Effective gluon mass and freezing of the QCD coupling

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Based on:

A. C. Aguilar and J. Papavassiliou, *In preparation*

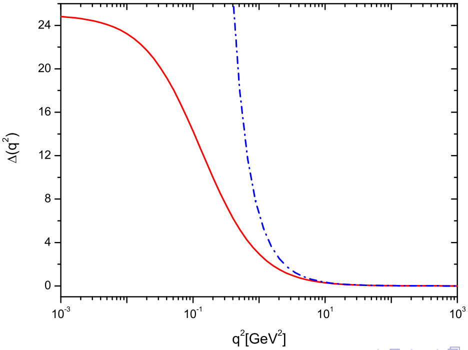
A. C. Aguilar and J. Papavassiliou
"Gluon mass generation in the PT-BFM scheme,"
JHEP **0612**, 012 (2006) [arXiv:hep-ph/0610040]

General Considerations

Gluon "propagator" $\Delta_{\mu\nu}(q)$

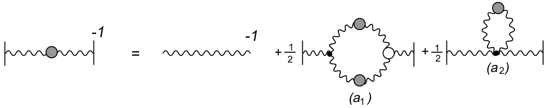
$$i\Delta_{\mu\nu}(q) = \left[\, g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] \Delta(q^2) + \frac{q_\mu q_\nu}{q^4}.$$

Dynamical generation of an infrared cutoff. The QCD dynamics allow for $\Delta^{-1}(0) \neq 0$. Acts as an effective "mass" for the gluons. Cornwall, Phys. Rev. D **26**, 1453 (1982)



- Does not correspond to a term $m^2 A_\mu^2$ in the QCD Lagrangian.
- The local gauge symmetry remains exact.
- Not hard but momentum dependent mass: $m = m(q^2)$.
- Drops off in the UV "sufficiently fast". $\Longrightarrow$ QCD remains renormalizable
- Purely non-perturbative effect:
  - Lattice (discretized space-time)
  - Schwinger-Dyson equations (continuum).

Schwinger-Dyson Equation



$$\Delta^{-1}(q^2)\mathrm{P}_{\mu\nu}(q) = q^2\mathrm{P}_{\mu\nu}(q) + i\left[\Pi^{(\mathbf{a_1})}_{\mu\nu}(q) + \Pi^{(\mathbf{a_2})}_{\mu\nu}\right]$$

$$\begin{aligned}
\Pi^{(\mathbf{a_1})}_{\mu\nu}(q) &= \frac{1}{2}\, C_A\, g^2 \int [dk]\Gamma_{\mu\alpha\beta}\Delta^{\alpha\alpha'}(k)\widetilde{\mathbb{\Gamma}}_{\nu\alpha'\beta'}\Delta^{\beta\beta'}(k+q) \\
\Pi^{(\mathbf{a_2})}_{\mu\nu} &= -C_A\, g^2\, g_{\mu\nu} \int [dk]\Delta(k)
\end{aligned}$$

Vertex

The expression for the vertex that we will use is given by

$$\widetilde{\mathbb{\Gamma}}^{\mu\alpha\beta} = L^{\mu\alpha\beta} + T_1^{\mu\alpha\beta} + T_2^{\mu\alpha\beta}$$

with

$$L^{\mu\alpha\beta}(q,p_1,p_2) = \widetilde{\Gamma}^{\mu\alpha\beta}(q,p_1,p_2) + ig^{\alpha\beta}\,\frac{q^\mu}{q^2}\,[\Pi(p_2) - \Pi(p_1)]$$

$$T_1^{\mu\alpha\beta}(q,p_1,p_2) = -i\,\frac{c_1}{q^2}\left(q^\beta g^{\mu\alpha} - q^\alpha g^{\mu\beta}\right)[\Pi(p_1) + \Pi(p_2)]$$

$$T_2^{\mu\alpha\beta}(q,p_1,p_2) = -ic_2\left(q^\beta g^{\mu\alpha} - q^\alpha g^{\mu\beta}\right)\left[\frac{\Pi(p_1)}{p_1^2} + \frac{\Pi(p_2)}{p_2^2}\right]$$

$$\widetilde{\Gamma}_{\mu\alpha\beta}(q,p_1,p_2) = (p_1 - p_2)_\mu g_{\alpha\beta} + 2q_\beta g_{\mu\alpha} - 2q_\alpha g_{\mu\beta}$$

SD equation

$$\Delta^{-1}(x) = Kx + bg^2 \sum_{i=1}^{8} a_i A_i(x) + \Delta^{-1}(0)$$

$$A_1(x) = a_1\, x \int_x^\infty dy y \Delta^2(y)$$

$$A_2(x) = a_2\, x \int_x^\infty dy \Delta(y)$$

$$A_3(x) = a_3\, x \Delta(x) \int_0^x dy y \Delta(y)$$

$$A_4(x) = a_4 \int_0^x dy y^2 \Delta^2(y)$$

$$A_5(x) = a_5\, \Delta(x) \int_0^x dy y^2 \Delta(y)$$

$$A_6(x) = a_6 \int_0^x dy y \Delta(y)$$

$$A_7(x) = a_7 \frac{\Delta(x)}{x} \int_0^x dy y^3 \Delta(y)$$

$$A_8(x) = a_8 \frac{1}{x} \int_0^x dy y^3 \Delta^2(y)$$

The renormalization condition K is fixed by $\Delta^{-1}(\mu^2) = \mu^2$

The UV behavior of effective gluon mass

$$m^2(x)\ln x = g^{-2}\Delta^{-1}(0) + \gamma_1 \int_0^x dy\, m^2(y)\tilde{\Delta}(y) + \frac{\gamma_2}{x}\int_0^x dy\, y m^2(y)\tilde{\Delta}(y)$$

with

$$\tilde{\Delta}(q^2) = \frac{1}{q^2 + m^2(q^2)},$$

The asymptotic solutions:

$$m^2(x) = \frac{\lambda_2^4}{x}(\ln x)^{\gamma_2 - 1} \quad \Longrightarrow \langle G^a_{\mu\nu} G^{\mu\nu}_a \rangle$$

The RG quantity, $d(q^2) = g^2\Delta(q^2)$, has the general form:

$$d(q^2) = \frac{\overline{g}^2(q^2)}{q^2 + m^2(q^2)}\,,$$

where the dynamical mass $m^2(q^2)$ and effective charge $\overline{g}^2(q^2)$ are

$$m^2(q^2) = \frac{m_0^4}{q^2 + m_0^2}\left[\ln\left(\frac{q^2 + \rho\, m_0^2}{\Lambda^2}\right) \Big/ \ln\left(\frac{\rho\, m_0^2}{\Lambda^2}\right)\right]^{\gamma_2 - 1}$$

$$\overline{g}^2(q^2) = \left[b\ln\left(\frac{q^2 + \rho m^2(q^2)}{\Lambda^2}\right)\right]^{-1}$$

Numerical Results