Landau-gauge gluon and ghost propagators from gauge-invariant Schwinger-Dyson equations

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Based on: A.C. Aguilar, D. Binosi, J. Papavassiliou, arXiv:0802.1870 [hep-ph]

Outline of the talk

- General considerations
- Gauge-invariant truncation
- System of Schwinger-Dyson equations
- Regularization of quadratic divergences
- Solutions
- Conclusions

Motivation

Study the infrared behaviour of the gluon and ghost propagators (in the Landau gauge) using Schwinger-Dyson equations .

Schwinger-Dyson equations:

- Infinite system of coupled non-linear integral equations for all Green's functions of the theory.
- Inherently non-perturbative
- Truncation scheme must be used

General Considerations

The gluon propagator $\Delta_{\mu\nu}(q)$ and the gluon self-energy $\Pi_{\mu\nu}(q)$ are related by

$$\Delta^{-1}_{\mu\nu}(q) = q^2\, g_{\mu\nu} + (\xi^{-1} - 1) q_\mu q_\nu - \Pi_{\mu\nu}(q)$$

with

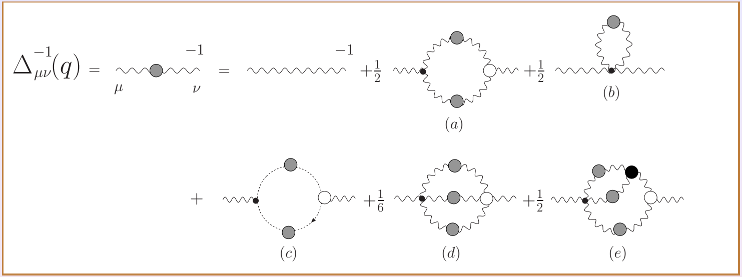
$$q^\mu \Pi_{\mu\nu}(q) = 0$$

The most fundamental statement at the level of Green's functions that one can obtain from the BRST symmetry .

It affirms the transversality of the gluon self-energy and is valid both perturbatively (to all orders) as well as non-perturbatively .

Any good truncation scheme ought to respect this property

Naive truncation violates it



$$q^{\mu}\Pi_{\mu\nu}(q)|_{(a)+(b)} \neq 0$$

$$q^{\mu}\Pi_{\mu\nu}(q)|_{(a)+(b)+(c)} \neq 0$$

Main reason : Full vertices satisfy complicated Slavnov-Taylor identities.

Pinch Technique

The pinch technique defines a good truncation scheme.

$\downarrow$

Diagrammatic rearrangement of perturbative expansion (to all orders) gives rise to effective Green's functions with special properties.

J. M. Cornwall , Phys. Rev. D **26**, 1453 (1982)
J. M. Cornwall and J. Papavassiliou , Phys. Rev. D **40**, 3474 (1989)
D. Binosi and J. Papavassiliou , Phys. Rev. D **66**, 111901 (2002).

Pinch Technique

- Simple, QED-like Ward Identities , instead of Slavnov-Taylor Identities, to all orders

$$q_1^{\mu}\widetilde{\mathbb{\Gamma}}^{abc}_{\mu\alpha\beta}(q_1,q_2,q_3) = gf^{abc}\left[\Delta^{-1}_{\alpha\beta}(q_2) - \Delta^{-1}_{\alpha\beta}(q_3)\right]$$

$$q_1^{\mu}\widetilde{\mathbb{\Gamma}}^{acb}_{\mu}(q_2,q_1,q_3) = gf^{abc}\left[D^{-1}(q_2) - D^{-1}(q_3)\right]$$

- Profound connection with

  Background Field Method $\Longrightarrow$ easy to calculate

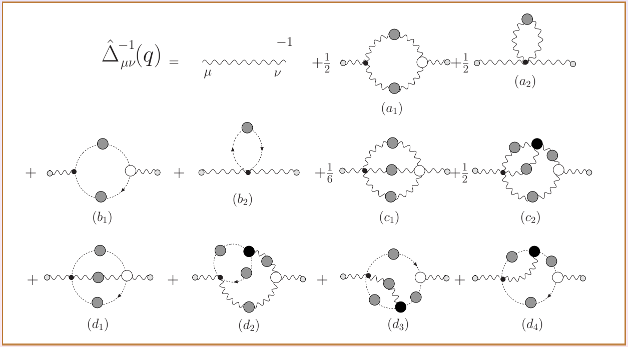
  D. Binosi and J. Papavassiliou , arXiv:0712.2707 [hep-ph] [to appear in PRD (RC)]

- Can move consistently from one gauge to another (from Landau to Feynman, etc)

  A. Pilaftsis , Nucl. Phys. B **487**, 467 (1997)

New series

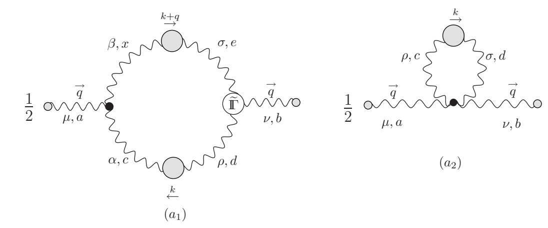
The new Schwinger-Dyson series based on the pinch technique



Transversality is enforced separately for gluon- and ghost-loops, and order-by-order in the "dressed-loop" expansion!

A. C. Aguilar and J. Papavassiliou , JHEP **0612**, 012 (2006)

Transversality

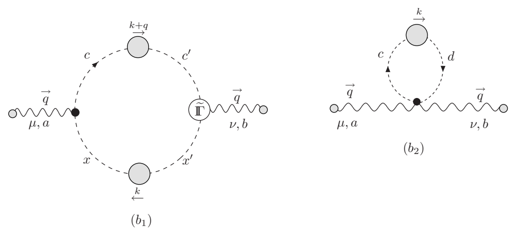


The gluonic contribution

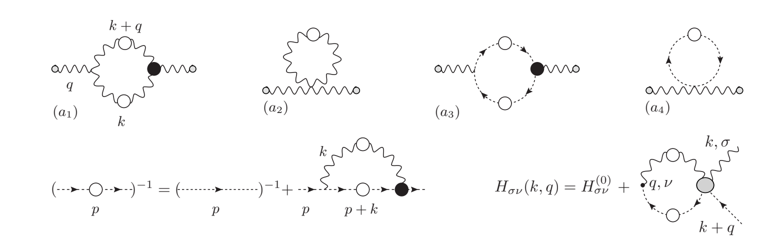
$$q^{\mu}\Pi_{\mu\nu}(q)|_{(a_1)+(a_2)} = 0$$

The ghost contribution

$$q^{\mu}\Pi_{\mu\nu}(q)|_{(b_1)+(b_2)} = 0$$



The system of SD equations



Gauge-technique Ansatz for the full vertex:

$$\widetilde{\mathbb{\Gamma}}_{\mu\alpha\beta} = \Gamma_{\mu\alpha\beta} + i\,\frac{q_\mu}{q^2}\left[\Pi_{\alpha\beta}(k+q) - \Pi_{\alpha\beta}(k)\right],$$

- Satisfies the correct Ward identity
- Contains longitudinally coupled massless poles $\sim 1/q^2$.
  Instrumental for obtaining an IR finite solution

R. Jackiw and K. Johnson , Phys. Rev. D **8**, 2386 (1973)
J. M. Cornwall and R. E. Norton , Phys. Rev. D **8** (1973) 3338
E. Eichten and F. Feinberg , Phys. Rev. D **10**, 3254 (1974)

IR-finiteness

Setting $\Delta^{-1}(q^2) = q^2 + i\Pi(q^2)$, IR-finiteness means that

$$\Delta^{-1}(0) \neq 0$$

The system of SD equations has the form

$$\begin{aligned}
\Delta^{-1}(q^2) &= q^2 + c_1 \int_k \Delta(k)\Delta(k+q) f_1(q,k) + c_2 \int_k \Delta(k) f_2(q,k) \\
D^{-1}(p^2) &= p^2 + c_3 \int_k \left[p^2 - \frac{(p\cdot k)^2}{k^2} \right] \Delta(k)\, D(p+k)\,,
\end{aligned}$$

Regularization

The crux of the matter is the limit as $q^2 \to 0$:

$$\Delta^{-1}(0) \sim \frac{15}{4}\int_k \Delta(k) - \frac{3}{2}\int_k k^2\Delta^2(k),$$

The integrals on the rhs are quadratically divergent

- Perturbatively the rhs vanishes because

$$\int_k \frac{\ln^n k^2}{k^2} = 0\,,\ \ n = 0, 1, 2, \dots$$

- Ensures the masslessness of the gluon to all orders in perturbation theory.
- Non-perturbatively $\Delta^{-1}(0)$ does not have to vanish, provided that the quadratically divergent integrals defining it can be properly regulated and made finite, without introducing counterterms of the form $m_0^2(\Lambda^2_{UV})A^2_\mu$, forbidden by the local gauge invariance.

Regularization

- This is indeed possible: the divergent integrals can be regulated by subtracting an appropriate combination of dimensional regularization "zeros"
For large enough k^2:

$$\Delta(k^2) \rightarrow \Delta_{\text{pert}}(k^2)$$

$$\Delta_{\text{pert}}(k^2) = \sum_{n=0}^{N} a_n \frac{\ln^n k^2}{k^2},$$

a_n known from perturbative expansion:
$a_0 \approx 1.7,\ a_1 \approx -0.1,\ a_3 \approx 2.5 \times 10^{-3}$.

Regularization

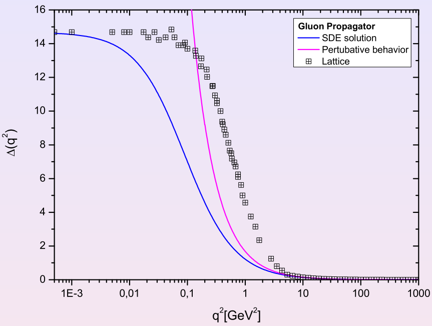
Then, subtracting from both sides

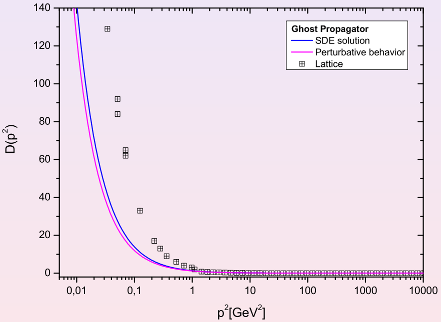
$$0 = \int_k \Delta_{\rm pert}(k^2)$$

$$\Delta^{-1}_{\rm reg}(0) \sim \frac{15}{4}\int_0^s dy\;\; y\,[\Delta(y) - \Delta_{\rm pert}(y)] - \frac{3}{2}\int_0^s dy\;\; y^2\left[\Delta^2(y) - \Delta^2_{\rm pert}(y)\right]\,.$$

s: the point where the perturbative expansion ceases to be valid.

Solution





P. O. Bowman et al. ,arXiv:hep-lat/0703022
A. Cucchieri and T. Mendes , arXiv:0710.0412 [hep-lat].
I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker and A. Sternbeck , arXiv:0710.1968 [hep-lat].

Conclusions

- Gauge-invariant treatment of SD equations. The transversality of the gluon self-energy is preserved.
- The gluon propagator is (and always has been) finite in the IR. In qualitative agreement with the early description by Cornwall (generation of a dynamical gluon mass)

J.M.Cornwall, Nucl. Phys. B **157**, 392 (1979); Phys. Rev. D **26**, 1453 (1982)
G.Parisi and R.Petronzio, Phys. Lett. B **94**, 51 (1980).
C.W.Bernard, Phys. Lett. B **108**, 431 (1982); Nucl. Phys. B **219**, 341 (1983).
J.F.Donoghue, Phys. Rev. D **29**, 2559 (1984).
M.Lavelle, Phys. Rev. D **44**, 26 (1991).
F.Halzen, G.I.Krein and A.A.Natale, Phys. Rev. D **47**, 295 (1993).
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C.Alexandrou, P.de Forcrand and E.Follana, Phys. Rev. D **63**, 094504 (2001); Phys. Rev. D **65**, 117502 (2002); Phys. Rev. D **65**, 114508 (2002).
A.C.Aguilar, A.A.Natale and P.S.Rodrigues da Silva, Phys. Rev. Lett. **90**, 152001 (2003).
A. C. Aguilar and J. Papavassiliou, JHEP **0612**, 012 (2006); Eur.Phys.J.A35:189-205 (2008).

and many more ...

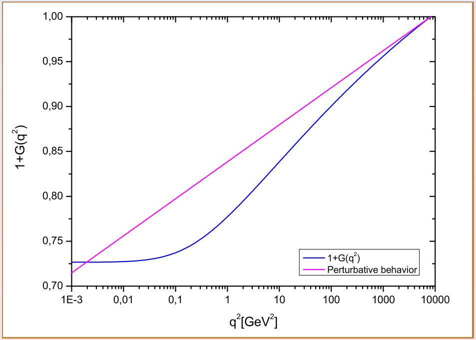
Conclusions

- In the Landau gauge the ghosts don't do much.
  (ghost submission)

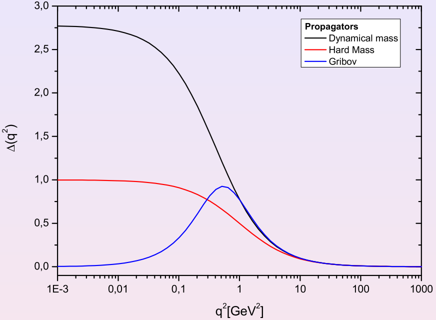
- Gauge-dependent quantities (like ghost propagators) have the right (and the obligation!) to behave gauge-dependently
  Challenge and bet : The ghost propagator in the
  Feynman gauge is IR-finite !

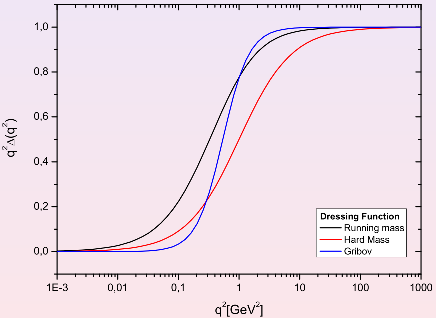
A.C.Aguilar and J.Papavassiliou , arXiv:0712.0780 [hep-ph]

G function


1,00
0,95
0,90
0,85
0,80
0,75
0,70
1+G(q²)
1E-3
0,01
0,1
1
10
100
1000
10000
q²[GeV²]
1+G(q²)
Perturbative behavior

Propagator versus Dressing function





$$D(k) = \frac{G(k^2)}{k^2}\,, \qquad \textbf{and} \qquad \Delta_{\mu\nu}(k) = \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right]\frac{Z(k^2)}{k^2}\,.$$

where $G(k^2)$ and $Z(k^2)$ are the ghost and the gluon dressing functions respectively.
in the deep IR, $G(k^2)$ and $Z(k^2)$ satisfy

$$Z(k^2) \to (k^2)^{2\kappa} \qquad G(k^2) \to (k^2)^{-\kappa}\,.$$

(same κ!). With the approximations they employ, their SD equations yields for κ the value $\kappa = 0.59$;
define the QCD coupling as

$$\alpha(k^2) = \alpha(\mu^2)G^2(k^2)Z(k^2)\,.$$

Clearly, we can see that Eq.(1) will lead to a IR fixed point if and only if the ghost and gluon are parametrized by the same κ.