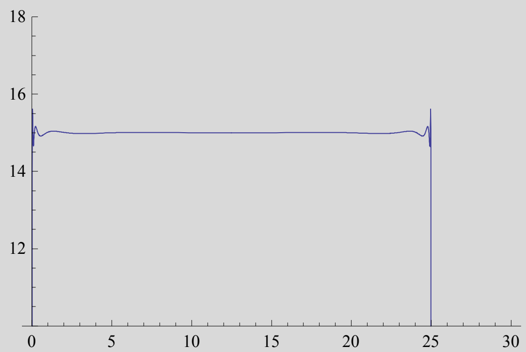
Exercise 4 (unit 2):

4. Consider the function $f(x)$ of Exercise 3 for $a = 1.5$ and $b = 0.1$. Plot the eigth iterate of f. Which is the attracting fixed point of f? What can you guess about its attraction basin? Draw two cobweb diagrams in the interval $[-30, 30]$ starting from $x_0 = 24$ and $x_0 = 26$ respectively. Argue graphically (by looking carefully at those cobweb diagrams) that, in the general case $a, b > 0$ in which there is an attractor, its attraction basin is exactly $]0, \frac{1+a}{b}[$.

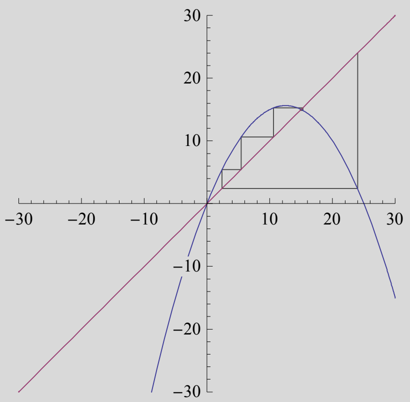
```
Remove["Global`*"]
```

```
a = 1.5; b = 0.1;
f = Function[x, (1 + a) x - b x^2];
```

```
Plot[Nest[f, x, 8], {x, 0, 30}, PlotRange → {10, 18}]
```



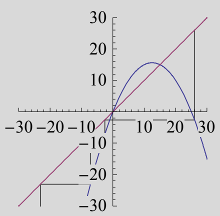
```
Clear[f, x];
a = 1.5;
b = 0.1;
f = Function[x, (1 + a) x - b x^2];
StartingValue = 24;
FirstIt = 0;
LastIt = 100;
xmin = -30;
xmax = 30;
i = 0;
y = N[StartingValue];
While[i < FirstIt, y = f[y]; i = i + 1];
DataTable = {{y, y}, {y, f[y]}};
While[i < LastIt,
  y = f[y];
  AppendTo[DataTable, {y, y}];
  AppendTo[DataTable, {y, f[y]}];
  i = i + 1];
AppendTo[DataTable, {f[y], f[y]}];
Cobweb = ListPlot[DataTable, Joined → True,
   PlotRange → {{xmin, xmax}, {xmin, xmax}},
   AspectRatio -> 1, PlotStyle → GrayLevel[.3],
   DisplayFunction → Identity];
Graf = Plot[{f[x], x}, {x, xmin, xmax},
   PlotRange → {xmin, xmax}, AspectRatio → 1,
   DisplayFunction → Identity];
Show[Cobweb, Graf]
```



```
Clear[f, x];
a = 1.5;
b = 0.1;
f = Function[x, (1 + a) x - b x^2];
StartingValue = 26;
FirstIt = 0;
LastIt = 100;
xmin = -30;
xmax = 30;
i = 0;
y = N[StartingValue];
While[i < FirstIt, y = f[y]; i = i + 1];
DataTable = {{y, y}, {y, f[y]}};
While[i < LastIt,
  y = f[y];
  AppendTo[DataTable, {y, y}];
  AppendTo[DataTable, {y, f[y]}];
  i = i + 1];
AppendTo[DataTable, {f[y], f[y]}];
Cobweb = ListPlot[DataTable, Joined → True,
   PlotRange → {{xmin, xmax}, {xmin, xmax}},
   AspectRatio -> 1, PlotStyle → GrayLevel[.3],
   DisplayFunction → Identity];
Graf = Plot[{f[x], x}, {x, xmin, xmax},
   PlotRange → {xmin, xmax}, AspectRatio → 1,
   DisplayFunction → Identity];
Show[Cobweb, Graf]
```

General::ovfl : Overflow occurred in computation. ≫

General::ovfl : Overflow occurred in computation. ≫




```
Clear[a, b];
Solve[f[x] == 0, x]
```

$\{\{x \to 0\}, \{x \to \frac{1 + a}{b}\}\}$

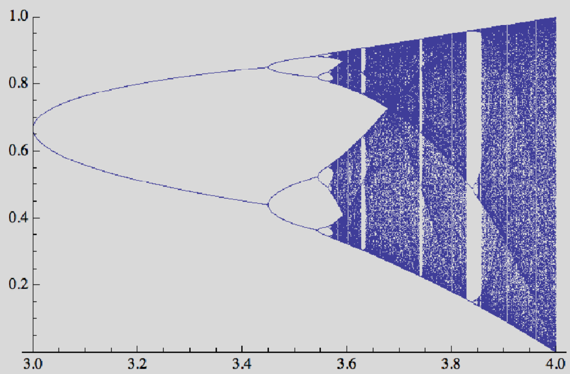
The function meets the x-axis at $\frac{1+a}{b}$. We see that if the initial value is less than this value (25 in this case), the orbit converges to the fixed point $\frac{a}{b}$ since in the first iteration we meet the positive side of the diagonal, but if the initial value is greater then in the first iteration we meet the negative side of the diagonal and the orbit diverges to -∞.

Exercise 7 (unit 2):

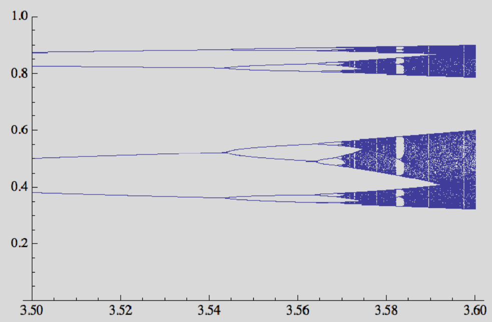
7. Consider the bifurcation diagram of the logistic map $f(x) = r\,x(1 - x)$. Localize a value for r corresponding to a 8-fold window by zooming the diagram appropriately as in the previous exercise. Draw the corresponding cobweb diagram for $x_0 = 0.5$.

```
Remove["Global`*"]
```

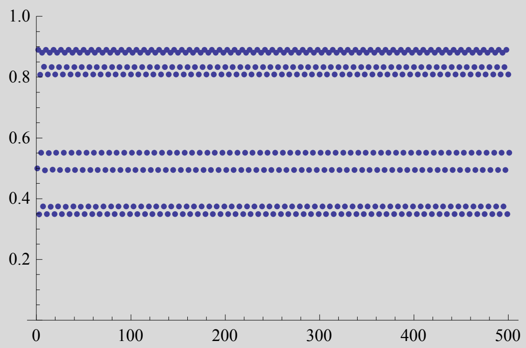
```
Clear[f, a, data, U];
firstIt = 500;     (*First value of t to be plotted*)
iterates = 500;
(*Number of iterations to be calculated after firstIt*)
p0 = 0.5;          (*Initial value*)
amin = 3;          (*Minimum value for the parameter*)
amax = 4;          (*Maximum vale for the parameter*)
asteps = 1000;
(*Number of points evaluated between amin and amax*)
f = Function[x, a x (1 - x)];
data = {};
Do[a = amin + (amax - amin)/asteps * d;
  U = Union[Drop[NestList[f, p0, firstIt + iterates], firstIt]];
  data = Join[data, Table[{a, U[[i]]}, {i, 1, Length[U]}]],
  {d, 0, asteps}];
ListPlot[data, PlotRange → {{amin, amax}, {0, 1}},
 PlotStyle -> {PointSize[.001]}]
```



```
Clear[f, a, data, U];
firstIt = 500;     (*First value of t to be plotted*)
iterates = 500;
(*Number of iterations to be calculated after firstIt*)
p0 = 0.5;          (*Initial value*)
amin = 3.5;          (*Minimum value for the parameter*)
amax = 3.6;         (*Maximum vale for the parameter*)
asteps = 1000;
(*Number of points evaluated between amin and amax*)
f = Function[x, a x (1 - x)];
data = {};
Do[a = amin + (amax - amin)/asteps * d;
  U = Union[Drop[NestList[f, p0, firstIt + iterates], firstIt]];
  data = Join[data, Table[{a, U[[i]]}, {i, 1, Length[U]}]],
  {d, 0, asteps}];
ListPlot[data, PlotRange → {{amin, amax}, {0, 1}},
 PlotStyle -> {PointSize[.001]}]
```




```
a = 3.56;
ListPlot[NestList[f, 0.5, 500], PlotRange -> {0, 1}]
```



```
Clear[f, x];
f = Function[x, 3.56 x (1 - x)];
StartingValue = 0.5;
FirstIt = 0;
LastIt = 200;
xmin = 0;
xmax = 1;
i = 0;
y = N[StartingValue];
While[i < FirstIt, y = f[y]; i = i + 1];
DataTable = {{y, y}, {y, f[y]}};
While[i < LastIt,
  y = f[y];
  AppendTo[DataTable, {y, y}];
  AppendTo[DataTable, {y, f[y]}];
  i = i + 1];
AppendTo[DataTable, {f[y], f[y]}];
Cobweb = ListPlot[DataTable, Joined → True,
   PlotRange → {{xmin, xmax}, {xmin, xmax}},
   AspectRatio -> 1, PlotStyle → GrayLevel[.3],
   DisplayFunction → Identity];
Graf = Plot[{f[x], x}, {x, xmin, xmax},
   PlotRange → {xmin, xmax}, AspectRatio → 1,
   DisplayFunction → Identity];
Show[Cobweb, Graf]
```