Gravitational collapse in quantum gravity

Viqar Husain

Valencia - March 2009
(VH, arXiv:0808.0949)

Outline

1. Motivation and approach
2. Classical collapse: scalar field model
3. Quantization and qg corrected equations
4. Numerical simulation
5. Conclusions and outlook

Some basic questions

What is a quantum black hole?

How does it form?

What role is played by fundamental discreteness?

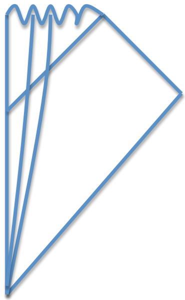
How does Hawking radiation show up in a suitable approximation?

Is there information loss?

What is black hole entropy in a dynamical setting?

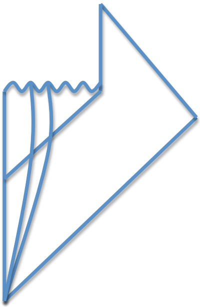
. . . . . .

Spherically symmetric black hole formation



-- a verified diagram: Vaidya, scalar field collapse, etc.

Black hole formation and evaporation



-- not a verified diagram

Some approaches

In classical theory: Metric $g_{\mu\nu}$ and matter fields ϕ.

*Non-perturbative: background independent

$$g,\ \phi \rightarrow (q,\pi)\ (\phi, P_\phi) \qquad H(q,\pi,\phi,P_\phi) \rightarrow \hat{H}$$

– attempt to follow evolution of a matter-geometry initial state

* Perturbative: fix background

$$g = g_0 + h, \qquad \phi = \phi_0 + \chi$$

$$h \rightarrow \hat{h}, \qquad \chi \rightarrow \hat{\chi}$$

– compute $\langle \hat{h}(x)\hat{h}(x')....\rangle$, $\langle \hat{h}(x)\hat{h}(x')\hat{\chi}(x'')....\rangle$

* AdS/CFT: so far no approach to bh formation – a first step is to study gravitational collapse with qg corrections in asymptotically AdS spacetimes.

* Other: g, ϕ are "emergent" collective degrees of freedom and shouldn't be quantized ... so a collective motion ansatz such as cooper pairs, Laughlin wavefunction, BE condensate needed ... for an unknown "fundamental" QG Hamlitonian.

Approach: motivated by LQG – apply polymer quantization to ADM theory.

The model

$$G_{ab} = 8\pi T_{ab}$$

$$T_{ab} = \partial_a\phi\partial_b\phi - \frac{1}{2}(\partial\phi)^2 g_{ab}$$

$$ds^2 = -f^2(r,t)dt^2 + g^2(r,t)dr^2 + r^2 d\Omega^2$$

or

$$ds^2 = -4\alpha(u,v)dudv + r^2(u,v)d\Omega^2$$

We use the latter form for simulations.

In the 2nd. parametrization, with $\alpha(u, v) := g(u, v) r'(u, v)$, where $'$ denotes the derivative with respect to v, the field equations may be written in the compact form

$$\dot{r} = -\frac{\bar{g}}{2} \tag{1}$$

$$\dot{h} = \frac{1}{2r^2}(h - \phi)(gr - 4\bar{g}) \tag{2}$$

where dot denotes partial derivative with respect to u, and we have defined

$$h = \phi + \frac{1}{4}\, r\phi', \tag{3}$$

$$g = \exp\left[8\pi \int_u^v \frac{1}{r}(h - \phi)^2 \, dv\right], \tag{4}$$

$$\bar{g} = \frac{1}{2}\int_u^v g \, dv \tag{5}$$

* $\phi = 0 \rightarrow$ flat space or Schwarzschild.

* $\phi(r, t)$ is the source of local degrees of freedom.

* complicated 2d field theory

* no known analytic collapse solutions that are asymptotically flat

* solvable collapse models (Oppenheimer-Snyder, Vaidya, CGHS, and variations) have only matter inflows.

scalar field model is much richer

PROBLEM

Find the quantum theory of this model, or at least some approximation that includes quantum gravity corrections to the equations of motion.

Classical results

* There are two classes of initial data $\phi(r, t = 0)$:

Weak data $\rightarrow$ no black hole formation in the long time limit.

Strong data $\rightarrow$ black holes form above threshold initial data parameters.

– Result of hard analysis (Christdoulou 1976)

* Details of transition weak $\rightarrow$ strong done by numerical simulation. (Choptuik 1993)

– with $\pm\Lambda$ (VH, M. Olivier, G. Kunstatter ... (2001))

Simulation procedure

* Specify $\phi(r, t=0) = ar^2 e^{-(r-r_0)^2/\sigma^2}$, $P_\phi(r, t=0) = 0$.

* Geometry data (q_{ab}, π^{ab}) determined by constraints.

* Evolve data and check for trapped surface formation at each time step: compute light expansions $\theta_\pm = D_a l^a_\pm$ on spheres S^2 embedded in time slice Σ_t.

$$\theta_\pm(\text{data on slice}) = \theta_\pm(r, t)$$

Normal: $\theta_+ > 0$, $\theta_- < 0$

Marginally trapped: $\theta_+ > 0$, $\theta_- < 0$

Trapped: $\theta_\pm < 0$

* Look for roots $\theta_+(r, t) = 0$ as simulation proceeds. Search for outermost root: this gives location of evolving horizon

$$r_H(t)$$

Results

$$M_{BH} = 2r_H(a, \sigma, r_0)$$

$a > a_* : M_{BH} \sim (a - a_*)^\gamma$

$a = a_*$: critical solution – naked singularity

$a < a*$: no horizon forms.

Results

$$M_{BH} = 2r_H(a, \sigma, r_0)$$

$a > a_* : M_{BH} \sim (a - a_*)^\gamma$

$a = a_*$: critical solution – naked singularity

$a < a*$: no horizon forms.

Classically black holes form without a mass gap

Results

$$M_{BH} = 2r_H(a, \sigma, r_0)$$

$a > a_* : M_{BH} \sim (a - a_*)^\gamma$

$a = a_*$: critical solution – naked singularity

$a < a*$: no horizon forms.

Classically black holes form without a mass gap

In QG we expect fundamental discreteness, and singularity avoidance:

How are these results modified by quantum effects?

Are there potential experimental signatures?

Quantization

Use an ADM variables: phase space variables (q_{ab}, π^{ab}) for geometry and (ϕ, P_ϕ) for matter.

$$S = \int d^3x dt \left(\pi^{ab} \dot{q}_{ab} + P_\phi \dot{\phi} - NH - N^a C_a \right)$$

* Realize constraints as self-adjoint operators.
  - H is Hamiltonian constraint $\rightarrow \hat{H}$
  - C_a diffeomorphism constraint $\rightarrow \hat{C}_a$

* Ideal: Compute $\langle \psi | \hat{H} | \psi \rangle$, $\langle \psi | \hat{C}_a | \psi \rangle$ for states $|\psi\rangle$ such that

$$H^{qg} \equiv \langle \psi | \hat{H} | \psi \rangle = H_{\text{classical}}(q, \pi, \phi, P_\phi) + \left(\frac{l_P}{L} \right)^k f(q, \pi, \phi, P_\phi) + \cdots$$

* State $|\psi\rangle$ is peaked on the phase space point q, π, ϕ, P_ϕ, and L is a scale in the state – its width.

Quantum corrected collapse:

Evolve initial data using H^{qg} and C_a^{qg}

Quantum corrected collapse:

Evolve initial data using H^{qg} and C_a^{qg}

Two types of corrections are present in H^{qg}, C_a^{qg}.

* No momentum operators – these must be written using translation operators $T_\lambda = e^{ip\lambda}$

$$p \to \hat{p}_\lambda = \frac{1}{i\lambda}(\hat{T}_\lambda - \hat{T}_\lambda^\dagger)$$

Quantum corrected collapse:

Evolve initial data using H^{qg} and C_a^{qg}

Two types of corrections are present in H^{qg}, C_a^{qg}.

* No momentum operators – these must be written using translation operators $T_\lambda = e^{ip\lambda}$

$$p \to \hat{p}_\lambda = \frac{1}{i\lambda}(\hat{T}_\lambda - \hat{T}_\lambda^\dagger)$$

* Inverse configuration operators written using Thiemann idea

$$\widehat{\left(\frac{1}{q}\right)}_\lambda = \left(\frac{1}{i\lambda}\left[\widehat{\sqrt{|q|}}, T_\lambda\right] T_\lambda^\dagger\right)^2$$

* In spherical symmetry the 3-metric is

$$ds^2 = \Lambda(r,t)dr^2 + R^2(r,t)d\Omega^2$$

so the geometry phase space variables are the pairs (R, P_R) and (Λ, P_Λ), and the matter variables are (ϕ, P_ϕ).

* Basic operators

$$\hat{R}(r_k,t)|a_1, a_2, \cdots a_n\rangle = a_k|a_1 \cdots a_N\rangle$$

$$e^{i\widehat{\lambda P_R(r_k,t)}}|a_1, a_2, \cdots a_n\rangle = |a_1, \cdots a_k + \lambda, \cdots a_n\rangle$$

Similar definitions of the other fields – LQG-like representation (VH, O. Winkler, gr-qc/0410125,CQG.22:L127)

The kinematical Hilbert space is the tensor product of geometry and matter Hilbert spaces with basis

$$|\underbrace{a_1, \ldots, a_N}_{gravity}; \underbrace{b_1, \ldots, b_N}_{matter}\rangle \tag{6}$$

Numerical simulation

* A code to evolve equations implemented with quantum corrected equations in double null coordinates.

* Only one type of qg correction – inverse triad: $1/R(r,t)$ factors in classical equations replaced by expectation values of the corresponding operator.

$$\left\langle \widehat{\frac{1}{R}} \right\rangle \to \frac{1}{R} \left(1 - e^{-(R/L)^2}\right) \tag{7}$$

This is a smoothened version of basis state result – to avoid numerical problem.

Numerical simulation

* A code to evolve equations implemented with quantum corrected equations in double null coordinates.

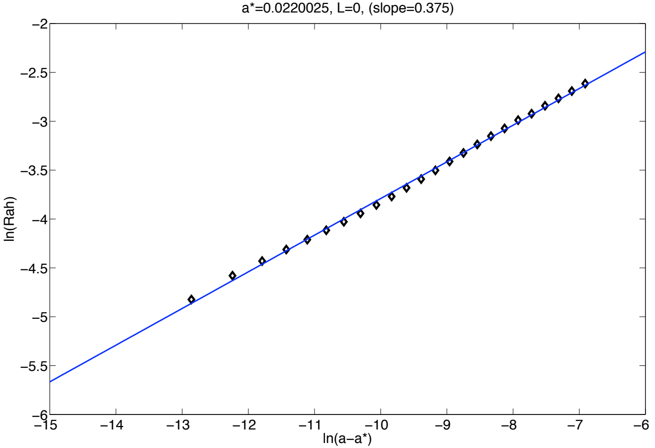
* Only one type of qg correction – inverse triad: $1/R(r,t)$ factors in classical equations replaced by expectation values of the corresponding operator.

$$\left\langle \widehat{\frac{1}{R}} \right\rangle \rightarrow \frac{1}{R} \left(1 - e^{-(R/L)^2}\right) \tag{7}$$

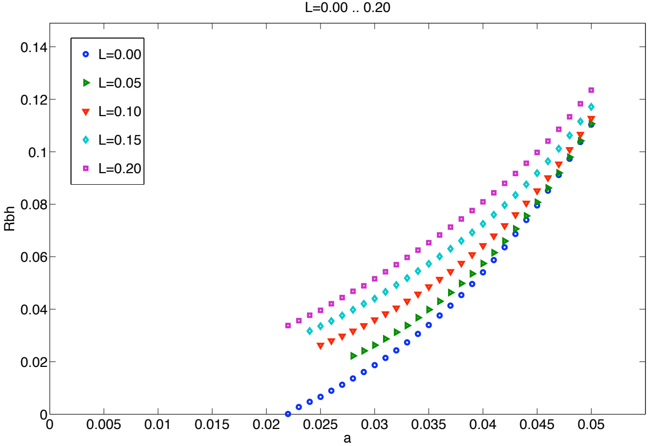
This is a smoothened version of basis state result – to avoid numerical problem.

* Horizon detection using same procedure: compute $\theta_{\pm}$ at each time step of simulation.

* Initial data is scalar field profile $\phi(r, t=0) = ar^2 e^{-(r-r_0)^2/\sigma^2}$



$L = 0$: this is the known classical result $M_{BH} = k(a - a*)^{0.37}$



* $L \neq 0$: mass gaps evident at threshold of bh formation

* points converge to classical case for large amplitude data

* mass gaps increase with increasing L

Mass formula

The graph can be summarized in the black hole mass formula

$$M_{BH} = m_0(L, a) + k\left[a - a^*(L)\right]^{\gamma(L,a)} \qquad (8)$$

in the supercritical region $a > a^*$, where m_0 is the mass gap and k and γ are numerically determined constants.

Summary

- A procedure for computing quantum gravity corrections to gravitational collapse.

- Mass gap at the onset of black hole formation – quantum gravity corrections to Choptuik result.

  (Mass gap known in the homogeneous case of Oppenheimer-Snyder model (Bojowald, Maartens, Singh), but no critical behaviour. This requires both inflow and outflow and interaction between flows.)

- Long to do list: put in momentum corrections, continue evolution beyond horizon formation (do horizons begin to shrink?), black hole entropy, Hawking radiation, $\cdots$.

Recent: J. Ziprick, G. Kunstatter (arXiv:0902.3224) repeated this calculation in flat slice coord. – verified mass gap; were able to see formation and evolution of trapping horizons.

What happens to the Choptuik's critical solution?
Conjecture from playing around with simulation: an unstable boson star for which "singularity avoidance repulsion" delicately balances attraction. Under study ...

Momentum corrections? (Like holonomy corrections in lqg) Will change details of mass formula and scaling graphs.