Gluon masses without seagull divergences

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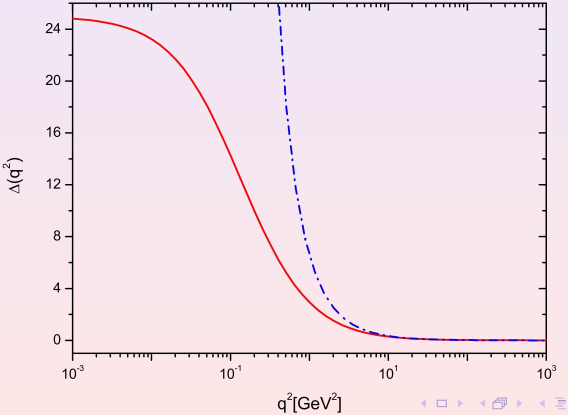
Outline of the talk

- Gluon mass generation: general considerations
- Schwinger mechanism and the problem of seagull divergences
- Cancellation of seagull divergences in scalar QED
- QCD and the correct Ansatz for the three-gluon vertex
- Separate equations for the gluon mass and the running coupling
- Conclusions

Gluon mass generation (J.M.Cornwall, Phys. Rev. D **26**, 1453 (1982))

$$i\Delta_{\mu\nu}(q) = \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right]\Delta(q^2) + \xi\frac{q_\mu q_\nu}{q^4}$$

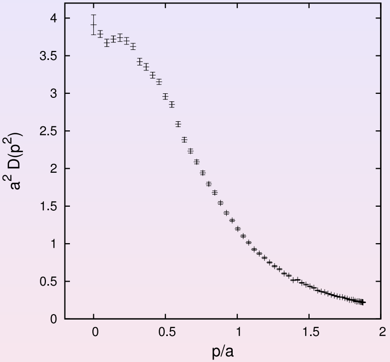
- The non-perturbative QCD dynamics allow for $\Delta^{-1}(0) \neq 0$.



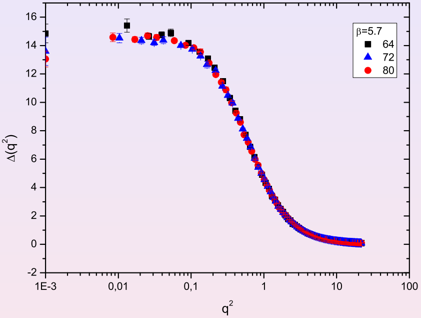
- Dynamical generation of an infrared cutoff; acts as an effective “mass” for the gluons.
- Does not correspond to a term $m^2 A_\mu^2$ in the $\mathcal{L}_{QCD}$. The local gauge symmetry remains exact.
- Not hard but momentum dependent mass: $m = m(q^2)$.
- Inextricably connected to the center vortex picture of confinement.
- Purely non-perturbative effect:
  - Lattice (discretized space-time)
  - Schwinger-Dyson equations (continuum).

Lattice results for the gluon propagator

$SU(2)$



$SU(3)$



A. Cucchieri and T. Mendes ,PoS **LAT2007**, 297 (2007).

I. L. Bogolubsky, *et al* ,PoS **LAT2007**, 290 (2007)

... looks like a massive propagator.

Dynamical mass generation: Schwinger mechanism in d=4

J. S. Schwinger, Phys. Rev. **125**, 397 (1962); Phys. Rev. **128**, 2425 (1962).

$$\Delta^{-1}(q^2) = q^2[1 + \Pi(q^2)]$$

- If $\Pi(q^2)$ has a pole at $q^2 = 0$ the vector meson is massive , even though it is massless in the absence of interactions.

$$\Pi(q^2) = \mu^2/q^2 \qquad \Rightarrow \qquad \Delta^{-1}(q^2) = q^2 + \mu^2$$

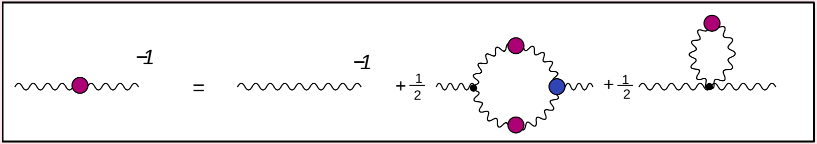
- Requires massless, longitudinally coupled, Goldstone-like poles $\sim 1/q^2$
- Such poles can occur dynamically, even in the absence of canonical scalar fields. Composite excitations in a strongly-coupled gauge theory.

R. Jackiw and K. Johnson, Phys. Rev. D **8**, 2386 (1973)
J. M. Cornwall and R. E. Norton, Phys. Rev. D **8** (1973) 3338
E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254 (1974)

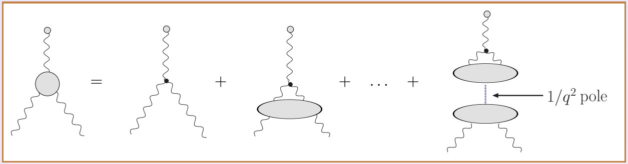
Schwinger-Dyson equations

- Equations of motion for off-shell Green's functions.
- Derived formally from the generating functional.
- Infinite system of coupled non-linear integral equations.
- Inherently non-perturbative.
- Self-consistent truncation scheme must be used. (gauge-invariant PT-BFM framework)

A. C. Aguilar and J. P. , JHEP **0612**, 012 (2006)
D. Binosi and J. P. , Phys. Rev. D **77**, 061702 (2008); JHEP 0811:063,2008.



- Schwinger mechanism enters through the three-gluon vertex:



- Satisfies the correct Ward identity

$$q_1^{\mu}\widetilde{\mathbb{I}\Gamma}^{abc}_{\mu\alpha\beta}(q_1, q_2, q_3) = gf^{abc}\left[\Delta^{-1}_{\alpha\beta}(q_2) - \Delta^{-1}_{\alpha\beta}(q_3)\right]$$

- longitudinally coupled massless bound-state poles $\sim 1/q^2$, instrumental for $\Delta^{-1}(0) \neq 0$

. . . **BUT**

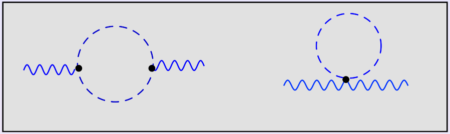
- The value of $\Delta^{-1}(0)$ is expressed in terms of (quadratically) divergent integrals (seagull-like terms).

$$\Delta^{-1}(0) = c_1 \int_k \Delta(k) + c_2 \int_k k^2 \Delta^2(k)$$

- Cannot be eliminated by counterterms of the form $m_0^2(\Lambda_{\rm UV}^2) A_\mu^2$ (forbidden by the local gauge invariance).
- A regularization needs be introduced, to obtain a finite $\Delta_{\rm reg}^{-1}(0)$... However, the actual value of $\Delta_{\rm reg}^{-1}(0)$ remains largely undetermined.

$\Rightarrow$ Imperfect implementation of the Schwinger mechanism at the level of the three-gluon vertex.

Scalar QED at one loop



The one loop expression for $\Pi(q^2)$

$$\Pi^{(1)}(q^2) = \frac{-ie^2}{d-1}\left[\int_k (4k^2 - q^2)\mathcal{D}_0(k)\mathcal{D}_0(k+q) - 2d\int_k \mathcal{D}_0(k)\right]$$

$$\mathcal{D}_0(k) = \frac{1}{k^2 - m^2}$$

Taking the limit $q \to 0$, we have

$$\Pi^{(1)}(0) = \frac{-4ie^2}{d-1}\left[\int_k k^2 \mathcal{D}_0^2(k) - \frac{d}{2}\int_k \mathcal{D}_0(k)\right]$$

Due to the dimensional regularization identity

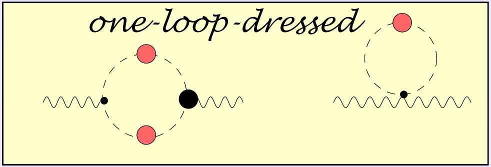
$$\int_k \frac{k^2}{(k^2-m^2)^2} = \frac{d}{2}\int_k \frac{1}{k^2-m^2}$$

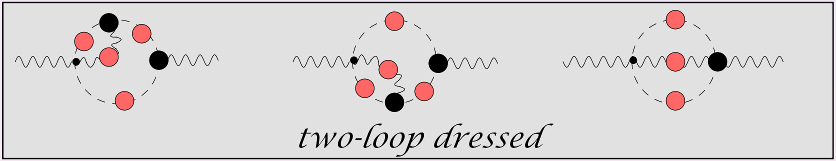
Photon remains massless $\to$ $\Pi^{(1)}(0) = 0$

The identity may be cast in the form

$$\int_k k^2 \frac{\partial \mathcal{D}_0(k)}{\partial k^2} = -\frac{d}{2}\int_k \mathcal{D}_0(k)$$

SD equation for the photon propagator





"One-loop dressed" SDE for the photon self-energy:

$$\Pi_{\mu\nu}(q) = e^2 \int_k \Gamma^{(0)}_{\mu} \mathcal{D}(k) \mathcal{D}(k+q) \mathrm{I\!\Gamma}_{\nu} + e^2 \int_k \Gamma^{(0)}_{\mu\nu} \mathcal{D}(k)$$

$\mathbb{\Gamma}_\mu$ should satisfy:

- The Abelian all-order Ward identity

$$q^\nu \mathbb{\Gamma}_\nu = \mathcal{D}^{-1}(k+q) - \mathcal{D}^{-1}(k)$$

- No kinematic singularities
  The Ball-Chiu Ansatz Phys.Rev. D22, 2542 (1980)

$$\mathbb{\Gamma}_\mu \quad = \quad \frac{(2k+q)_\mu}{(k+q)^2 - k^2}\left[\mathcal{D}^{-1}(k+q) - \mathcal{D}^{-1}(k)\right]$$

$$\Pi(q^2) = \frac{ie^2}{d-1}\left[\int_k (4k^2 - q^2)\frac{\mathcal{D}(k+q) - \mathcal{D}(k)}{(k+q)^2 - k^2} + 2d\int_k \mathcal{D}(k)\right]$$

$$\Pi(0) = \frac{4ie^2}{d-1}\left[\int_k k^2\frac{\partial\mathcal{D}(k)}{\partial k^2} + \frac{d}{2}\int_k \mathcal{D}(k)\right]$$

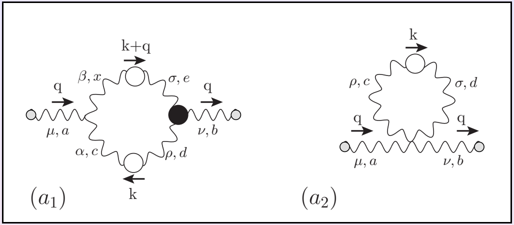
- Using the dim.reg. identity

$$\int_k k^2\frac{\partial\mathcal{D}(k)}{\partial k^2} = -\frac{d}{2}\int_k \mathcal{D}(k)$$

$\Pi(0) = 0 \quad \Rightarrow$ **photon remains massless**

The seagull terms cannot be set to zero individually. The only way to keep the photon massless is to employ the identity!

QCD: one-loop dressed approximation



Feynman rules of the Background Field Method

L. F. About, , Nucl. Phys. B185, 189 (1981); A. C. Aguilar, J. P. , JHEP 0612, 012 (2006).

$$\Delta^{-1}(q) = q^2 + \frac{ig^2 C_{\rm A}}{2(d-1)}\left[\int_k \widetilde{\Gamma}^{(0)}_{\mu\alpha\beta}\Delta(k)\Delta(k+q)\widetilde{\mathbb{\Gamma}}^{\mu\alpha\beta} + 2d^2\int_k \Delta(k)\right]$$

The form of the vertex

Write $\Delta^{-1}(q)$ in the form $\boxed{\Delta^{-1}(q) = q^2 H^{-1}(q) - \widetilde{m}^2(q)}$

$$i\widetilde{\Gamma}_{\mu\alpha\beta} = \overbrace{\left[\frac{(k+q)^2 H^{-1}(k+q) - k^2 H^{-1}(k)}{(k+q)^2 - k^2}\right] \widetilde{\Gamma}^{(0)}_{\mu\alpha\beta}}^{\text{triggers seagull identity}}$$
$$+ \underbrace{\frac{q_\mu}{q^2}\left[\widetilde{m}^2(k) - \widetilde{m}^2(k+q)\right] g_{\alpha\beta}}_{\text{triggers Schwinger mechanism.}}$$

satisfies the correct Ward identity

$$q^\nu \widetilde{\Gamma}_{\nu\alpha\beta} = [\Delta^{-1}(k+q) - \Delta^{-1}(k)] g_{\alpha\beta}$$

The implications for the SDE

$$\Delta^{-1}(q^2) = q^2 - \frac{ig^2 C_A}{2(d-1)}\left[\Pi(q) + \Pi_m(q)\right]$$

$$\Pi(q) = \underbrace{(7d-8)\, q^2 \int_k \frac{\Delta(k+q)-\Delta(k)}{(k+q)^2-k^2}}_{\text{"kinetic" term}} + 4d \underbrace{\left[\int_k k^2 \frac{\Delta(k+q)-\Delta(k)}{(k+q)^2-k^2} + \frac{d}{2}\int_k \Delta(k)\right]}_{q^2\to 0 \;\;\Rightarrow\;\; \text{seagull identity}}$$

remarkable conspiracy of coefficients, possible only in PT-BFM!

$$\Pi_m(q) = \underbrace{-\frac{2d}{q^2}\int_k k^2 \Delta(k)\Delta(k+q)[\widetilde{m}^2(k+q) - \widetilde{m}^2(k)]}_{\text{dynamical mass term}}$$

Therefore, **total seagull annihilation!**

$$\Rightarrow \boxed{\Pi(0) = 0}$$

On the other hand $(\tilde{b} = 10C_{\rm A}/48\pi^2)$

$$\Delta^{-1}(0) = \Pi_m(0) = -\frac{\tilde{b}g^2}{5}\int_k k^2\Delta^2(k^2)\left(\frac{d\widetilde{m}^2(k^2)}{dk^2}\right)$$

$$\Downarrow$$

- $\Pi_m(0)$ is UV finite , provided that $\widetilde{m}^2(k^2)$ drops sufficiently fast in the UV (to be determined from its own equation!)
- $$\frac{d\widetilde{m}^2(k^2)}{dk^2} < 0 \quad \Rightarrow \quad \Delta^{-1}(0) > 0$$

Remembering that the left hand side of SDE is

$$\Delta^{-1}(q) = q^2 H^{-1}(q) - \widetilde{m}^2(q)$$

the equation can be split unambiguously in two parts :

$$\begin{aligned} q^2 H^{-1}(q) &= q^2 - \frac{ig^2 C_{\mathrm{A}}}{2(d-1)}\Pi(q) \\ \widetilde{m}^2(q) &= \frac{ig^2 C_{\mathrm{A}}}{2(d-1)}\Pi_m(q) \end{aligned}$$

- The first determines the running of the **QCD coupling** (effective charge) $g^2 H(q^2) = \overline{g}^2(q^2)$
- The second gives the running of the **gluon mass** $m^2(q^2) = \widetilde{m}^2(q^2) H(q^2)$

A new system for $\overline{g}(q^2)$ and $m^2(q^2)$

Setting $\bar{\Delta}^{-1}(q^2) = q^2 + m^2(q^2)$

$$\frac{1}{\overline{g}^2(q^2)} = \frac{1}{\overline{g}^2(\mu^2)} + \tilde{b}\left[\int_0^{q^2/4} dz \left(1+\frac{4}{5}\frac{z}{q^2}\right)\left(1-\frac{4z}{q^2}\right)^{1/2}\bar{\Delta}(z)\right] - \tilde{b}\left[\int_0^{\mu^2/4} dz \left(1+\frac{4}{5}\frac{z}{\mu^2}\right)\left(1-\frac{4z}{\mu^2}\right)^{1/2}\bar{\Delta}(z)\right]$$

$$\frac{m^2(q^2)}{\overline{g}^2(q^2)} = \frac{2\tilde{b}}{5}\left[\bar{\Delta}(q^2)\int_0^{q^2} dy\, y\, m^2(y)\bar{\Delta}(y) + \frac{1}{2}\int_{q^2}^{\infty} dy\, y\, \bar{\Delta}^2(y)\overline{g}^2(y) m^2(y)\right]$$

Asymptotic running of the gluon mass

- For asymptotically large q^2 we have

$$m^2(q^2)\ln q^2 = \frac{2}{5}\left[\frac{1}{q^2}\int_0^{q^2} dy\, m^2(y) - \frac{1}{2}\int_{q^2}^{\infty} dy\,\overline{g}^2(y)[m^2(y)]'\right]$$

substituting $m^2(q^2)$ of the form

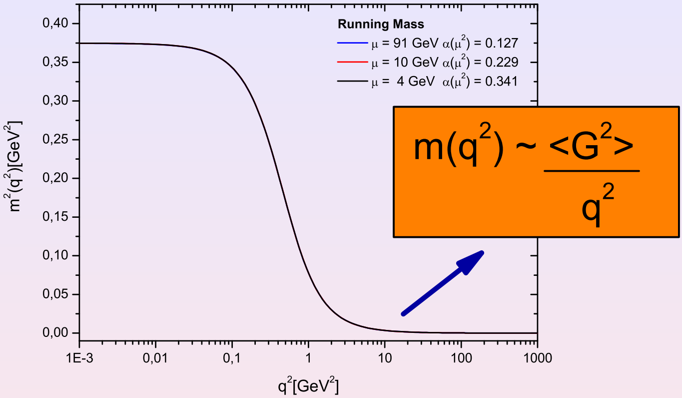
$$m^2(q^2) = \frac{\lambda_0^4}{q^2}(\ln q^2)^{\gamma-1}$$

we find

$$\gamma = 2/5$$

$$\Longrightarrow \quad m^2(q^2) = \frac{\lambda_0^4}{q^2}(\ln q^2)^{-3/5}$$

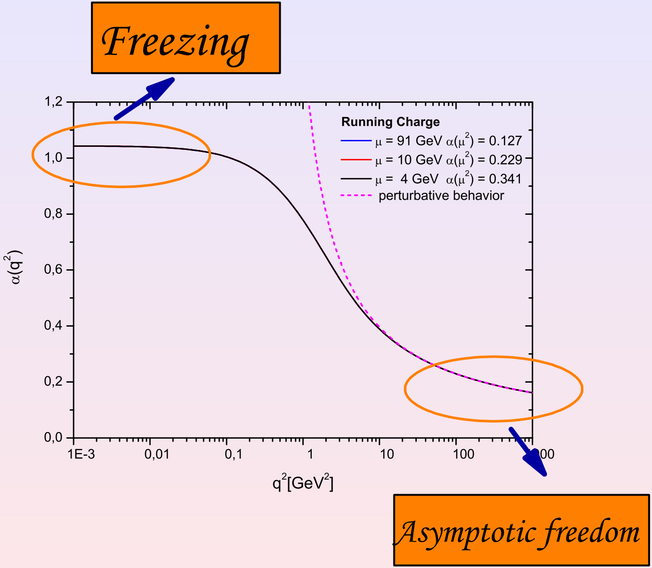
Gluon mass with power-law running



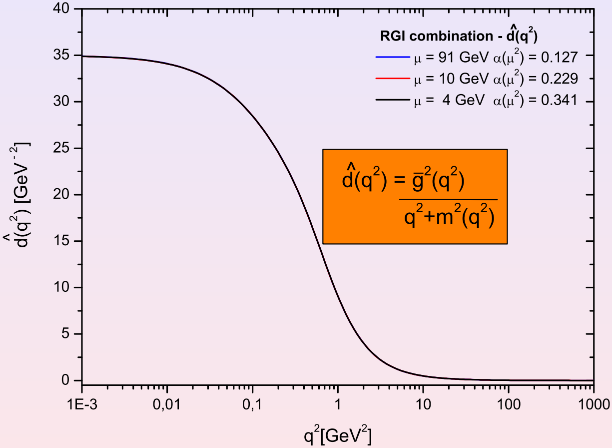
$\langle G^2 \rangle$: dimension four gauge-invariant gluon condensate

J. M. Cornwall , Phys. Rev. D **26**, 1453 (1982).
M. Lavelle , Phys. Rev. D **44**, 26 (1991).
A. C. Aguilar and JP , Eur. Phys. J. A **35**, 189 (2008).

Effective charge



Given $\overline{g}^2(q^2)$ and $m^2(q^2)$ $\Rightarrow$ $\widehat{d}(q^2)$



Conclusions

- Gluon mass generation without seagull divergences
- Crucial ingredient: Ansatz for the gluon vertex
  - Satisfies the right Ward identity
  - Contains massless poles
  - Triggers the seagull identity
- Individual (but coupled) equations determining the momentum dependence of the gluon mass and the effective charge.

Renormalization group invariant quantities

In the PT-BFM scheme

$$
\begin{aligned}
g(\mu^2) &= Z_g^{-1}(\mu^2)g_0, \\
\Delta(q^2;\mu^2) &= Z_A^{-1/2}(\mu^2)\Delta_0(q^2)
\end{aligned}
$$

The QED-like Ward identities imply

$$Z_g = Z_A^{-1/2}$$

$$\Downarrow$$

$$\widehat{d}_0(q^2) = g_0^2\Delta_0(q^2) = g^2\Delta(q^2) = \widehat{d}(q^2)$$

retains the same form before and after renormalization,
it forms a RG-invariant (μ-independent) quantity .

For asymptotically large momenta

$$\widehat{d}(q^2) = \frac{\overline{g}^2(q^2)}{q^2}$$

where $\overline{g}^2(q^2)$ is the effective charge of QCD

$$g^2 H(q^2) = \overline{g}^2(q^2)$$

at one-loop

$$\overline{g}^2(q^2) = \frac{g^2}{1 + bg^2 \ln(q^2/\mu^2)} = \frac{1}{b \ln\left(q^2/\Lambda^2_{\mathrm{QCD}}\right)}$$

where Λ_{QCD} denotes an RG-invariant mass scale of a few hundred MeV.

Non-perturbatively:

$$\widehat{d}(q^2) = \frac{\overline{g}^2(q^2)}{q^2 + m^2(q^2)}$$

where $m^2(q^2)$ is the RG-invariant dynamical gluon mass

$$m^2(q^2) = \widetilde{m}^2(q^2) H(q^2)$$