Gluon and Ghost Propagators from Schwinger-Dyson Equation and Lattice Simulations

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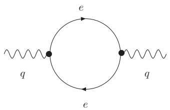
Outline of the talk

- Gauge-invariant gluon self-energy in perturbation theory
  Field theoretic framework: Pinch Technique
- Beyond perturbation theory: gauge-invariant truncation of Schwinger-Dyson equations
- Dynamical mass generation
- IR finite gluon propagator from SDE and comparison with the lattice simulations
- Obtaining physically meaningful quantities
- Conclusions

Vacuum polarization in QED (prototype)

$\Pi_{\mu\nu}(q) \quad =$ 

$\Pi_{\mu\nu}(q)$ is independent of the gauge-fixing parameter to all orders

$$\Delta(q^2) = \frac{1}{q^2[1+\Pi(q^2)]}$$

$e = Z_e^{-1} e_0$ and $1 + \Pi(q^2) = Z_A[1 + \Pi_0(q^2)]$

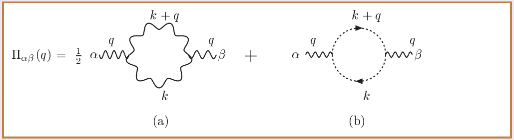
From QED Ward identity follows $Z_1 = Z_2$ and $Z_e = Z_A^{-1/2}$

RG-invariant combination

$$e_0^2 \Delta_0(q^2) = e^2 \Delta(q^2) \quad \Longrightarrow \quad \alpha(q^2) = \frac{\alpha}{1+\Pi(q^2)}$$

Gluon self-energy in perturbation theory

- $\Pi_{\mu\nu}(q,\xi)$ depends on the gauge-fixing parameter already at one-loop



- Ward identities replaced by Slavnov-Taylor identities involving ghost Green's functions. ($Z_1 \neq Z_2$ in general)

Difficulty with conventional SD series

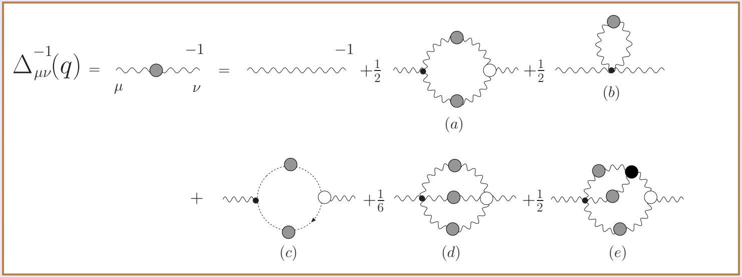
$$q^{\mu}\Pi_{\mu\nu}(q) = 0$$

The most fundamental statement at the level of Green's functions that one can obtain from the BRST symmetry .

It affirms the transversality of the gluon self-energy and is valid both perturbatively (to all orders) as well as non-perturbatively .

Any good truncation scheme ought to respect this property

Naive truncation violates it



$$q^{\mu}\Pi_{\mu\nu}(q)|_{(a)+(b)} \neq 0$$

$$q^{\mu}\Pi_{\mu\nu}(q)|_{(a)+(b)+(c)} \neq 0$$

Main reason : Full vertices satisfy complicated Slavnov-Taylor identities.

Pinch Technique

Diagrammatic rearrangement of perturbative expansion (to all orders) gives rise to effective Green's functions with special properties .

J. M. Cornwall , Phys. Rev. D **26**, 1453 (1982)
J. M. Cornwall and J.P. , Phys. Rev. D **40**, 3474 (1989)
D. Binosi and J.P. , Phys. Rev. D **66**, 111901 (2002).

In covariant gauges: $\longrightarrow$

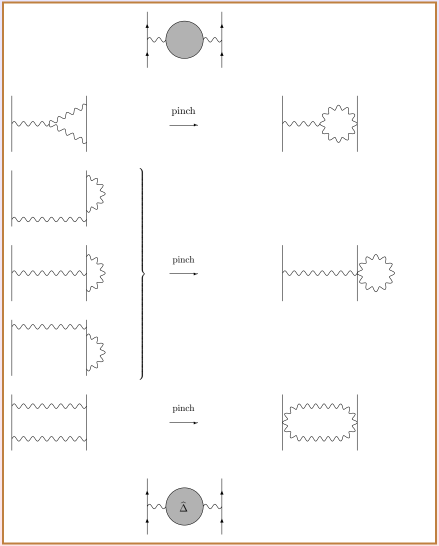
$$i\Delta^{(0)}_{\mu\nu}(k) = \left[g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2} \right] \frac{1}{k^2}$$

In light cone gauges: $\longrightarrow$

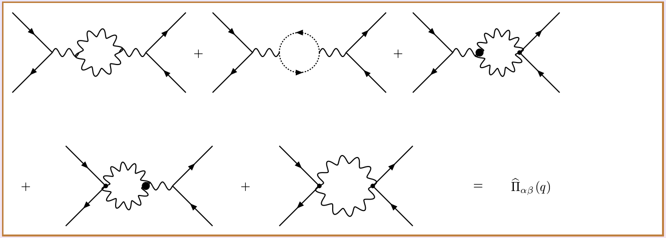
$$i\Delta^{(0)}_{\mu\nu}(k) = \left[g_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{nk} \right] \frac{1}{k^2}$$

$$\begin{aligned} k_\nu \gamma^\nu &= (\not k + \not p - m) - (\not p - m) \\ &= S_0^{-1}(k+p) - S_0^{-1}(p), \end{aligned}$$

Pinch Technique rearrangement



Gauge-independent self-energy



$$\widehat{\Delta}(q^2) = \frac{1}{q^2\left[1 + bg^2 \ln\left(\frac{q^2}{\mu^2}\right)\right]}$$

$b = 11C_A/48\pi^2$ first coefficient of the QCD β-function ($\beta = -bg^3$) in the absence of quark loops.

- Simple, QED-like Ward Identities , instead of Slavnov-Taylor Identities, to all orders

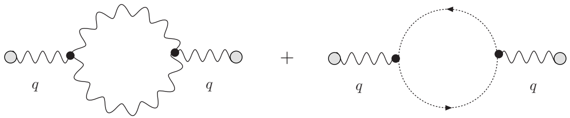
$$
\begin{aligned}
q^{\mu}\widetilde{\Gamma}_{\mu}(p_1,p_2) &= g\left[S^{-1}(p_2)-S^{-1}(p_1)\right] \\
q_1^{\mu}\widetilde{\Gamma}^{abc}_{\mu\alpha\beta}(q_1,q_2,q_3) &= gf^{abc}\left[\Delta^{-1}_{\alpha\beta}(q_2)-\Delta^{-1}_{\alpha\beta}(q_3)\right]
\end{aligned}
$$

- Profound connection with

Background Field Method $\Longrightarrow$ easy to calculate

D. Binosi and J.P. , Phys. Rev. D **77**, 061702 (2008); arXiv:0805.3994 [hep-ph]

$\widehat{\Pi}_{\mu\nu}(q) =$



- Can move consistently from one gauge to another (Landau to Feynman, etc) A. Pilaftsis , Nucl. Phys. B **487**, 467 (1997)

Restoration of:

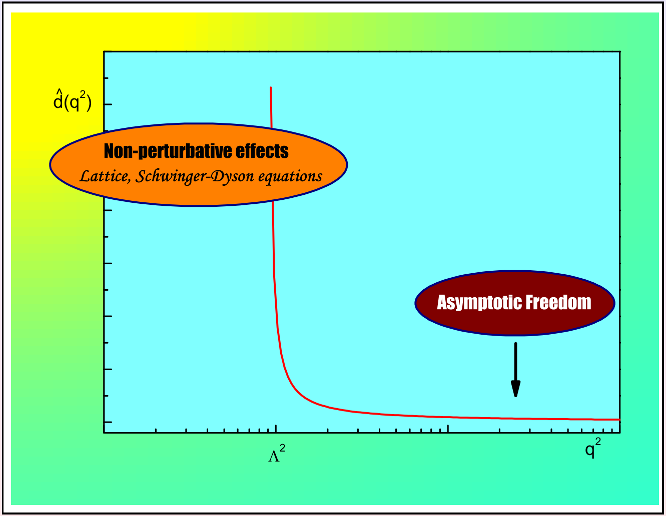
- Abelian Ward identities $\widehat{Z}_1 = \widehat{Z}_2,\ Z_g = \widehat{Z}_A^{-1/2}$

$\Longrightarrow$ RG invariant combination $g_0^2\,\widehat{\Delta}_0(q^2) = g^2\,\widehat{\Delta}(q^2)$

For large momenta q^2, define the RG-invariant effective charge of QCD,

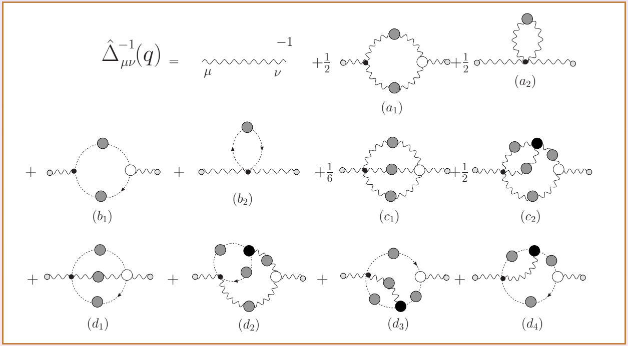
$$\overline{\alpha}(q^2) = \frac{g^2(\mu)/4\pi}{1 + bg^2(\mu)\ln\left(q^2/\mu^2\right)} = \frac{1}{4\pi b\ln\left(q^2/\Lambda^2\right)}$$

Beyond perturbation theory ...



New SD series

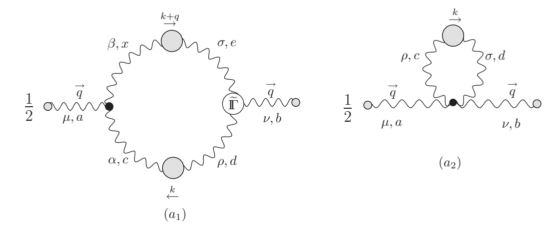
The new Schwinger-Dyson series based on the pinch technique



Transversality is enforced separately for gluon- and ghost-loops, and order-by-order in the "dressed-loop" expansion!

A. C. Aguilar and J. P. , JHEP **0612**, 012 (2006)
D. Binosi and J. P. , Phys. Rev. D **77**, 061702 (2008); arXiv:0805.3994 [hep-ph].

Transversality enforced loop-wise in SD equations

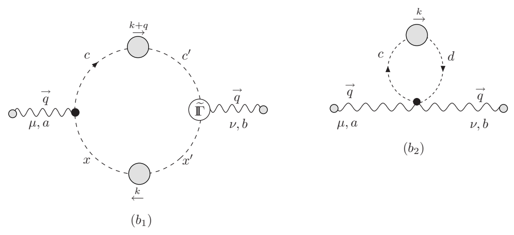


The gluonic contribution

$$q^{\mu}\Pi_{\mu\nu}(q)|_{(a_1)+(a_2)} = 0$$

The ghost contribution

$$q^{\mu}\Pi_{\mu\nu}(q)|_{(b_1)+(b_2)} = 0$$



$$\Delta(q^2) = \frac{1}{q^2[1 + \Pi(q^2)]}$$

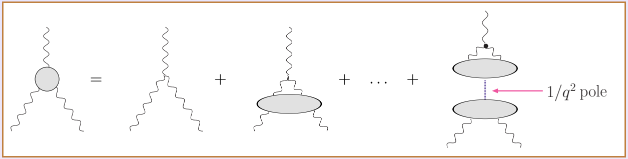
- If $\Pi(q^2)$ has a pole at $q^2 = 0$ the vector meson is massive, even though it is massless in the absence of interactions.

  J. S. Schwinger, Phys. Rev. **125**, 397 (1962); Phys. Rev. **128**, 2425 (1962).

- Requires massless, longitudinally coupled, Goldstone-like poles $\sim 1/q^2$
- Such poles can occur dynamically, even in the absence of canonical scalar fields. Composite excitations in a strongly-coupled gauge theory.

  R. Jackiw and K. Johnson, Phys. Rev. D **8**, 2386 (1973)
  J. M. Cornwall and R. E. Norton, Phys. Rev. D **8** (1973) 3338
  E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254 (1974)

Ansatz for the vertex



Gauge-technique Ansatz for the full vertex:

$$\widetilde{\Gamma}_{\mu\alpha\beta} = \Gamma_{\mu\alpha\beta} + i\,\frac{q_\mu}{q^2}\left[\Pi_{\alpha\beta}(k+q) - \Pi_{\alpha\beta}(k)\right],$$

- Satisfies the correct Ward identity

$$q_1^\mu \widetilde{\Gamma}^{abc}_{\mu\alpha\beta}(q_1, q_2, q_3) = g f^{abc}\left[\Delta^{-1}_{\alpha\beta}(q_2) - \Delta^{-1}_{\alpha\beta}(q_3)\right]$$

- Contains longitudinally coupled massless bound-state poles $\sim 1/q^2$, instrumental for $\Delta^{-1}(0) \neq 0$

System of coupled SD equations

$$\begin{aligned}\Delta^{-1}(q^2) &= q^2 + c_1 \int_k \Delta(k)\Delta(k+q) f_1(q,k) + c_2 \int_k \Delta(k) f_2(q,k) \\ D^{-1}(p^2) &= p^2 + c_3 \int_k \left[p^2 - \frac{(p\cdot k)^2}{k^2}\right] \Delta(k)\, D(p+k)\,,\end{aligned}$$

- Infrared finite

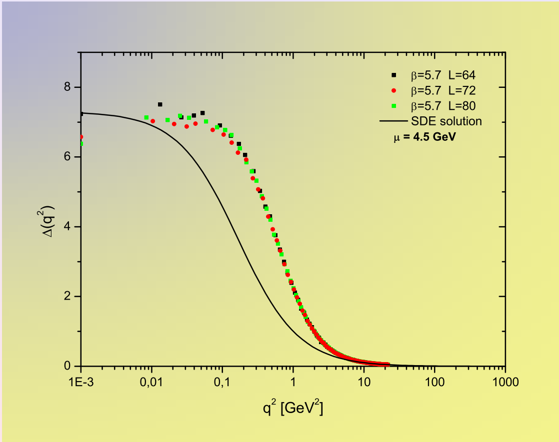
$$\Delta^{-1}(0) \neq 0$$

- Renormalize
- Solve numerically

A. C. Aguilar, D. Binosi and J. P. , Phys. Rev. D 78, 025010 (2008).

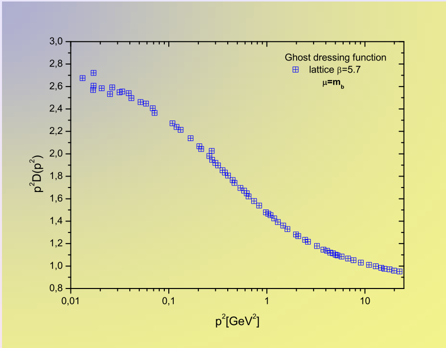
Numerical results and comparison with lattice

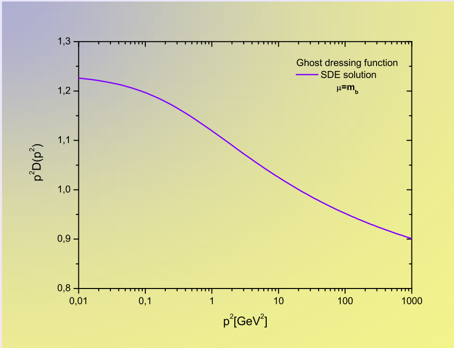
- Use lattice to calibrate the SDE solution.



I. L. Bogolubsky, *et al* , PoS **LAT2007**, 290 (2007)

Ghost propagator





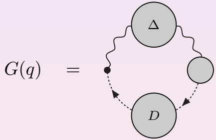
In the deep IR $p^2 D(p^2) \to \text{constant}$ $\Rightarrow$ **No power-law enhancement**

Making contact with physical quantities

- The conventional $\Delta(q^2)$ and the PT-BFM $\widehat{\Delta}(q^2)$ are related by

$$\Delta(q^2) = \left[1 + G(q^2)\right]^2 \widehat{\Delta}(q^2)$$

- Formal relation derived within Batalin Vilkovisky formalism D. Binosi and J. P., Phys. Rev. D 66, 025024 (2002) .



Auxiliary Green's function related to the full gluon-ghost vertex

$$G(q^2) = -\frac{C_A g^2}{3} \int_k \left[2 + \frac{(k \cdot q)^2}{k^2 q^2}\right] \Delta(k) D(k+q)\,.$$

- Enforces β function coefficient in front of UV logarithm.

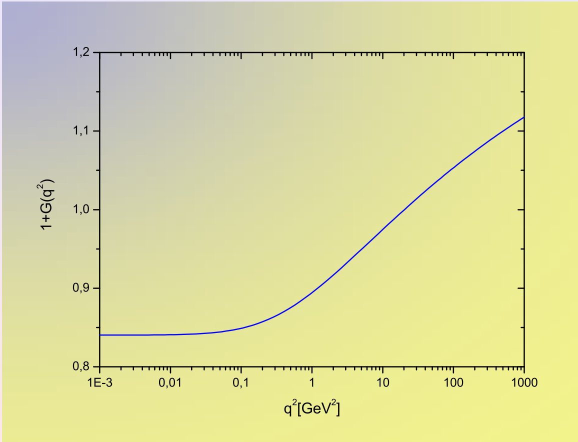
$$1 + G(q^2) = 1 + \frac{9}{4}\frac{C_{\rm A}\, g^2}{48\pi^2}\ln(q^2/\mu^2)$$

$$\Delta^{-1}(q^2) = q^2\left[1 + \frac{13}{2}\frac{C_{\rm A}\, g^2}{48\pi^2}\ln(q^2/\mu^2)\right]$$

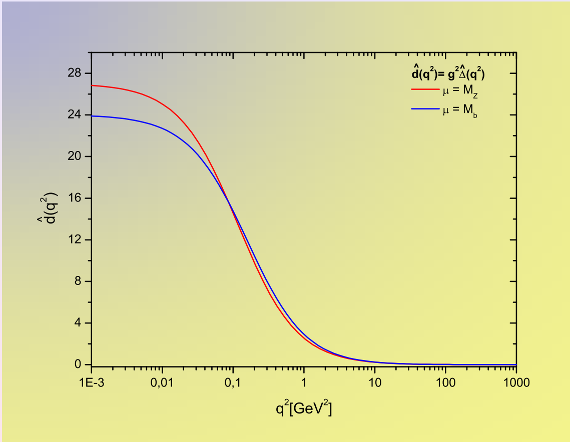
$$\Downarrow$$

$$\widehat{\Delta}^{-1}(q^2) = q^2\left[1 + 11\frac{C_{\rm A}\, g^2}{48\pi^2}\ln(q^2/\mu^2)\right]$$

Numerical Results



Numerical Results



Physically motivated fit: Cornwall's massive propagator

The RG invariant quantity, $\widehat{d}(q^2) = g^2 \widehat{\Delta}(q^2)$, has the form:

$$\widehat{d}(q^2) = \frac{g^2(q^2)}{q^2 + m^2(q^2)}$$

where the running charge is

$$g^2(q^2) = \frac{1}{b \ln\left(\frac{q^2 + 4m^2(q^2)}{\Lambda^2}\right)}$$

and the running mass

$$m^2(q^2) = m_0^2 \left[\ln\left(\frac{q^2 + 4\, m_0^2}{\Lambda^2}\right) \Big/ \ln\left(\frac{4\, m_0^2}{\Lambda^2}\right) \right]^{-12/11}$$

J. M. Cornwall , Phys. Rev. D **26**, 1453 (1982)

Phenomenological studies

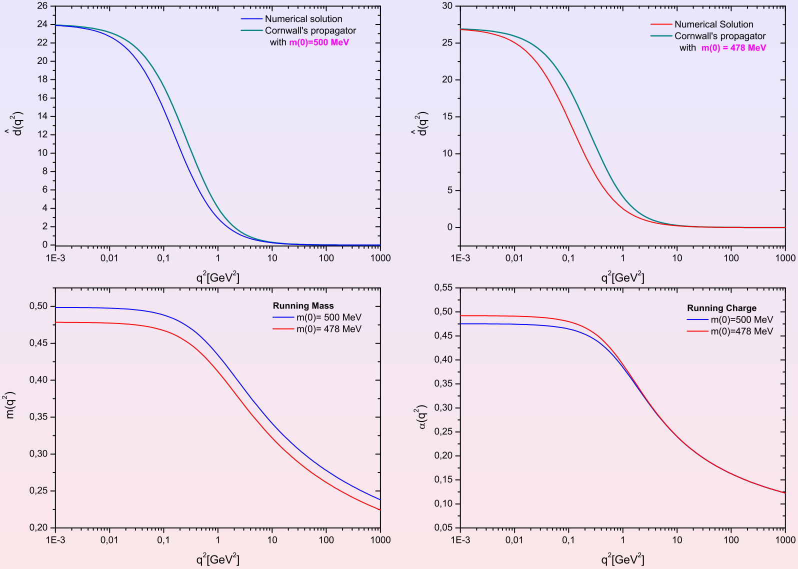
A. A. Natale , Braz. J. Phys. **37**, 306 (2007).
E. G. S. Luna and A. A. Natale , Phys. Rev. D **73**, 074019 (2006).
A. C. Aguilar, A. Mihara and A. A. Natale , Int. J. Mod. Phys. A **19**, 249 (2004).
S. Bar-Shalom, G. Eilam and Y. D. Yang , Phys. Rev. D **67**, 014007 (2003).
A.C.Aguilar, A.Mihara and A.A.Natale, Phys. Rev. D **65**, 054011 (2002).
F.Halzen, G.I.Krein and A.A.Natale, Phys. Rev. D **47**, 295 (1993).

$$m(0) = 500 \pm 200\,\text{MeV}$$

$$\alpha(0) = \frac{1}{4\pi b \ln\left(\frac{4m^2(0)}{\Lambda^2}\right)}$$

- Freezes at a finite value in the deep IR

$$\alpha(0) = 0.7 \pm 0.3$$


Numerical solution
Cornwall's propagator
with m(0)=500 MeV
d̂(q²)
q²[GeV²]
Numerical Solution
Cornwall's propagator
with m(0) = 478 MeV
d̂(q²)
q²[GeV²]
Running Mass
m(0)= 500 MeV
m(0)= 478 MeV
m(q²)
q²[GeV²]
Running Charge
m(0)=500 MeV
m(0)=478 MeV
α(q²)
q²[GeV²]

Conclusions

- Self-consistent description of the non-perturbative QCD dynamics in terms of an IR finite gluon propagator appears to be within our reach.
- Gauge invariant truncation of SD equations furnishes reliable non-perturbative information and strengthens the synergy with the lattice community.
- Meaningful contact with phenomenological studies