Infrared Finite
Effective Charge of QCD

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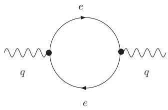
Outline of the talk

- Effective charge of QED (prototype)
- QCD effective charge in perturbation theory
  Field theoretic framework: Pinch Technique
- Beyond perturbation theory: Schwinger-Dyson equations and lattice
- Dynamical mass generation
- IR finite gluon propagator and effective charge
- The role of the quarks
- Conclusions

Effective charge of QED (prototype)

Textbook construction: $\bar{\alpha}(q^2)$ is defined from the vacuum polarization $\Pi(q)$.

$$\Pi_{\mu\nu}(q) =$$



$$\Delta_{\mu\nu}(q) = -i\mathrm{P}_{\mu\nu}(q)\Delta(q^2)$$

$$\mathrm{P}_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

$$\Delta(q^2) = \frac{1}{q^2[1+\Pi(q^2)]}$$

$e = Z_e^{-1} e_0$ and $1 + \Pi(q^2) = Z_A[1 + \Pi_0(q^2)]$

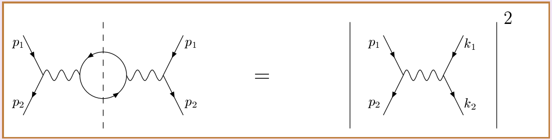
From QED Ward identity follows $Z_1 = Z_2$ and $Z_e = Z_A^{-1/2}$

RG-invariant combination

$$e_0^2 \Delta_0(q^2) = e^2 \Delta(q^2) \Longrightarrow \overline{\alpha}(q^2) = \frac{\alpha}{1+\Pi(q^2)}$$

Properties

- Gauge-independent (to all orders)
- Renormalization group invariant,
- Universal (process-independent)
- Non-trivial dependence on the masses m_i of the particles in the loop. Reconstruction from physical amplitudes, using optical theorem and dispersion relations.



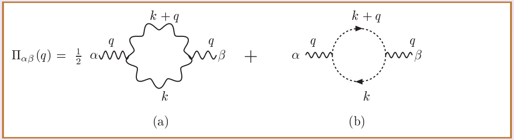
- For $q^2 \gg m_i^2$, the effectice charge coincides with the running coupling (solution of RG equation).

$$\overline{\alpha}(q^2) \rightarrow \frac{e^2/4\pi}{1 - e^2 b \log(q^2/m_f^2)}$$

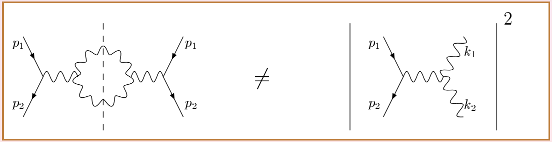
where $b = \frac{1}{6\pi^2} n_f$ [n_f = number of fermion flavors].

QCD effective charge in perturbation theory

- Ward identities replaced by Slavnov-Taylor identities involving ghost Green's functions. ($Z_1 \neq Z_2$ in general)
- $\Pi_{\mu\nu}(q)$ depends on the gauge-fixing parameter already at one-loop



- Optical theorem does not hold for individual Green's functions



Diagrammatic rearrangement of perturbative expansion (to all orders) gives rise to effective Green's functions with special properties .

J. M. Cornwall , Phys. Rev. D **26**, 1453 (1982)
J. M. Cornwall and J.P. , Phys. Rev. D **40**, 3474 (1989)
D. Binosi and J.P. , Phys. Rev. D **66**, 111901 (2002).

In covariant gauges: $\longrightarrow$

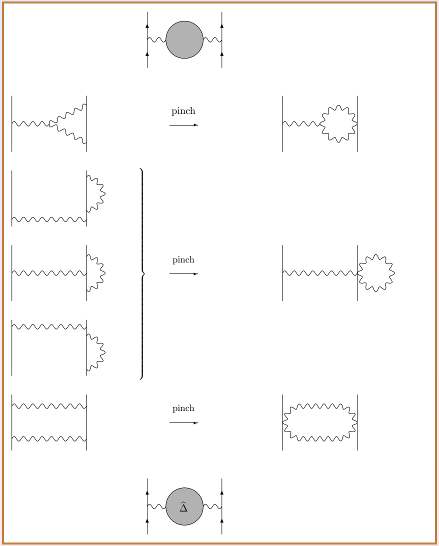
$$i\Delta^{(0)}_{\mu\nu}(k) = \left[g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2} \right] \frac{1}{k^2}$$

In light cone gauges: $\longrightarrow$

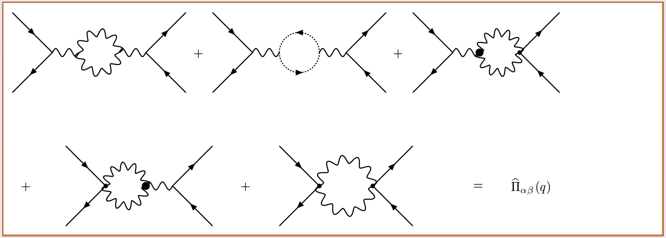
$$i\Delta^{(0)}_{\mu\nu}(k) = \left[g_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{nk} \right] \frac{1}{k^2}$$

$$\begin{aligned} k_\nu \gamma^\nu &= (\not k + \not p - m) - (\not p - m) \\ &= S_0^{-1}(k+p) - S_0^{-1}(p), \end{aligned}$$

Pinch Technique rearrangement



Gauge-independent self-energy



$$\widehat{\Delta}(q^2) = \frac{1}{q^2\left[1 + bg^2 \ln\left(\frac{q^2}{\mu^2}\right)\right]}$$

$b = 11C_A/48\pi^2$ first coefficient of the QCD β-function ($\beta = -bg^3$) in the absence of quark loops.

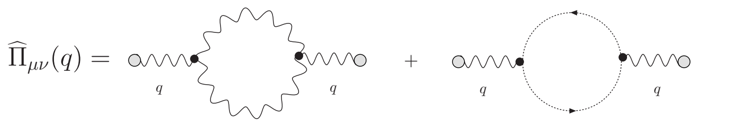
- Simple, QED-like Ward Identities , instead of Slavnov-Taylor Identities, to all orders

$$
\begin{aligned}
q^{\mu}\widetilde{\Gamma}_{\mu}(p_1,p_2) &= g\left[S^{-1}(p_2)-S^{-1}(p_1)\right] \\
q_1^{\mu}\widetilde{\Gamma}^{abc}_{\mu\alpha\beta}(q_1,q_2,q_3) &= gf^{abc}\left[\Delta^{-1}_{\alpha\beta}(q_2)-\Delta^{-1}_{\alpha\beta}(q_3)\right]
\end{aligned}
$$

- Profound connection with

  Background Field Method $\Longrightarrow$ easy to calculate

D. Binosi and J.P. , Phys. Rev. D **77**, 061702 (2008); arXiv:0805.3994 [hep-ph]



Restoration of:

- Abelian Ward identities $\widehat{Z}_1 = \widehat{Z}_2,\ Z_g = \widehat{Z}_A^{-1/2}$

$\Longrightarrow$ RG invariant combination $g_0^2 \widehat{\Delta}_0(q^2) = g^2 \widehat{\Delta}(q^2)$

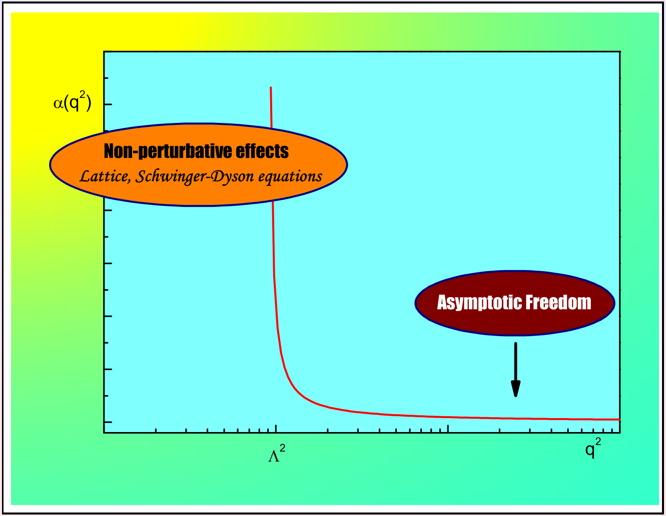
For large momenta q^2, define the RG-invariant effective charge of QCD,

$$\overline{\alpha}(q^2) = \frac{g^2(\mu)/4\pi}{1 + bg^2(\mu)\ln(q^2/\mu^2)} = \frac{1}{4\pi b \ln(q^2/\Lambda^2)}$$

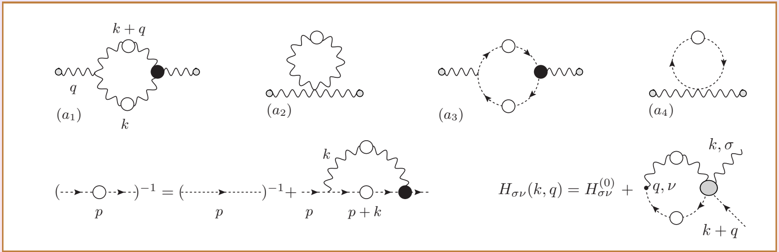
- Strong version of optical theorem

J.P., E. de Rafael and N.J.Watson, Nucl. Phys. B **503**, 79 (1997)

Beyond perturbation theory ...

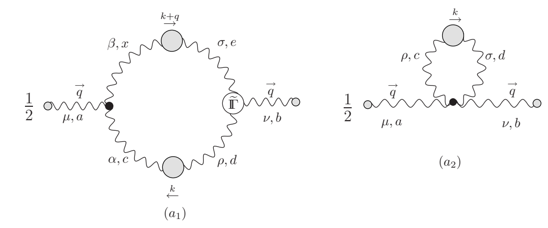


- Lattice QCD (discretization of space-time)
- Schwinger-Dyson equations (continuous approach)



A.C. Aguilar, D. Binosi, J. P. , arXiv:0802.1870 [hep-ph], Phys. Rev. D (in press)

Transversality enforced loop-wise in SD equations

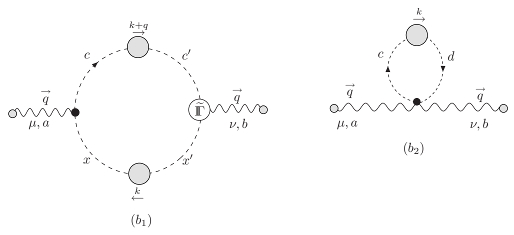


The gluonic contribution

$$q^{\mu}\Pi_{\mu\nu}(q)|_{(a_1)+(a_2)} = 0$$

The ghost contribution

$$q^{\mu}\Pi_{\mu\nu}(q)|_{(b_1)+(b_2)} = 0$$



$$\Delta(q^2) = \frac{1}{q^2[1+\Pi(q^2)]}$$

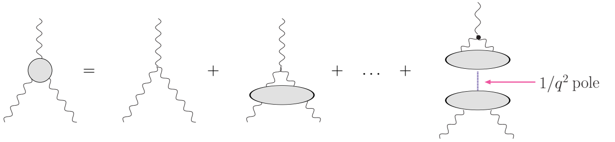
- If $\Pi(q^2)$ has a pole at $q^2 = 0$ the vector meson is massive , even though it is massless in the absence of interactions.

  J. S. Schwinger, Phys. Rev. **125**, 397 (1962); Phys. Rev. **128**, 2425 (1962).

- Requires massless, longitudinally coupled , Goldstone-like poles $\sim 1/q^2$
- Such poles can occur dynamically , even in the absence of canonical scalar fields. Composite excitations in a strongly-coupled gauge theory.

  R. Jackiw and K. Johnson, Phys. Rev. D **8**, 2386 (1973)
  J. M. Cornwall and R. E. Norton, Phys. Rev. D **8** (1973) 3338
  E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254 (1974)

Ansatz for the vertex



Gauge-technique Ansatz for the full vertex:

$$\widetilde{\Gamma}_{\mu\alpha\beta} = \Gamma_{\mu\alpha\beta} + i\,\frac{q_\mu}{q^2}\left[\Pi_{\alpha\beta}(k+q) - \Pi_{\alpha\beta}(k)\right],$$

- Satisfies the correct Ward identity

$$q_1^\mu \widetilde{\Gamma}^{abc}_{\mu\alpha\beta}(q_1, q_2, q_3) = g f^{abc}\left[\Delta^{-1}_{\alpha\beta}(q_2) - \Delta^{-1}_{\alpha\beta}(q_3)\right]$$

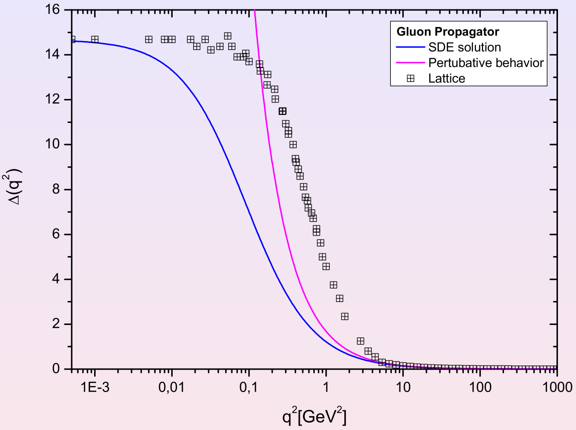
- Contains longitudinally coupled massless bound-state poles $\sim 1/q^2$, instrumental for $\Delta^{-1}(0) \neq 0$

System of coupled SD equations

$$\Delta^{-1}(q^2) = q^2 + c_1 \int_k \Delta(k)\Delta(k+q) f_1(q,k) + c_2 \int_k \Delta(k) f_2(q,k)$$
$$D^{-1}(p^2) = p^2 + c_3 \int_k \left[p^2 - \frac{(p\cdot k)^2}{k^2}\right] \Delta(k)\, D(p+k)\,,$$

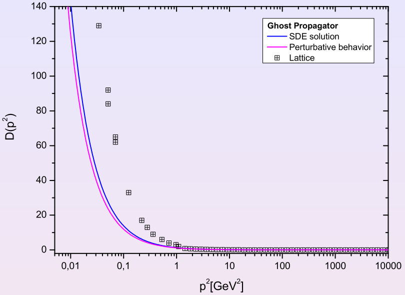
- Renormalize
- Solve numerically

Gluon propagator (Landau gauge)

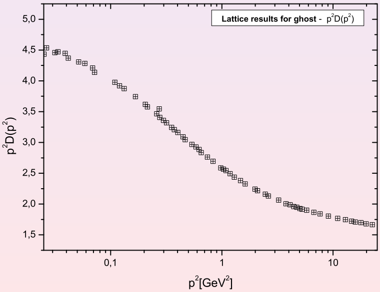


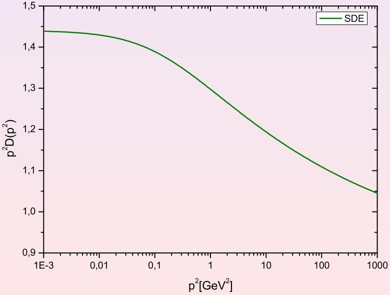
I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker and A. Sternbeck, PoS LATTICE, 290 (2007).
P. O. Bowman et al., Phys. Rev. D **76**, 094505 (2007)
A. Cucchieri and T. Mendes, PoS LATTICE, 297 (2007).

Ghost propagator



No power-law enhancement





The physical picture

- Dynamical generation of an infrared cutoff.

  J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982);
  A.C.Aguilar, A.A.Natale and P.S.R. da Silva, Phys. Rev. Lett. **90**, 152001 (2003);
  A. C.Aguilar and J.P., JHEP **0612**, 012 (2006);.
  A.C.Aguilar, D. Binosi, J.P., arXiv:0802.1870 [hep-ph], Phys. Rev. D (in press).

- Acts as an effective "mass" for the gluons.
  - Not hard but momentum dependent mass $m = m(q^2)$
  - Drops off "sufficiently fast" in the UV.

    A. C. Aguilar and J.P., Eur.Phys.J.A35:189-205 (2008).
  - Does not induce to a term $m^2 A_\mu^2$ in $\mathcal{L}_{QCD}$.
  - The local gauge symmetry remains exact.

The physical picture ...

- The "mass" is not directly measurable. Must be related to glueball masses, string tension, and condensates .
  J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982)
  M.Lavelle, Phys. Rev. D **44**, 26 (1991).
- Potential energy of a pair of heavy, static sources in the adjoint (adjoint Wilson Loop).
  Flux tube formed $\Longrightarrow$ Finite threshold for popping dynamical gluons out of the vacuum.
  C. Bernard , Nucl. Phys. B **219**:341,1983
- Bag Model : Gluon production requires a net energy cost because of confinement. Acts like a constituent quark mass
  John F. Donoghue , Phys.Rev.D29:2559,1984
- Phenomenological studies $\Longrightarrow m(0) = 500 \pm 200\,\mathrm{MeV}$
  F.Halzen, G.I.Krein and A.A.Natale, Phys. Rev. D **47**, 295 (1993).
  G.Parisi and R.Petronzio, Phys. Lett. B **94**, 51 (1980).
  A.C.Aguilar, A.Mihara and A.A.Natale, Phys. Rev. D **65**, 054011 (2002).

Effective charge

The RG invariant quantity, $\widehat{d}(q^2) = g^2\Delta(q^2)$, has the general form:

$$\widehat{d}(q^2) = \frac{4\pi\overline{\alpha}(q^2)}{q^2 + m^2(q^2)},$$

where the running charge is

$$\overline{\alpha}(q^2) = \frac{1}{4\pi b \ln\left(\frac{q^2+\rho m^2(q^2)}{\Lambda^2}\right)}$$

- It displays asymptotic freedom in the UV.
- Freezes at a finite value in the low energy regime

$\overline{\alpha}^{-1}(0) = 4\pi b \ln\left(\frac{\rho m^2(0)}{\Lambda^2}\right)$ $\Longrightarrow$ **Infrared Fixed Point for QCD**.

Running of the gluon mass

$$m_1^2(x) = \lambda_1^2 (\ln x)^{-1+\gamma_1} \Longrightarrow \langle A_\mu^2 \rangle$$

$$m_2^2(x) = \frac{\lambda_2^4}{x} (\ln x)^{\gamma_2 - 1} \Longrightarrow \langle G_{\mu\nu}^2 \rangle$$

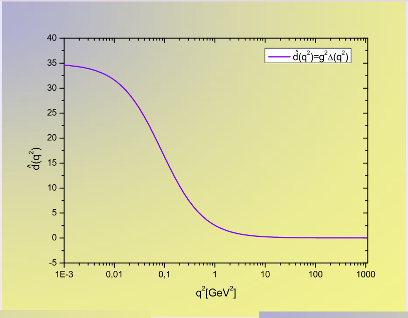
- Consistency with the OPE
- $\langle A_\mu^2 \rangle$: condensate of $d = 2$ (not gauge-invariant but becomes gauge-invariant when minimized over all local gauge transformations.

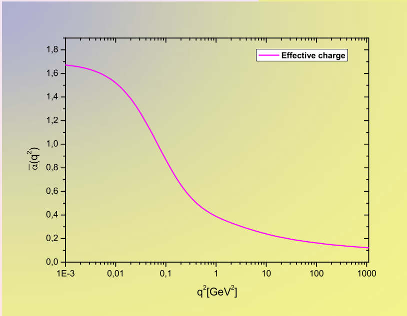
F.V.Gubarev, L.Stodolsky and V.I.Zakharov, Phys. Rev. Lett. **86**, 2220 (2001)
P. Boucaud *et al.*, Phys. Rev. D **66**, 034504 (2002)

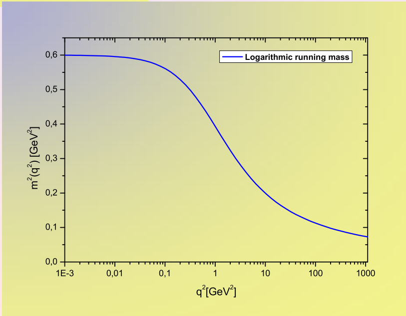
- $\langle G_{\mu\nu}^2 \rangle$: gluon condensate, $d = 4$ (gauge-invariant); standard term, related to the vaccum energy of QCD $E_{vac} = \frac{\beta}{8g} \langle G_{\mu\nu}^2 \rangle$

R. J. Crewther , Phys. Rev. Lett. **28** (1972) 1421;
M. S. Chanowitz and J. R. Ellis, Phys. Lett. B **40**, 397 (1972).

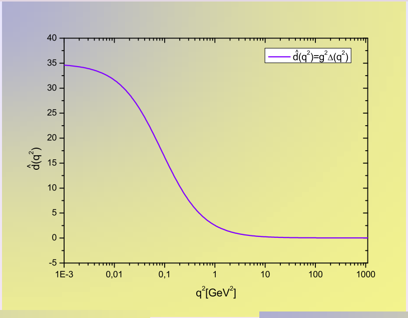
Log case

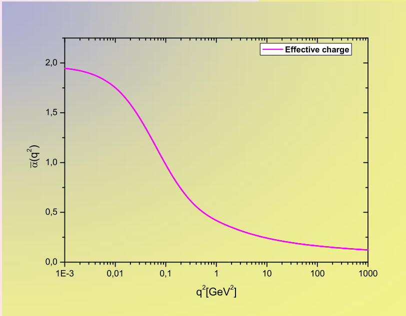

$\hat{d}(q^2)=g^2\Delta(q^2)$
$\hat{d}(q^2)$
40
35
30
25
20
15
10
5
0
-5
1E-3
0,01
0,1
1
10
100
1000
q^2[GeV^2]

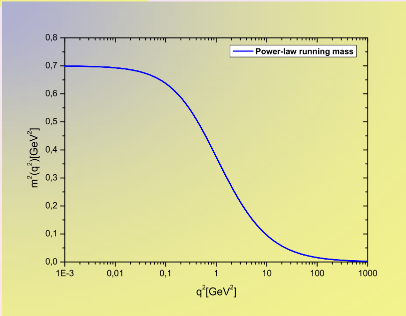

Effective charge
$\overline{\alpha}(q^2)$
1,8
1,6
1,4
1,2
1,0
0,8
0,6
0,4
0,2
0,0
1E-3
0,01
0,1
1
10
100
1000
q^2[GeV^2]


Logarithmic running mass
$m^2(q^2)$ [GeV^2]
0,6
0,5
0,4
0,3
0,2
0,1
0,0
1E-3
0,01
0,1
1
10
100
1000
q^2[GeV^2]

Power-law case

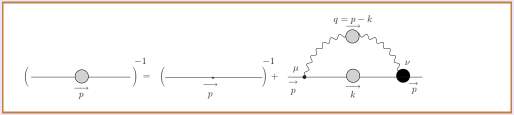


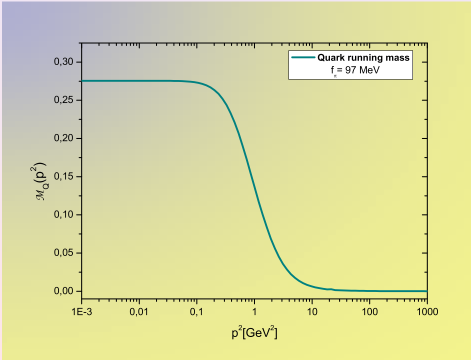




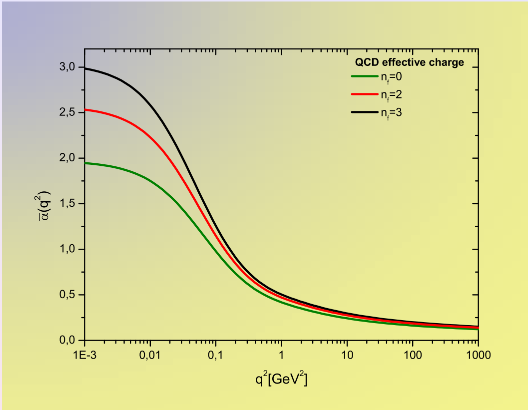
Putting quarks into the game

- Quarks introduce only quantitative changes, once the chiral symmetry has been dynamically broken and the constituent quark mass has been generated.





Effective charge with quarks included



$$4\pi\overline{\alpha}(q^2) = \frac{1}{b\ln\left(\frac{q^2+\rho m^2(q^2)}{\Lambda^2}\right) - b_f\ln\left(\frac{q^2+\mathcal{M}_Q^2(q^2)}{\Lambda^2}\right)}$$

$$b_f = 2n_f/48\pi^2$$

Conclusions

- Self-consistent description of the non-perturbative QCD dynamics in terms of an IR finite gluon propagator appears to be within our reach.
- Gauge invariant truncation of SD equations furnishes reliable non-perturbative information and strengthens the synergy with the lattice community.
- In the deep IR the QCD effective charge saturates at a finite value.
- Quarks do not interfere with the IR finiteness of the gluon propagator or of the effective charge.