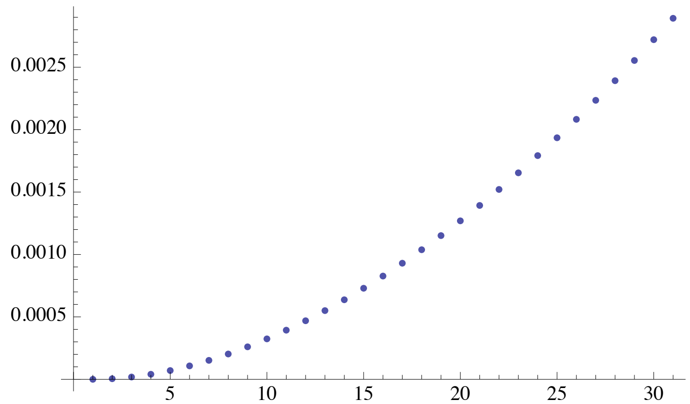
eigenvalues of the Hessian
corresponding to Hamiltonian
as function of time like length

Symmetries are not restored if we take only time like length to be small!

Take all lengths to be small!

corresponds to small lambda





Eigenvalues of the Hessian as function of lambda: at least quadratic.

↓

We obtain a first class algebra to first order in lambda!

This first order coincides with the first order constraints from the exact action.

Higher order terms are severely restricted by first order (uniquely?).

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Repercussions for canonical descriptions?

a) Canonical formalism reproducing solutions and symmetries of discretized action?

b) Constraints? Constraint algebra?

a) Yes.

b1) Constraints depend on lapse/shift.

b2) Continuum limit in time does not work.

b3) Exact constraints with closed algebra to first order in curvature.

Does an action with exact diffeo symmetry / a closed constraint algebra on lattice exist for 4d gravity?

Reparametrization invariant toy systems

[Marsden, West 01, "Symplectic Integrators"], [B.D., Bahr, to appear]

- take reparametrization invariant action, discretize, generically invariance broken
- But there is always a "discrete exact action"!
- trick: use the Hamilton-Jacobi functional of continuum theory as discrete action
- ⇒discrete theory captures exactly continuums dynamic
- can be obtained by integrating out almost all variables
  (" renormalization group flow")

$$
\begin{aligned}
S_e &= \sum_{n=0}^{N-1} S_{HJ}^{s_n, s_{n+1}}(t_n, q_n, t_{n+1}, q_{n+1}) \\
&= \sum_{n=0}^{N-1} \int_{s_n}^{s_{n+1}} ds \; L(t(s), q(s)) \quad .
\end{aligned}
$$

continuums solution

Gravity?

- discrete exact action exists for 3d with cc, but topological theory

- 4d Regge: can we obtain constraints to linear order in curvature?
- 3d/4d Regge: renormalization group flow [wip]

Do actions with exact diffeo symmetry / first class constraint algebras on lattice exist for 4d?

We hope so. And their might be a constructive algorithm.

Conclusion

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bad news

- Repercussions for canonical descriptions?

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Thank you for this great workshop!