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**Separable quotient problem for spaces  $C(X)$  of continuous functions**

by

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Abstract

One of the famous unsolved problems of functional analysis asks (Mazur's problem (1932)) if every (infinite-dimensional) Banach space  $E$  has an (infinite-dimensional) separable quotient. Many concrete Banach spaces are known to have separable quotient, for example, reflexive Banach spaces, or even weakly compactly generated Banach spaces. Quite recently Agriros, Dodos and Kanellopoulos, proved that every dual Banach space has a separable quotient. On the other hand, V. Rosenthal (independently Lacey) showed that all Banach space  $C(X)$  of continuous (real-valued) functions on  $X$  have a separable quotient. We provide several useful methods to examine which Banach spaces admit a separable quotient. The talk gathers also quite new results due to T. Banach, J. Kakol and W. Sliwa concerning the separable quotient problem for spaces  $C_p(X)$  of continuous functions endowed with the pointwise topology. A connection with Efimov compact spaces  $X$  will be also discussed.