BH STATE COUNTING

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STATE COUNTING

- Number of punctures carrying label j, m is $s_{j,m}$
- Conditions on each configuration $s_{j,m}$:

$$2\sum s_{j,m}\sqrt{j(j+1)} \in [A-\epsilon, A+\epsilon]$$

$$\sum m s_{j,m} = 0$$

- To get the dominant configuration the entropy

$$S = \log d[s_{j,m}] = \log\left[\frac{(\sum s_{j,m})!}{\prod s_{j,m}!}\right]$$

is to be maximized subject to above conditions

- $2\sum \delta s_{j,m}\sqrt{j(j+1)} \in [-\epsilon, \epsilon],\ \sum m\delta s_{j,m} = 0$

CONFIG. LABELLED BY [J,m]

- This gives : $\bar{s}_{j,m} = (\sum \bar{s}_{j,m}) e^{-2\lambda\sqrt{j(j+1)} - \alpha m}$
- The m-constraint gives $\alpha = 0$
  λ is determined by the Eq. $1 = \sum e^{-2\lambda\sqrt{j(j+1)}}$
- However, one is really interested in the total no. of states $d = \sum d[s_{j,m}]$
- Assumption : Expand d around the dominant configuration $s_{j,m} = \bar{s}_{j,m} + \delta s_{j,m}$ where $\delta s_{j,m}$ must satisfy the two constraints, to 2nd order
- Sum over $\delta s_{j,m}$: $d = \frac{\text{const}}{\sqrt{A}} e^{\lambda A}, \ \text{const} \sim o(1)$

CONFIG. LABELLED BY [m]

- No. of punctures carrying label m, no matter what j they come from :
  $$s_m = \sum_j s_{j,m}, \;\; j = |m|, |m|+1, |m|+2, \ldots$$
- Maximize $$\log[d(s_m)] = \log\left[\frac{(\sum s_m)!}{\prod s_m!}\right]$$
  subject to the same two earlier conditions
- One has to be more careful here : For each m there exists one j(m) which turns out to be the minimum value of j that yields the m, that is
  $$j(m) = |m| \; \forall \; m \neq 0, \quad j(0) = 1$$

CONFIG. [m]

- This gives : $s_m = (\sum s_m) e^{-2\lambda\sqrt{j(m)(j(m)+1)} - \alpha m}$
- As before $\alpha = 0$ from the m-constraint and λ is determined by the Eq. (3=2+1)

$$1 = \sum_{|m| \neq 1} 2e^{-2\lambda\sqrt{|m|(|m|+1)}} + 3e^{-2\lambda\sqrt{2}}$$

- The result is very close to Meissner et al except $\lambda = 0.79$ not 0.746 in $S = \lambda A - \frac{1}{2} \log A$
- [j,m] = Physics Letters B 616 (2005) 114
  [m] = Physical Review D 74 (2006) 064026

PRECISE COUNTING

- Strict area condition $A = 2\sum s_{j,m}\sqrt{j(j+1)}$ implies :
$$0 = \sum \delta s_{j,m}\sqrt{j(j+1)}$$
- The 1st factor is an integer, whereas the 2nd one is an irrational number
- So any two arbitrary terms cannot cancel each other, only those with rational ratios can
- As a result the sum splits into several classes, each class contains terms having rational ratios, which are separately zero – this implies the following :

MORE MULTIPLIERS...

- The area condition is actually not one condition, but several :

$$A_{[j]} = 2\sum_{j\in[j]} s_{j,m}\sqrt{j(j+1)} = const.$$

where $[j]$ is a class of spins for which $\sqrt{j(j+1)}$ has rational ratios between any two spins

- This means there are $[j]$ number of Lagrange's multipliers $\lambda_{[j]}$ satisfying

$$\sum_{[j]}\sum_{j\in[j]} e^{-2\lambda_{[j]}\sqrt{j(j+1)}}\sum_m e^{-\alpha m} = 1$$

$$\sum_m m e^{-\alpha m} = 0$$

DEPARTURE FROM LINEARITY

- Since for each $j \in [j]$

$$s_{j,m} = (\sum s_{j,m}) e^{-2\lambda_{[j]}\sqrt{j(j+1)}}$$

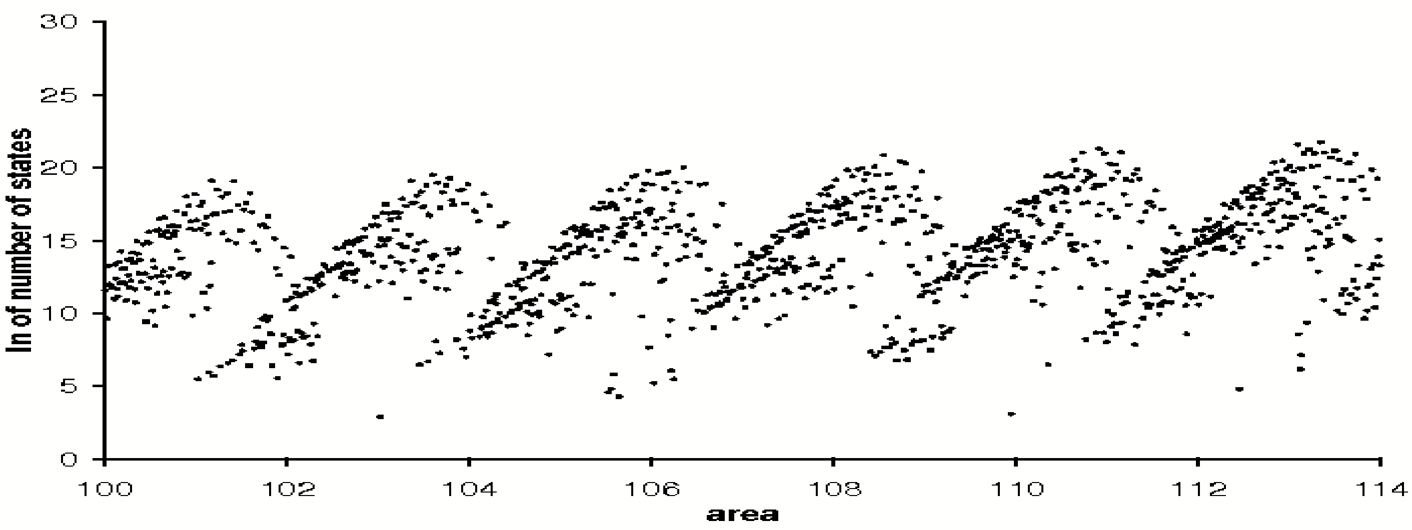
there are exactly $[j]$ equations to determine all the multipliers

$$\frac{A_{[j]}}{A} = \frac{\sum_{j\in[j]} \sqrt{j(j+1)}(2j+1)e^{-2\lambda_{[j]}\sqrt{j(j+1)}}}{\sum_{[j]}\sum_{j\in[j]} \sqrt{j(j+1)}(2j+1)e^{-2\lambda_{[j]}\sqrt{j(j+1)}}}$$

- The entropy : $S = \sum_{[j]} \frac{\lambda_{[j]} A_{[j]}}{4\pi\gamma\ell_P^2}$
- In general this entropy is non-linear in area

NUMERICAL COMPUTATIONS

- $A_{[j]}$ depends on the total area A and $\lambda_{[j]}$ are determined by $A_{[j]}, A$ - so $\lambda_{[j]} = \lambda_{[j]}[A]$
- Most values of A cannot be expressed as a sum of any spin, for such values there are no states! This is depicted in the graph :



HOW BIG ARE THESE OSCL?

- This is clearly counter-intuitive from the point of view of semi-classical Physics, but this is a true spectrum
  - semi-classically one should have a large enough width which should average out these oscillations
  - But how big these oscillations are? Is there a scale?
  - Since λ_S depend on the ratios of sub-areas, the entropy is non-universal
  - Usually $E \sim \sqrt{A}$ and so the temperature is also non-universal
  - Averaging is essential