Black hole entropy in Loop Quantum Gravity: Inclusion of distortions and rotation

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Outline

- Background and motivation
- Definition of multipoles
- Phase space we are quantizing; U(1) connections at the horizon
- Quantization strategy
- Quantum operators $\hat{\zeta}$, $\hat{\Psi}_2$
- Multipole operators
- Ensemble and entropy

Background and motivation

⇒ Black hole thermodynamics (Bekenstein, Hawking):

$$S = \frac{1}{4}A, \quad T = \kappa/8\pi G$$
$$\mathrm{d}M = T\mathrm{d}S + \Omega\mathrm{d}J$$

was argued for globally stationary space-times, using classical GR, and QFT in curved space-time.

⇒ Extended to isolated horizons (Ashtekar, Beetle, Fairhurst, Lewandowski): same thermodynamics with only stationarity of *intrinsic horizon* geometry required ("*isolated horizon* boundary conditions").

⇒ Entropy calculation in LQG (Ashtekar, Baez, Corichi, Krasnov): Statistical mechanical derivation of the entropy, assuming intrinsic geometry of horizon is spherically symmetric ("*type I* isolated horizon")

⇒ Present work: extend to axisymmetric case ("*type II* isolated horizon").

For entropy calculation, need diffeomorphism-invariant observables to characterize horizon geometry (q_{ab}, D_a). Free initial data: $(\tilde{q}_{ab}, \tilde{\omega}_a)$.

Definition of Multipoles. When S is a cross-section of an axisymmetric ("type II") isolated horizon,

$$I_n + iL_n := -\int_S Y_{n,0}(\zeta)\Psi_2 \, {}^2\epsilon$$

where

- (ζ, ϕ) are the unique coordinates in which $\tilde{q}_{ab} = R^2 \left(\frac{1}{f(\zeta)} \partial_a \zeta \partial_b \zeta + f(\zeta) \partial_a \phi \partial_b \phi \right)$
- $\Psi_2 = \frac{-1}{4} \tilde{\mathcal{R}} + \frac{i}{2} \tilde{\epsilon}^{ab} \tilde{\mathcal{D}}_a \tilde{\omega}_b$.

They are not just useful for entropy calculation — they are also used in numerical relativity! [5]

Reconstruction:

- Choose (ζ, ϕ) (diffeo freedom)
- $\Psi_2 := \frac{-1}{R^2} \sum_n (I_n + iL_n) Y_{n,0}(\zeta)$
- $f(\zeta) = 4 \left[R^2 \int_{-1}^{\zeta} \mathrm{d}\zeta_1 \int_{-1}^{\zeta_1} \mathrm{d}\zeta_2 \mathrm{Re}\Psi_2(\zeta_2) \right] + 2(\zeta + 1)$
- $\tilde{q}_{ab} = R^2 (\ldots)$
- $\tilde{\mathcal{D}}_{[a} \tilde{\omega}_{b]} = \mathrm{Im}\Psi_2 \tilde{\epsilon}_{ab}$ and $\tilde{q}^{ab} \tilde{\mathcal{D}}_a \tilde{\omega}_b = 0$ determine $\tilde{\omega}_a$.

Phase space we are quantizing

- Basic variables: Ashtekar-Barbero variables $({}^{\gamma}A_a^i, {}^{\gamma}\Sigma_{ab}^i = \epsilon_{abc}{}^{\gamma}E^{ci})$.
- Boundary conditions: internal boundary, S, is type II isolated horizon with fixed multipoles $\mathring{I}_n$,$\mathring{L}_n$ and fixed area a_o.

Partial gauge-fixing condition ($r^i E_i^a$ = the normal to S) reduces $SU(2)$ gauge, at S, to $U(1)$. Physical U(1) connection:

$$V := \frac{1}{2}{}^{\gamma}\underleftarrow{A}^i r_i = \frac{1}{2}\left(-\Gamma_a + \gamma\omega_a\right)$$

Canonical type I geometry and assoc. U(1) connection

$$\mathring{q}_{ab} = R^2\left(\frac{1}{\mathring{f}}\partial_a\zeta\partial_b\zeta + \mathring{f}\partial_a\phi\partial_b\phi\right)$$

where $\mathring{f} := 1 - \zeta^2$. We also define

$$V_a^o := V_a - \frac{1}{4}\left(f' - \mathring{f}'\right)\partial_a\phi - \frac{\gamma}{2}\omega_a$$

Manifestly $U(1)$ and diffeo cov:
imp. for solving Gauss and diffeo constraints

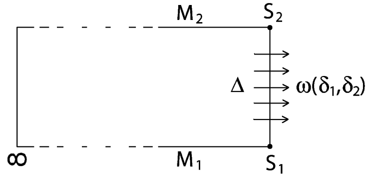
where ζ, ϕ, f are as on last slide.

Horizon boundary condition:

$$\mathrm{d}V^o = -\frac{2\pi}{a_o}\left({}^2\epsilon\right) = -\frac{16\pi^2\gamma}{a_o}\left({}^{\gamma}\underleftarrow{\Sigma}^i r_i\right)$$

Symplectic structure

Can calculate symplectic current $\omega(\delta_1, \delta_2)$ from Lagrangian. On-shell $d\omega = 0$ ("locally conserved"). Usually this is enough for $\int_\Sigma \omega$ to be a good definition of symplectic structure that is independent of Σ. But in present case, symplectic current "escapes" across the horizon:



To fix this, decompose $\int_\Delta \omega = \left(\oint_{S_1} - \oint_{S_2}\right)\lambda$, and define $\Omega_\Sigma = \int_\Sigma \omega + \oint_S \lambda$ so that onshell

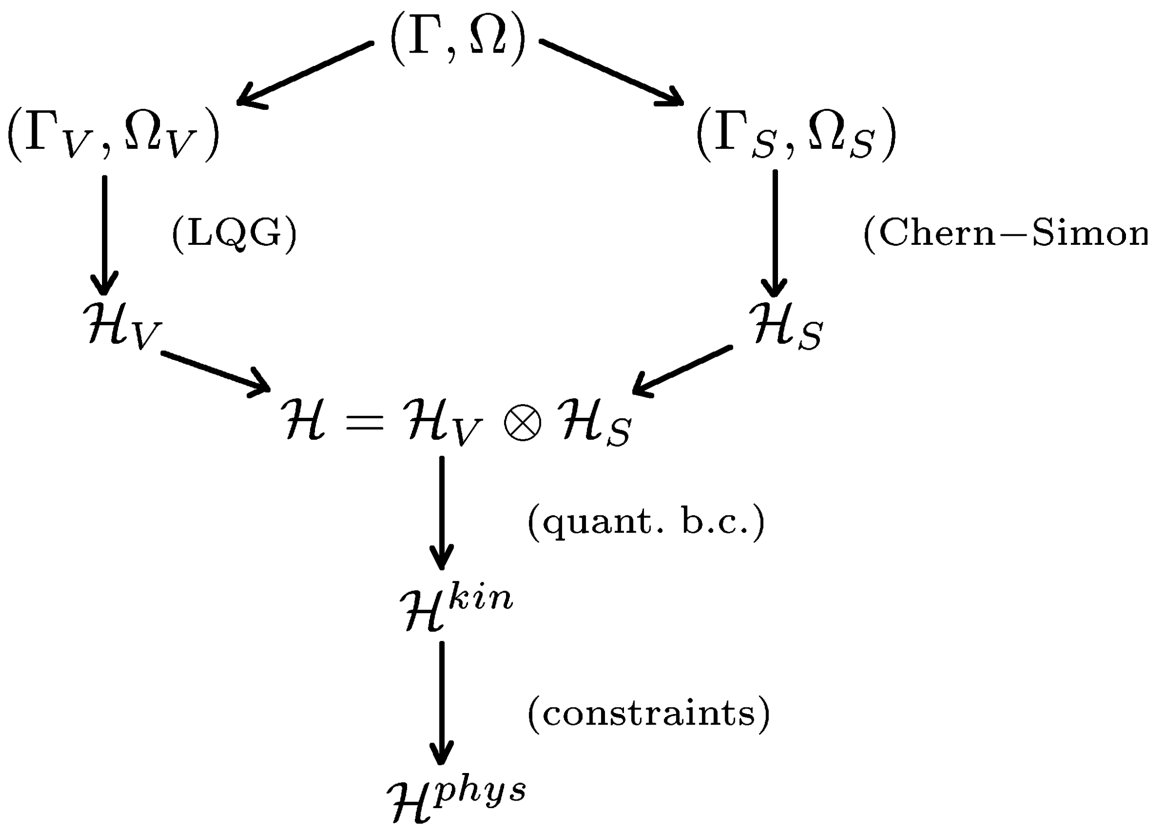
$$\Omega_{\Sigma_1} - \Omega_{\Sigma_2} = \left(\oint_{\Sigma_2} - \oint_{\Sigma_1}\right)\omega + \int_\Delta \omega = 0.$$

Final result:

$$\Omega(\delta_1, \delta_2) = -\int_M \mathrm{Tr}(\delta_1{}^\gamma A \wedge \delta_2{}^\gamma \Sigma - \delta_2{}^\gamma A \wedge \delta_1{}^\gamma \Sigma) + \frac{1}{8\pi G}\frac{a_o}{\gamma\pi}\oint_S \delta_1 V^o \wedge \delta_2 V^o$$

Note: Canonically associated *type I* connection appears in horizon symplectic structure. The type II connection cannot be used.

Quantization Strategy



Imposing the constraints

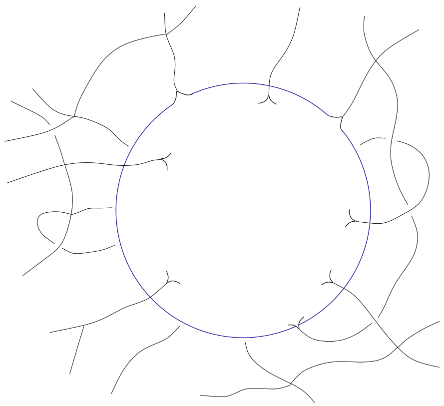
Solution to quant. b.c. and Gauss constr.

$$\mathcal{H}_{\text{kin}} = \bigoplus_{\substack{\mathcal{P},\vec{m},\vec{b} \\ 2\vec{m}=\vec{b} \bmod k}} \mathcal{H}_V^{\mathcal{P},\vec{m}} \otimes \mathcal{H}_S^{\mathcal{P},\vec{b}}$$

$\mathcal{P}$:punctures

$\vec{m}$:label quant. excit'ns of $\underleftarrow{r_i} \hat{\Sigma}^i$

$\vec{b}$:label quant. excit'ns of Chern-Simons curvature (holonomies)



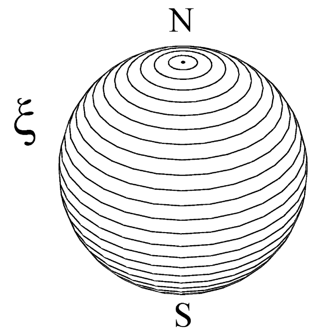
Diffeo. constr.

Group average over ["divide by"] all diffeos preserving M and S.

Hamiltonian constr.

Is imposed in bulk as usual. Is not imposed on the horizon b/c $C(N)$ does not generate gauge unless lapse vanishes at S.

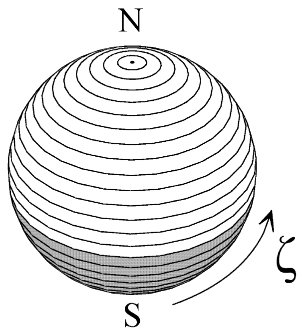
- Fix **axial foliation** ξ — gauge-fixing in the sense of being used to interpret the physics.
  For convenience: introduce ζ_0 as background coordinate labeling leaves of ξ.



- preferred coordinate $\hat{\zeta}$ **operator**:

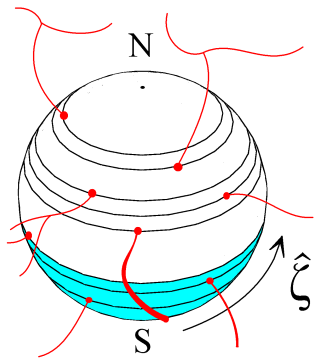
$$\hat{\zeta}(\zeta_0) = -1 + \frac{2\hat{a}_{\zeta<\zeta_0}}{\hat{a}_S}$$

  (taken over directly from classical theory.)
  Area e-vals are discrete $\rightarrow \hat{\zeta}$ e-vals are discrete.



- $\mathring{\Psi}_2(x) = -\frac{1}{R_0^2}\sum_n(\mathring{I}_n + i\mathring{L}_n)Y_{n,0}(x), \quad \left(\mathring{\Psi}_2 : [-1,1] \rightarrow \mathbb{C}\right)$

$$\hat{\Psi}_2(p) := \mathring{\Psi}_2(\hat{\zeta}(p))$$



Multipole operators.

Classically: $I_n + iL_n = -\oint \Psi_2 Y_{n,0}(\zeta)\, {}^2\epsilon = -\frac{a_S}{2}\int_{-1}^{1} \Psi_2 Y_{n,0}(\zeta) \mathrm{d}\zeta$

Motivates: "$\hat{I}_n + i\hat{L}_n = -\frac{\hat{a}_S}{2}\int_{-1}^{1} \hat{\Psi}_2 Y_{n,0}(\hat{\zeta}) \mathrm{d}\hat{\zeta}$"

regularize: set $\zeta = \lim_{i\to\infty} \zeta_i$, ζ_i smooth.

$$\hat{I}_n + i\hat{L}_n = -\lim_{i\to\infty} \frac{\hat{a}_S}{2}\int_{-1}^{1} \mathring{\Psi}_2(\hat{\zeta}_i) Y_{n,0}(\hat{\zeta}_i) \mathrm{d}\hat{\zeta}_i = \boxed{\frac{\hat{a}_S}{a_o}\left(\mathring{I}_n + i\mathring{L}_n\right)} \qquad (1)$$

Def'n of ensemble.

$$a_o - \delta < a_S < a_o + \delta$$

From eq'n (1): $$\frac{\Delta \hat{a}_S}{a_o} = \frac{\Delta \hat{I}_n}{\mathring{I}_n} = \frac{\Delta \hat{L}_n}{\mathring{L}_n}$$

Answer for entropy: Same as in type (I) case!

$$\boxed{S = \frac{1}{4}\frac{\gamma_0}{\gamma} a_o, \qquad \gamma_0 = 0.2375329579\ldots}$$

Synopsis

- Type II case **reduces to type I case** :
  - Surface symplectic str. is Chern-Simons with V_a^o
  - Relation b/w V_a^o and bulk is again given by
    $\mathrm{d}V^o = -\frac{16\pi^2\gamma}{a_o}\left({}^\gamma\underleftarrow{\Sigma}\cdot r\right)$

  Do same quantization as before and get same entropy.

- However, **physical interp.** of V_a^o in type II case is different: concentrations of $\mathrm{d}V^o$ at punctures are no longer simply deficit angles. $\hat{\zeta}$, $\hat{\Psi}_2$,$\hat{I}_n$,$\hat{L}_n$ introduced to recover physical interp. in type II case.

- Note: ***Takes us far beyond Kerr Isolated Horizons.*** Kerr is a 2-parameter family, whereas the multipoles are an infinite set of parameters.

References

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[2] A. Ashtekar, J.E., T. Pawlowski and C. Van Den Broeck 2004 Multipole moments of isolated horizons, Class. Quantum Gravity **21**, 2549-2570

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[5] E. Schnetter, B. Krishnan and F. Beyer 2006 Introduction to dynamical horizons in numerical relativity, *Preprint:* `gr-qc/0604015`