Scrutinizing the Green's functions of QCD: Lattice meets Schwinger-Dyson

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Outline of the talk

- Schwinger-Dyson equations in non-Abelian gauge theories: difficulties with the conventional formulation
- Gauge-invariant truncation scheme: Pinch Technique
- Lattice results for gluon propagator
- Dynamical mass generation (Schwinger mechanism)
- Comparing SD results with lattice simulations
- IR finite effective charge
- Kugo-Ojima revisited
- Conclusions

The (quarkless) QCD Lagrangian

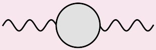
$$\mathcal{L} = -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \frac{1}{2\xi}(\partial^\mu A_\mu^a)^2 + \partial^\mu \bar{c}^a \partial_\mu c^a + g f^{abc} (\partial^\mu \bar{c}^a) A_\mu^b c^c$$

where gluonic field strength tensor

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

$$\Downarrow$$

$$< 0 | T[A^a(x)_\mu A^b(y)_\nu] | 0 >= \Delta_{\mu\nu}^{ab}(x-y)$$



$$< 0 | T[\bar{c}^a(x) c^b(y)] | 0 >= D^{ab}(x-y)$$

Off-shell Green's functions

- Gauge dependent
- Renormalization point (μ) and scheme dependent

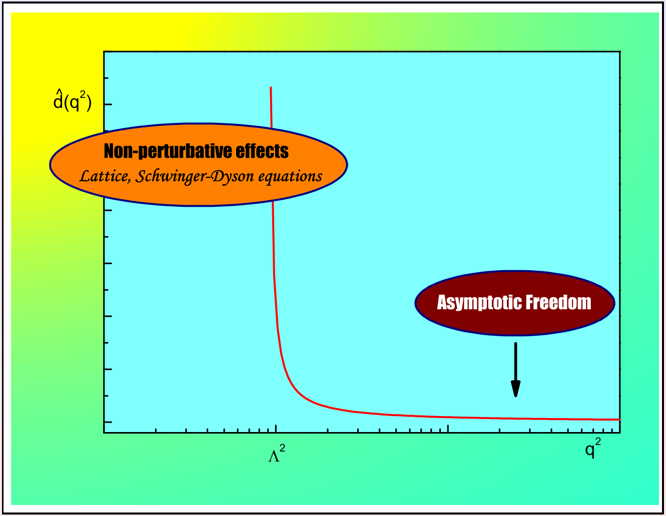
  ⇒ **Not really physical**

However...

- They capture characteristic ingredients of the underlying dynamics (perturbative and non-perturbative)
- When appropriately combined give rise to physical observables

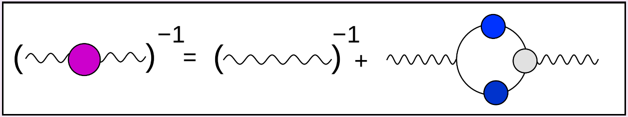
  ⇒ crucial pieces for completing the QCD puzzle .

Beyond perturbation theory ...



Schwinger-Dyson equations

- Equations of motion for off-shell Green's functions.
- Derived formally from the generating functional.



- Infinite system of coupled non-linear integral equations .
- Inherently non-perturbative .
- Self-consistent truncation scheme must be used.

Difficulties with conventional SD series

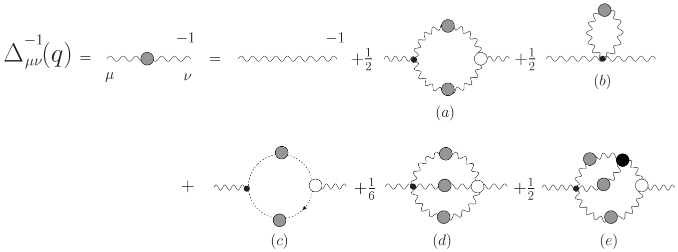
$$q^{\mu}\Pi_{\mu\nu}(q) = 0$$

The most fundamental statement at the level of Green's functions that one can obtain from the BRST symmetry .

It affirms the transversality of the gluon self-energy and is valid both perturbatively (to all orders) as well as non-perturbatively .

Any good truncation scheme ought to respect this property.

Naive truncation violates it.



$$q^{\mu}\Pi_{\mu\nu}(q)|_{(a)+(b)} \neq 0$$

$$q^{\mu}\Pi_{\mu\nu}(q)|_{(a)+(b)+(c)} \neq 0$$

Main reason : Full vertices satisfy complicated Slavnov-Taylor identities.

Pinch Technique

The pinch technique defines a good truncation scheme.

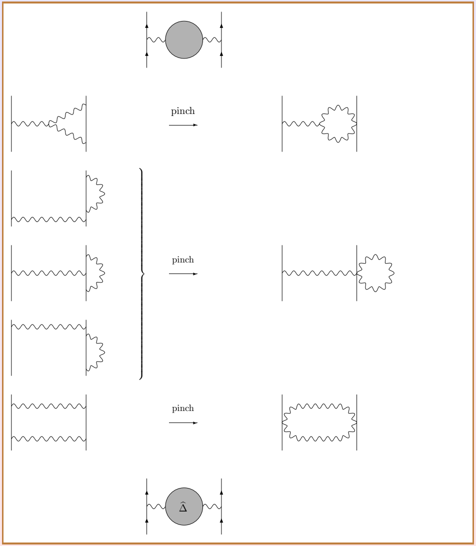
Diagrammatic rearrangement of perturbative expansion (to all orders) gives rise to effective Green's functions with special properties .

J. M. Cornwall , Phys. Rev. D **26**, 1453 (1982)
J. M. Cornwall and J.P. , Phys. Rev. D **40**, 3474 (1989)
D. Binosi and J.P. , Phys. Rev. D **66**, 111901 (2002)
M. Binger and S.J.Brodsky , Phys. Rev. D **74**:054016 (2006)

Longitudinal momenta trigger
Slavnov-Taylor identities inside diagrams:

$$\begin{aligned} k_\nu \gamma^\nu &= (\not{k} + \not{p} - m) - (\not{p} - m) \\ &= S_0^{-1}(k+p) - S_0^{-1}(p), \end{aligned}$$

Pinch technique rearrangement:

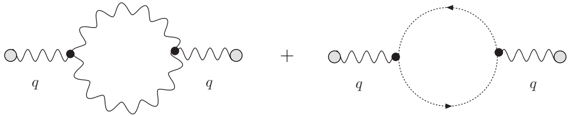


- Simple, QED-like Ward Identities , instead of Slavnov-Taylor Identities, to all orders

$$q_1^{\mu}\widetilde{\mathbb{\Gamma}}_{\mu\alpha\beta}^{abc}(q_1,q_2,q_3) \quad = \quad gf^{abc}\left[\Delta_{\alpha\beta}^{-1}(q_2)-\Delta_{\alpha\beta}^{-1}(q_3)\right]$$

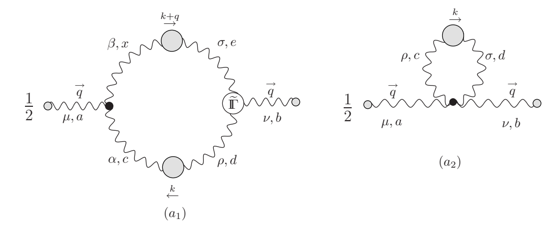
- Profound connection with Background Field Method

$$\widehat{\Pi}_{\mu\nu}(q) =$$



- Special transversality properties

Transversality enforced loop-wise and field-wise

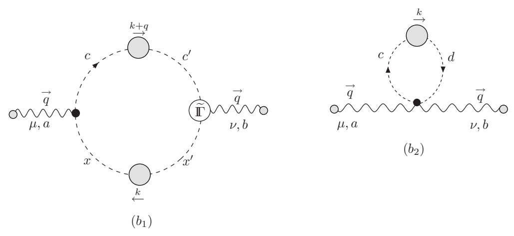


The gluonic contribution

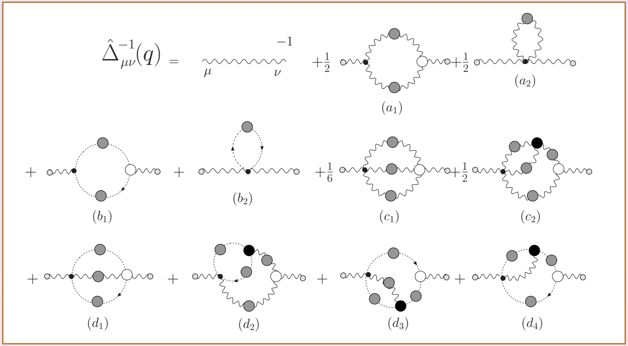
$$q^\mu \Pi_{\mu\nu}(q)|_{(a_1)+(a_2)} = 0$$

The ghost contribution

$$q^\mu \Pi_{\mu\nu}(q)|_{(b_1)+(b_2)} = 0$$



New Schwinger-Dyson series

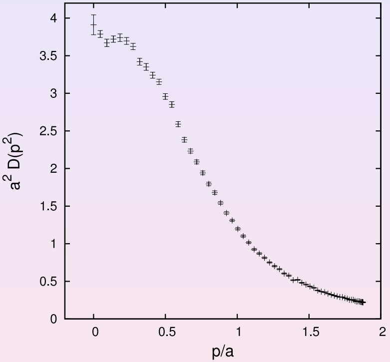


Transversality is enforced separately for gluon- and ghost-loops, and order-by-order in the "dressed-loop" expansion!

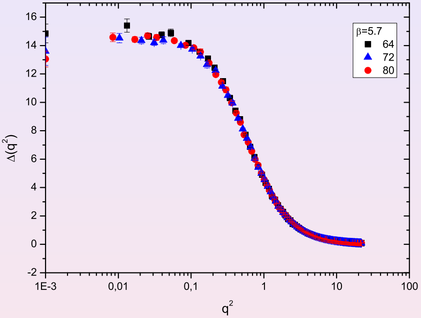
A. C. Aguilar and J. P. , JHEP **0612**, 012 (2006)
D. Binosi and J. P. , Phys. Rev. D **77**, 061702 (2008); JHEP 0811:063,2008.

Lattice predictions for the gluon propagator

$SU(2)$



$SU(3)$



A. Cucchieri and T. Mendes ,PoS **LAT2007**, 297 (2007).

I. L. Bogolubsky, *et al* ,PoS **LAT2007**, 290 (2007)

... looks like a massive propagator.

$$\Delta(q^2) = \frac{1}{q^2[1+\Pi(q^2)]}$$

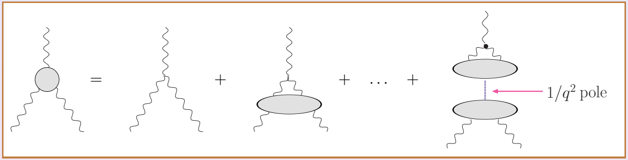
- If $\Pi(q^2)$ has a pole at $q^2 = 0$ the vector meson is massive , even though it is massless in the absence of interactions.

  J. S. Schwinger, Phys. Rev. **125**, 397 (1962); Phys. Rev. **128**, 2425 (1962).

- Requires massless, longitudinally coupled , Goldstone-like poles $\sim 1/q^2$
- Such poles can occur dynamically , even in the absence of canonical scalar fields. Composite excitations in a strongly-coupled gauge theory.

  R. Jackiw and K. Johnson, Phys. Rev. D **8**, 2386 (1973)
  J. M. Cornwall and R. E. Norton, Phys. Rev. D **8** (1973) 3338
  E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254 (1974)

Dynamics enters through the three-gluon vertex:

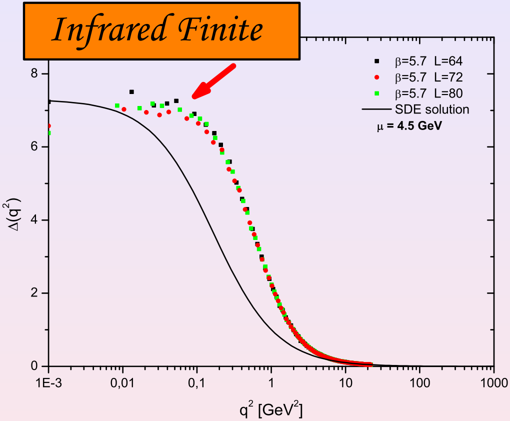


- Satisfies the correct Ward identity

$$q_1^{\mu}\widetilde{\Gamma}^{abc}_{\mu\alpha\beta}(q_1, q_2, q_3) = gf^{abc}\left[\Delta^{-1}_{\alpha\beta}(q_2) - \Delta^{-1}_{\alpha\beta}(q_3)\right]$$

- longitudinally coupled massless bound-state poles $\sim 1/q^2$, instrumental for $\Delta^{-1}(0) > 0$.

Numerical results and comparison with lattice

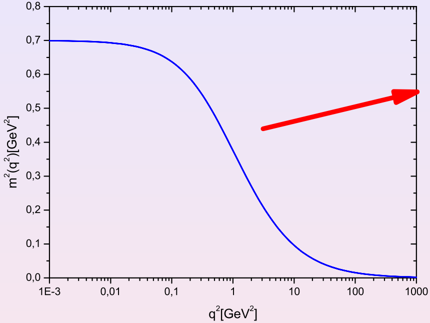


A. C. Aguilar, D. Binosi and J. P. , Phys. Rev. D 78, 025010 (2008).

I. L. Bogolubsky, *et al* , PoS **LAT2007**, 290 (2007)

A. Cucchieri and T. Mendes ,PoS **LAT2007**, 297 (2007); Phys. Rev. Lett. **100**, 241601 (2008)

The gluon “mass” is not “hard” but momentum-dependent

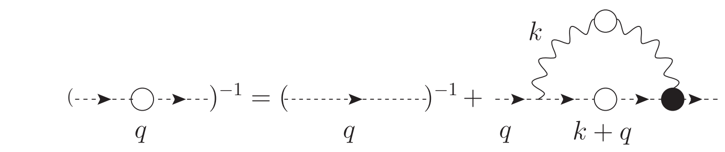


$$m^2(q^2) \sim \frac{\langle G^2_{\mu\nu} \rangle}{q^2}$$

$\langle G^2_{\mu\nu} \rangle$: dimension four gauge-invariant gluon condensate

J. M. Cornwall , Phys. Rev. D **26**, 1453 (1982).
M. Lavelle , Phys. Rev. D **44**, 26 (1991).
A. C. Aguilar and JP , Eur. Phys. J. A **35**, 189 (2008).

The ghost sector: SD equation

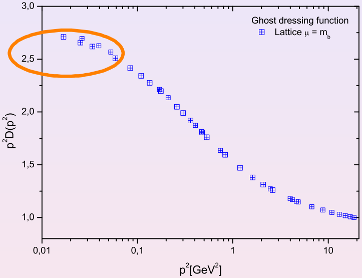


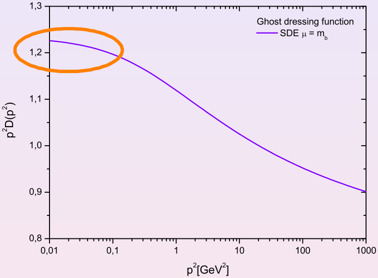
- Landau gauge $\Rightarrow \Gamma_\mu =$ tree level

$$D^{-1}(p^2) = p^2 - g^2 C_A \int_k \left[p^2 - \frac{(p\cdot k)^2}{k^2} \right] \Delta(k)\, D(p+k)$$

The ghost sector: results

Dressing is infrared finite





In the deep IR: $p^2 D(p^2) \to$ constant $\Rightarrow$ **No "power-law enhancement"!**

$\Rightarrow$ At odds with the "ghost-dominance" picture.

C. S. Fischer, J. Phys. G **32**, R253 (2006)

The pinch technique effective charge

- Abelian Ward identities $\widehat{Z}_1 = \widehat{Z}_2,\; Z_g = \widehat{Z}_A^{-1/2}$

$\Longrightarrow$ Renormalization-group invariant combination

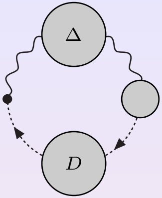
$$\widehat{d}_0(q^2) \equiv g_0^2 \widehat{\Delta}_0(q^2) = g^2 \widehat{\Delta}(q^2) \equiv \widehat{d}(q^2)$$

- $\Delta(q^2)$ and $\widehat{\Delta}(q^2)$ are connected by the formal relation:

$$\Delta(q^2) = \left[1 + G(q^2)\right]^2 \widehat{\Delta}(q^2)$$

D. Binosi and J. P., Phys. Rev. D 66, 025024 (2002) .

where

$G(q) =$ 

Important Green's function !

- Its SDE is (Landau gauge)

$$G(q^2) = -\frac{C_A g^2}{3} \int_k \left[2 + \frac{(k \cdot q)^2}{k^2 q^2} \right] \Delta(k) D(k+q) \, .$$

Checking the perturbative behavior of $\widehat{d}(q^2)$

- Enforces β function coefficient in front of UV logarithm ($b = 11C_A/48\pi^2$).

$$g^2\widehat{\Delta}(q^2) = \frac{g^2\Delta(q^2)}{[1+G(q^2)]^2}$$

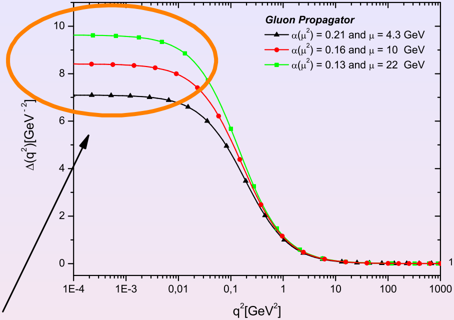
$$1 + G(q^2) = 1 + \frac{9}{4}\frac{C_{\rm A}g^2}{48\pi^2}\ln(q^2/\mu^2)$$

$$\Delta^{-1}(q^2) = q^2\left[1 + \frac{13}{2}\frac{C_{\rm A}g^2}{48\pi^2}\ln(q^2/\mu^2)\right]$$

$$\Downarrow$$

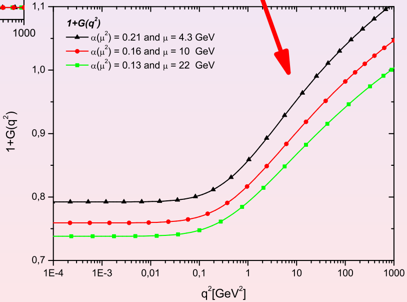
$$\widehat{\Delta}^{-1}(q^2) = q^2\left[1 + bg^2\ln(q^2/\mu^2)\right]$$

The μ-dependent ingredients from the SDE

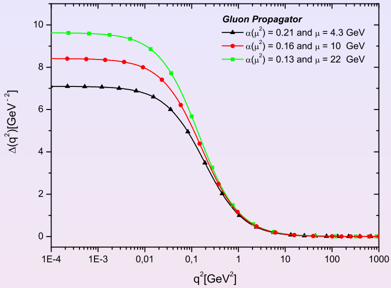


Dependence on μ

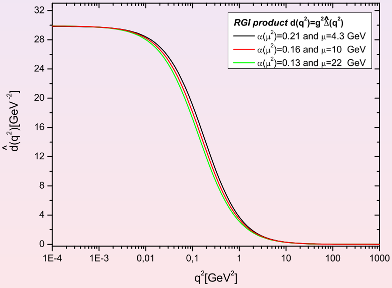
Dependence on μ



Forming the μ-independent $\hat{d}(q^2)$



$$\times \; \frac{g^2(\mu)}{[1+G(q^2)]^2}$$



No μ - dependence

Defining the effective charge

Cast the dimensionful $\widehat{d}(q^2) = g^2\widehat{\Delta}(q^2)$ in the form:

$$\widehat{d}(q^2) = \frac{4\pi\overline{\alpha}(q^2)}{q^2 + m^2(q^2)},$$

where the dimensionless effective charge is

$$\overline{\alpha}(q^2) = \frac{1}{4\pi b \ln\left(\frac{q^2+\rho m^2(q^2)}{\Lambda^2}\right)}$$

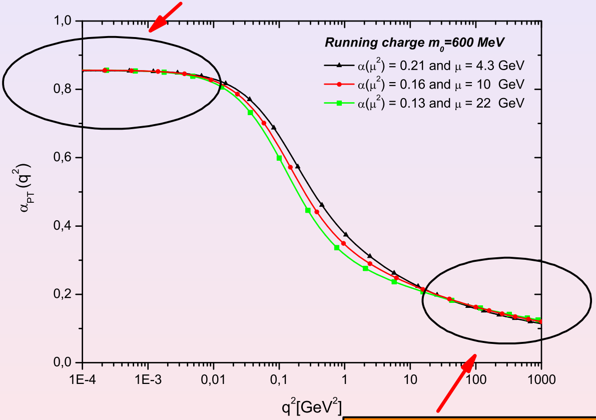
- It displays asymptotic freedom in the UV.
- Freezes at a finite value in the low energy regime

$\overline{\alpha}^{-1}(0) = 4\pi b \ln\left(\frac{\rho m^2(0)}{\Lambda^2}\right)$ $\Longrightarrow$ **Infrared Fixed Point for QCD**

.

Infrared finite effective charge

Infrared fixed point



Asymptotic freedom

The Kugo-Ojima story

Consider the Green's function

$$\int d^4x \ \mathrm{e}^{-iq\cdot(x-y)} \langle T \left[(\mathcal{D}_\mu c)^m_x \, (f^{nrs} A^n_\nu \bar{c}^s)_y \right] \rangle = P_{\mu\nu}(q) \delta^{mn} u(q^2)$$

A heavy-duty formal study concludes that if

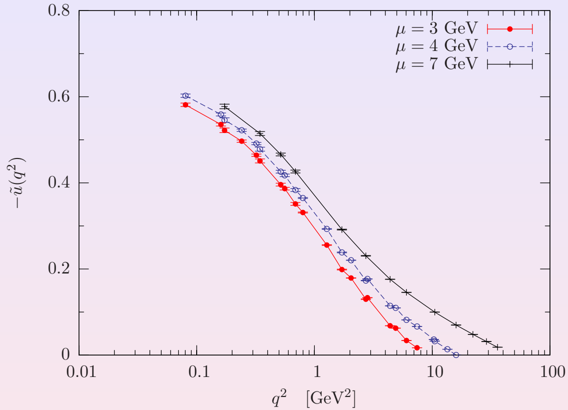
$$u(0) = -1$$

⇒ Color Confinement and infrared-enhanced ghost

T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. **66**, 1 (1979) .

However, the ghost IS NOT enhanced . And, in addition ...

Direct lattice calculation



No evidence of $u(0) \to -1$

A. Sternbeck, arXiv:hep-lat/0609016

Indirect symbiotic approach

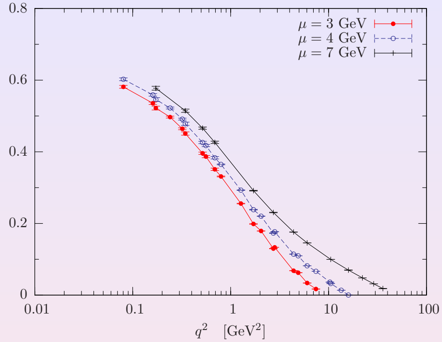
A. C. Aguilar, D. Binosi and JP, arXiv:0907.0153 [hep-ph].

A special relation allows us to write

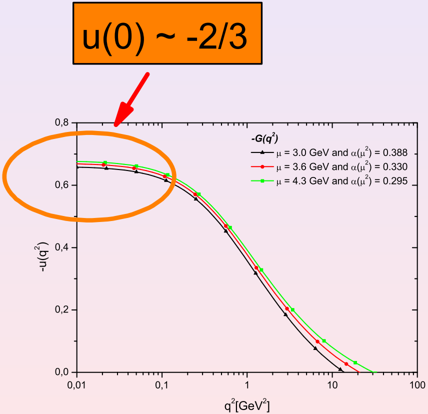
$$u(q^2) = -\frac{1}{3} C_{\mathrm{A}} g^2 \int_k \left[2 + \frac{(k \cdot q)^2}{k^2 q^2} \right] \Delta(k) D(k+q) \, .$$

Plug in the lattice results for Δ and D, and see what happens ..

Our results



μ = 3 GeV
μ = 4 GeV
μ = 7 GeV
0.8
0.6
0.4
0.2
0
0.01
0.1
1
10
100
q² [GeV²]



u(0) ~ -2/3
-G(q²)
μ = 3.0 GeV and α(μ²) = 0.388
μ = 3.6 GeV and α(μ²) = 0.330
μ = 4.3 GeV and α(μ²) = 0.295
-u(q²)
0,8
0,6
0,4
0,2
0,0
0,01
0,1
1
10
100
q²[GeV²]

A prediction and a challenge

$$u(q^2) = \sqrt{\frac{\Delta(q^2)}{\widehat{\Delta}(q^2)}} - 1$$

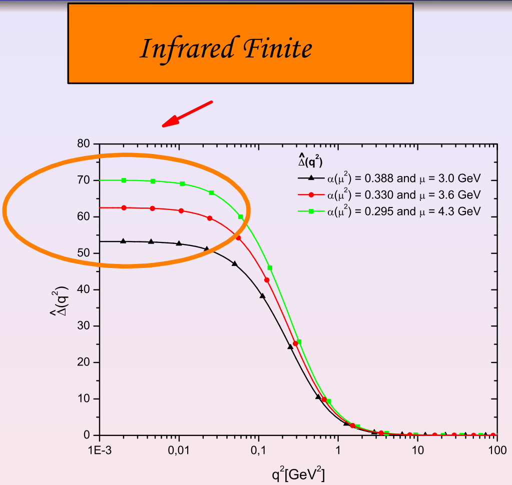
Relates a "ghostly" Green's function to two gluon propagators, defined at two completely different quantization schemes.

For the Kugo-Ojima criterion [$u(0) = -1$] to be valid, we must have: $\widehat{\Delta}(0) \to \infty$ ...

No way !

Instead ...

Our prediction:



The challenge : Compute $\widehat{\Delta}$ on the lattice !

Lattice formulation of Background Field Method exists.

R. F. Dashen and D. J. Gross, Phys. Rev. D **23**, 2340 (1981)

Conclusions

- Self-consistent description of the non-perturbative QCD dynamics in terms of an IR finite gluon propagator appears to be within our reach.
- Gauge invariant truncation of SD equations furnishes reliable non-perturbative information and strengthens the synergy with the lattice community .
- Meaningful contact with phenomenological studies.

The big picture

J. M. Cornwall, Phys. Rev. D **57**, 7589 (1998); Nucl. Phys. B **157**, 392 (1979)

- Effective low-energy field theory describing the gluon mass:
  massive gauge-invariant Yang-Mills

$$\mathcal{L}_{MYM} = \frac{1}{2} G^2_{\mu\nu} - m^2 \mathrm{Tr}\left[A_\mu - g^{-1} U(\theta) \partial_\mu U^{-1}(\theta)\right]^2$$

$U(\theta) = \exp\left[i \frac{1}{2} \lambda_a \theta^a\right]$, θ_a: scalar (Goldstone-like) fields

- Locally gauge-invariant under combined

$$A'_\mu = V A_\mu V^{-1} - g^{-1}\left[\partial_\mu V\right] V^{-1}, \qquad U' = U(\theta') = V U(\theta)$$

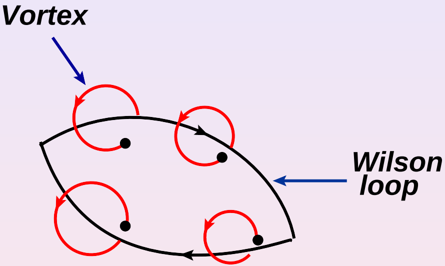
- Gauged non-linear sigma model:
  $\Rightarrow$ non-renormalizable (because $m = \text{const}$).
- But, from the SD analysis, $m = m(q^2)$, vanishes in the UV
  $\Rightarrow$ renormalizability restored .

What about confinement?

If gluons are "massive", where does the long range force associated with confinement come from ?

- $\mathcal{L}_{MYM}$ admits vortex solutions , with a long-range pure gauge term in their potentials (like Nielsen-Olesen)
- Vortices have topological quantum number corresponding to the center of the gauge group, Z_N for $SU(N)$
- Center vortices of thickness $\sim m^{-1}$ form a condensate : their entropy is larger than their action $\Rightarrow \langle G^2_{\mu\nu} \rangle$

- The topological linking (Gauss linking) between the (fundamental representation) Wilson loop and center vortices with a finite density in the vacuum



$\Rightarrow$ area law

$\Rightarrow$ quark confinement

(not fully demonstrated!)