

Covariant Effective Action for LQC à la Palatini

Gonzalo J. Olmo

Instituto de Estructura de la Materia - CSIC (Spain)

G.O. and P.Singh, JCAP (2009)



About this talk . . .

- Modified theories of gravity of the $f(R)$ type have been thoroughly studied in the recent literature in connection with the **cosmic speedup** problem.

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- I will show here that **Palatini** $f(R)$ theories can also be used to address issues of the very early Universe such as the **Big Bang singularity**.

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- Structure of the talk:
 - ◆ Introduction to **LQG**, **LQC**, and the Big Bounce.
 - ◆ Advantages of a covariant action for **LQC**.
 - ◆ Finding a covariant action for **LQC**: do it yourself.

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- LQG and LQC
- Covariant Action for LQC
- Effective LQC Dynamics
- Palatini $f(R)$ theories
- Finding the CEA
- Numerics and Fits
- Summary and Conclusions

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LQG, LQC and Palatini $f(R)$ gravity



LQG and LQC

- **LQG** is a **non-perturbative** canonical quantization of GR in which the fundamental variables are triads and holonomies.

(which contrasts with the Wheeler-DeWitt program based on 3D-metrics and their momenta)

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- The main successes of loop quantum gravity are:
 - ◆ It replaces the classical notion of a smooth diff. geometry by a **discrete quantum geometry** with quantized area and volume operators.
 - ◆ It provides a microscopic calculation of the **entropy of black holes**.

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 - ◆ It provides a microscopic calculation of the **entropy of black holes**.
- **LQC** is a symmetry-reduced homogeneous and isotropic model based on **LQG** in which the **Big Bang singularity** is replaced by a **quantum bounce**.
 - ◆ The fundamental description in **LQC** is discrete.
 - ◆ It admits an effective continuum spacetime description which successfully captures the quantum gravity effects at high energies and becomes classical at low energies.

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Covariant Action for LQC

- Why do we need a Covariant Effective Action for LQC?
 - ◆ If a CEA did not exist, it would be a bad symptom for LQC.

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 - ◆ One could test the resulting CEA in black hole spacetimes and other regimes.

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 - ◆ One could test the resulting CEA in black hole spacetimes and other regimes.

- This problem will also force us to use our knowledge on modified theories of gravity to reproduce an explicitly given set of dynamical equations.
 - ◆ Is the LQC dynamics of scalar-tensor type?
 - ◆ Can it be written as an $f(R)$ theory?
 - ◆ Is it something more complicated than these candidates?

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Effective LQC Dynamics

- Using coherent state techniques, from the fundamental difference equations one can find the following effective o.d.e. :

$$H^2 = \frac{8\pi G}{3} \rho \left(1 - \frac{\rho}{\rho_{crit}} \right), \text{ with } \rho_{crit} = 0.41 \rho_{Planck}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho \left(1 - 4 \frac{\rho}{\rho_{crit}} \right) - 4\pi G P \left(1 - 2 \frac{\rho}{\rho_{crit}} \right)$$

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- **No new degrees of freedom.** Matter alone can cure the singularity.

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- **No new degrees of freedom.** Matter alone can cure the singularity.
- **There is a problem:** key new insights needed to find an effective action.

Requiring second-order equations and covariance one is uniquely led to the Einstein-Hilbert lagrangian density (modulo a cosmological constant) and hence to the Einstein field equations.

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- Any $f(R, R_{\mu\nu}R^{\mu\nu}, \dots)$ action in **metric formalism** and any scalar-tensor theory introduce additional degrees of freedom, not present in **LQC**.
- Palatini $f(R)$ theories have the same number of d.o.f. as GR and **LQC**: they seem a natural candidate to produce an effective action.

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Palatini $f(R)$ theories

- Action and field equations of Palatini $f(R)$ theories:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m(g_{\mu\nu}, \Psi) , \text{ where } (g_{\mu\nu}, \Gamma_{\beta\gamma}^\alpha) \text{ are independent.}$$

$$f_R R_{\mu\nu}(\Gamma) - \frac{1}{2} g_{\mu\nu} f(R) = \kappa^2 T_{\mu\nu} , \text{ where } f_R \equiv df/dR.$$

$$\nabla_\alpha \left(\sqrt{-g} f_R g^{\beta\gamma} \right) = 0 \Rightarrow \Gamma_{\beta\gamma}^\alpha = \frac{t^{\alpha\rho}}{2} \left[\partial_\beta t_{\rho\gamma} + \partial_\gamma t_{\rho\beta} - \partial_\rho t_{\beta\gamma} \right] , \text{ where}$$

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- Trace Equation: $R f_R - 2f = \kappa^2 T \Rightarrow R = \mathcal{R}(T)$

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- Resulting equations for the metric $g_{\mu\nu}$:

$$G_{\mu\nu}(g) = \frac{\kappa^2}{f_R} T_{\mu\nu} - \frac{\mathcal{R} f_R - f}{2f_R} g_{\mu\nu} - \frac{3}{2f_R^2} \left(\partial_\mu f_R \partial_\nu f_R - \frac{1}{2} g_{\mu\nu} (\partial f_R)^2 \right) + \frac{1}{f_R} \left(\nabla_\mu \nabla_\nu f_R - g_{\mu\nu} \square f_R \right)$$

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- Palatini $f(R)$ looks like GR with a modified source !!!

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Finding the CEA

- For a massless scalar, the Hubble function is given by

- ◆ In LQC: $3H^2 = 8\pi G\rho \left(1 - \frac{\rho}{\rho_{crit}}\right)$, with $\rho_{crit} = 0.41\rho_{Planck}$.

- ◆ In Palatini $f(R)$: $3H^2 = \frac{f_R(\kappa^2\rho + (\mathcal{R}f_R - f)/2)}{\left(f_R - \frac{12\kappa^2\rho f_{RR}}{2(\mathcal{R}f_{RR} - f_R)}\right)^2}$.

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- Equating the R.H.S. of these equations: $8\pi G\rho \left(1 - \frac{\rho}{\rho_{crit}}\right) = \frac{f_R(\kappa^2\rho + (\mathcal{R}f_R - f)/2)}{\left(f_R - \frac{12\kappa^2\rho f_{RR}}{2(\mathcal{R}f_{RR} - f_R)}\right)^2}$

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- We find the following o.d.e.: $f_{RR} = -f_R \left(\frac{Af_R - B}{2(\mathcal{R}f_R - 3f)A + \mathcal{R}B} \right)$,

where $A = \sqrt{2(\mathcal{R}f_R - 2f)(2\mathcal{R}_c - [\mathcal{R}f_R - 2f])}$,

$$B = 2\sqrt{\mathcal{R}_c f_R (2\mathcal{R}_c f_R - 3f)}, \text{ and } \mathcal{R}_c \equiv \kappa^2 \rho_c.$$

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- The trace Equation, $Rf_R - 2f = 2\kappa^2\rho$, implies $\Rightarrow \rho = \rho(\mathcal{R})$

- Equating the R.H.S. of these equations: $8\pi G\rho \left(1 - \frac{\rho}{\rho_{crit}}\right) = \frac{f_R(\kappa^2\rho + (\mathcal{R}f_R - f)/2)}{\left(f_R - \frac{12\kappa^2\rho f_{RR}}{2(\mathcal{R}f_{RR} - f_R)}\right)^2}$

- We find the following o.d.e.: $f_{RR} = -f_R \left(\frac{Af_R - B}{2(\mathcal{R}f_R - 3f)A + \mathcal{R}B} \right)$,

where $A = \sqrt{2(\mathcal{R}f_R - 2f)(2\mathcal{R}_c - [\mathcal{R}f_R - 2f])}$,

$B = 2\sqrt{\mathcal{R}_c f_R (2\mathcal{R}_c f_R - 3f)}$, and $\mathcal{R}_c \equiv \kappa^2 \rho_c$.

- There is a **unique solution** with $f_R \rightarrow 1$ when $R \rightarrow 0$ satisfying $\ddot{a}_{LQC} = \ddot{a}_{Pal}$ at $\rho = \rho_c$.

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LQG LQC and Palatini

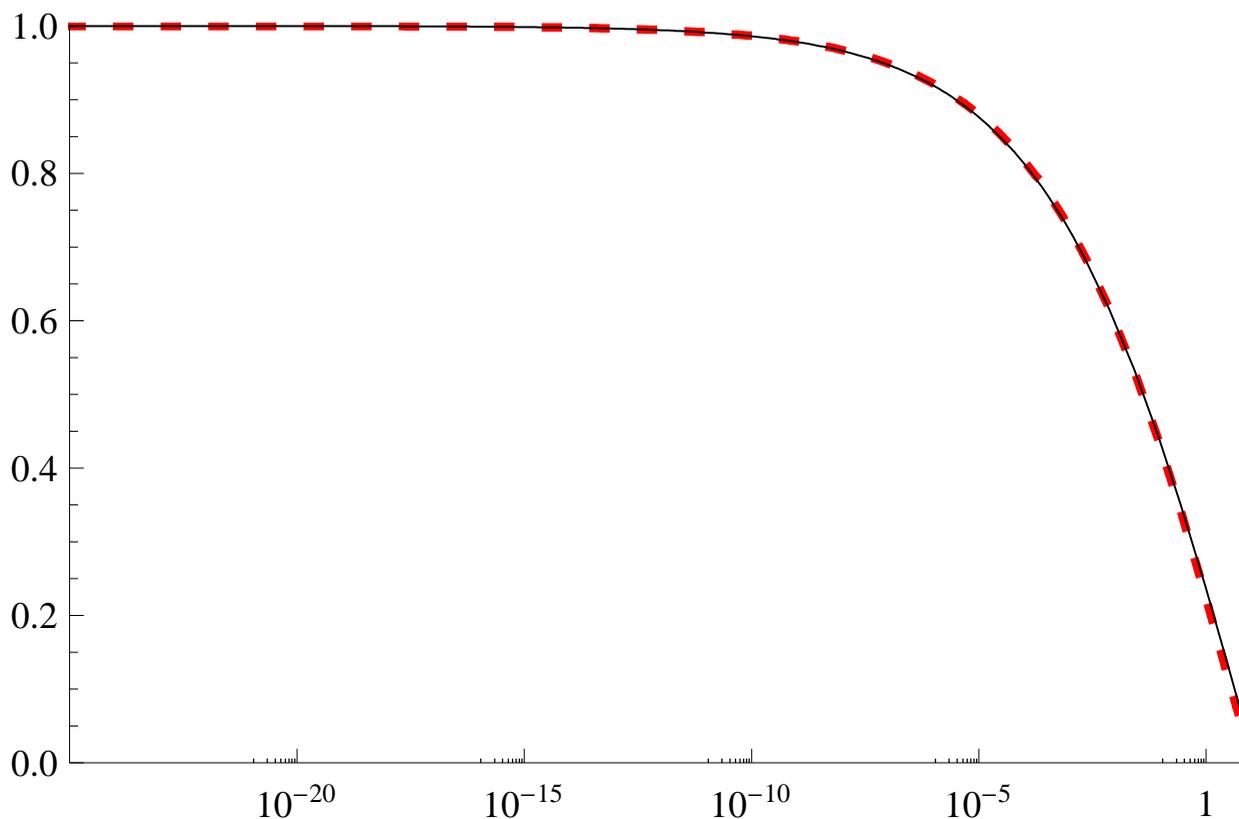
- LQG and LQC
- Covariant Action for LQC
- Effective LQC Dynamics
- Palatini $f(R)$ theories
- Finding the CEA
- Numerics and Fits
- Summary and Conclusions

The End

Numerics and Functional Fits

■ Dashed red line: Numerical Curve.

■ Solid line: $\frac{df}{dR} = -\tanh\left(\frac{5}{103} \ln\left[\left(\frac{R}{12R_c}\right)^2\right]\right)$



Vertical axis: df/dR ; Horizontal axis: R/R_c

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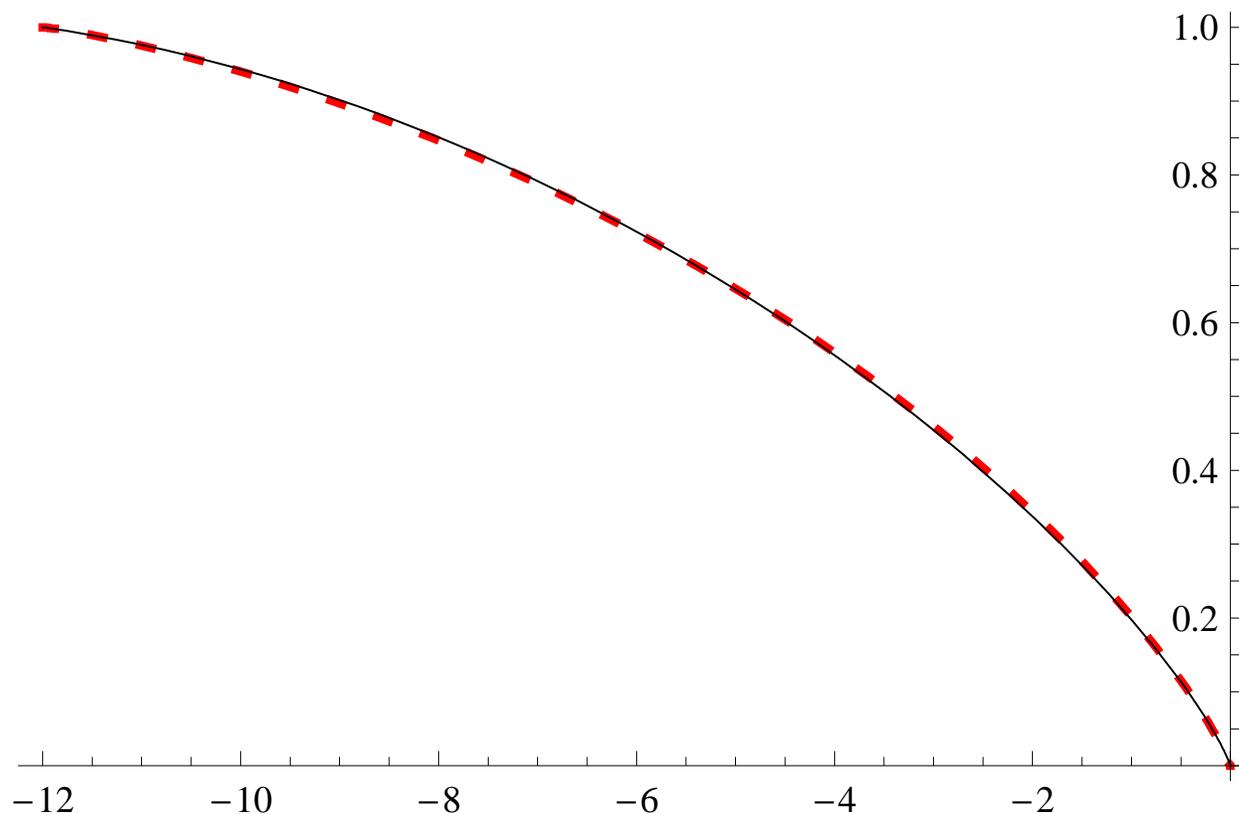
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Numerics and Functional Fits

■ Dashed red line: Numerical Curve.

■ Solid line: $2\kappa^2\rho = Rf_R - 2f$ with $\frac{df}{dR} = -\tanh\left(\frac{5}{103}\ln\left[\left(\frac{R}{12R_c}\right)^2\right]\right)$



Vertical axis: ρ ; Horizontal axis: R/R_c

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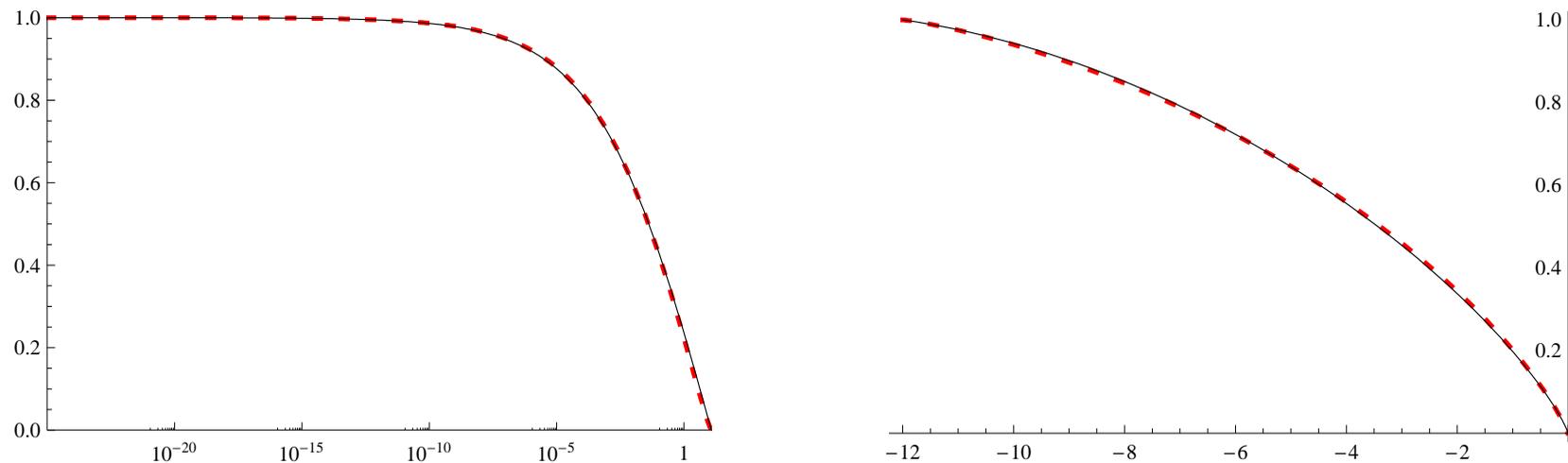
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Numerics and Functional Fits

■ Dashed red line: Numerical Curve.

■ Solid line: functional fits.



■ This $f(R)$ lagrangian exactly reproduces the dynamics of isotropic LQC.

■ The cosmic bounce occurs at $R = -12R_c$, where $f_R \rightarrow 0$.

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Summary and Conclusions

- The dynamics of isotropic LQC can be derived from a covariant action.

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Summary and Conclusions

- The dynamics of isotropic LQC can be derived from a covariant action.
- We have found a unique Palatini $f(R)$ lagrangian which exactly reproduces its dynamics.

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Summary and Conclusions

- The dynamics of isotropic LQC can be derived from a covariant action.
- We have found a unique Palatini $f(R)$ lagrangian which exactly reproduces its dynamics.
- At low curvatures the lagrangian is almost linear, but near the bounce the modified dynamics is non-trivial. The lagrangian,

$$f(R) = - \int dR \tanh \left(\frac{5}{103} \ln \left[\left(\frac{R}{12\mathcal{R}_c} \right)^2 \right] \right),$$

requires an infinite series in R to capture the full non-perturbative dynamics.

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Summary and Conclusions

- The dynamics of isotropic LQC can be derived from a covariant action.
- We have found a unique Palatini $f(R)$ lagrangian which exactly reproduces its dynamics.
- At low curvatures the lagrangian is almost linear, but near the bounce the modified dynamics is non-trivial. The lagrangian,
$$f(R) = - \int dR \tanh \left(\frac{5}{103} \ln \left[\left(\frac{R}{12\mathcal{R}_c} \right)^2 \right] \right),$$
requires an infinite series in R to capture the full non-perturbative dynamics.
- Our results provide new insights on the kind of fields that an action must contain to capture non-perturbative quantum gravity effects:

Unlike in the classical spacetime of GR, the metric might not be the sole fundamental geometric entity, which shares similarities with the effective continuum geometry of crystals.

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Thanks !!!