THE NATURE OF THE QUANTITIES IN CONDITIONAL PROBABILITY PROBLEMS. ITS INFLUENCE ON PROBLEM SOLVING BEHAVIOUR

M. Pedro Huerta, *Universitat de València, Spain*
Mª Ángeles Lonjedo, *IES Montserrat, Spain*

Abstract: *In order to solve verbal conditional probability problems, students are involved in a process in which we can identify several steps or phases. One of them is that of the translation from the text of the problem, generally written in everyday language, to that of mathematics. In translating sentences, students should recognize events and probabilities. But, in much of those problems data are not explicitly mentioned in terms of probability. In this case, students can solve these problems with the help of arithmetical thinking and not necessarily with the help of probabilistic reasoning. Other works (Ojeda 1996, Huerta-Lonjedo 2003) already referred to that but not adequately. In this piece of work we investigate, through an exploratory study of 166 students from different school levels, the extent to which the nature of quantities in conditional probability influences the way in which students solve these problems.*

Introduction

We use the term problem in a Puig (1996) sense, that is, any problematic situation in a school context. Probability problems are problems in which the question is about the probability of an event and conditional probability problems involve at least one conditional probability, either as data and question or both. In this report we consider conditional probability problems written in everyday language.

In addition to the nature of data, there are, as we know, some others factors that also have an influence on problem solving of conditional probability tasks. One of these factors is not necessarily previous knowledge about relationships between probabilities but, for example, the identification of the events and their probabilities. The prior identification of events and the corresponding assignment of their probabilities have to do with semiotic and semantic aspects as well as the right correlation between data and events. In this piece of work we are not going to deal with this issue, instead we investigate the nature of quantities in problems, and its influence on the problem solving process.

When data in conditional probability problems are expressed in terms of frequencies, percentages or rates, students do not necessarily interpret them as probabilities. Consequently, relationships between probabilities are not used when students are solving problems, at least in an explicit way. However, this does not mean that no student can solve these problems. Of course, there are students that succeed in solving, but they mainly use arithmetic thinking and not probabilistic thinking. It is only at the end of the problem solving process that students answer the question in terms of a required probability, usually by assignment methods. In this paper, we will show the results of an exploratory study with 166 students from different school levels solving conditional probability problems, in which the type of data is varied systematically so that its impact on solving strategies of individuals may be studied.

Nature of quantities in conditional probability problems

From an investigation of conditional probability problems in textbooks we noticed that data involved are not always expressed in terms of probabilities. Previous studies (Ojeda 1996; Huerta, Lonjedo 2003; Lonjedo, 2003) show that problems could be solved just by using numeric thinking. We, too often observed students using arithmetic instead of probabilistic thinking when solving conditional probability problems. This is because data are not being interpreted consciously as probabilities and consequently, students do not need to use relationships between probabilities to solve the problem. It is only at the end of the problem solving process that students try to express their answer in terms of the required probability and assign a probability to their arithmetic solution. We will term this strategy as "solving by numeric assignment". On the other hand, we will use "solving by probability calculations" to denote the strategy to use probability relations to derive at a solution (see next example)

According to these different strategies, we will subsequently classify probability problems into assignment and calculation problems.For a conditional probability problem this means that it will be classified as assignment problem if the quantities involved are presented as frequencies or percentages, and it will be classified as calculation problem if data involved in the problem are given as probabilities and, consequently relationships between probabilities are needed in order to answer the posed question.

Nevertheless, teachers and textbooks usually present problem situations asking for this type of probabilistic thinking, which is by no means really requested by the posed task. The following example illustrates matters.

Two machines A and B produce respectively 100 and 200 pieces. It is known that machine A produces 5% faulty pieces and machine B produces 6% faulty pieces. If you took a piece at random calculate: a) The probability that this piece would be faulty. b) Knowing that the piece is faulty, the probability that it is made by machine A.

Dos máquinas A y B han producido respectivamente, 100 y 200 piezas. Se sabe que A produce un 5% de piezas defectuosas y B un 6%. Se toma una pieza y se pide: a) Probabilidad de que sea defectuosa. b) Sabiendo que es defectuosa, probabilidad de que proceda de la primera máquina (Cuadras 1983, p. 55).

Solución

Indiquemos por: M_A = {la pieza procede de la máquina A}

M_B = {la pieza procede de la máquina B}

Entonces,

$$\Omega = \{300 \text{ piezas}\} = M_A + M_B$$

$$P(M_A) = \frac{1}{3} \qquad P(M_B) = \frac{2}{3}$$

1) Sea D = {la pieza es defectuosa}

$$P(D) = P(D/M_A)\cdot P(M_A) + P(D/M_B)\cdot P(M_B) = (0.05)\cdot\frac{1}{3} + (0.06)\cdot\frac{2}{3} = 0.0567$$

2) Es la probabilidad de M_A, condicionada a la presencia de D

$$P(M_A/D) = \frac{P(D/M_A)\cdot P(M_A)}{P(D/M_A)\cdot P(M_A) + P(D/M_B)\cdot P(M_B)} = \frac{(0.05)\cdot 1/3}{0.0567} = 0.2941$$

Fig. 1. Solution of the schoolbook

The solution of this problem can be seen in Figure 1. The textbook considers this as a problem of calculation, consistent with its placement in the unit of "Total Probability and Bayes' Theorem". However, some students (Lonjedo, 2003) solved problems similar to that both in data nature and in data structure by the numeric assignment strategy – according to the nature of presented data:

If we have 100 pieces from machine A and 5% are faulty, in 100 pieces we have 5 faulty ones. If we have 200 pieces from machine B and 6% are faulty, in 200 pieces we have 12 faulty ones. In total, among 300 pieces we have 17 (5+12) faulty pieces. So, if we take a piece at random, the probability that it will be faulty is 17 out of 300 or 17/300 or $0.05\dot{6}$. We have 17 faulty pieces, 5 of them are from machine A and 12 from machine B. If we know that the piece is faulty, the probability it is made by machine A is 5 out of 17 or 5/17 or 0.2941.

Si tenemos 100 piezas de la máquina A y el 5% son defectuosas: tenemos 5 piezas defectuosas de las 100 de A. Si tenemos 200 piezas de la máquina B y el 6% son defectuosas: tenemos 12 piezas defectuosas de las 200 de B. En total, de 300 piezas de las dos máquinas, 5+12=17 son defectuosas. Luego la probabilidad de ser defectuosa es: 17 de 300 o $0.05\dot{6}$. Para la segunda cuestión, tenemos 17 piezas defectuosas, de donde 5 vienen de la máquina primera, luego la probabilidad pedida es de: 5 de 17 o 0.2941.

Different nature of data

Conditional probability problems may be classified according to the nature of the data presented in the formulation of the problem. We distinguish the following types:

Data expressed in probability terms

If quantities are expressed in terms of probability, they quantify the probability of a certain event A by a number $p(A) \in [0,1]$, as in the following example:

Complete the next contingency table. From this table build a tree diagram and calculate p (B|A), p (noB|A), p (B|noA) and p (noB|noA) - p(noB|A) means $p(\overline{B} \mid A)$.

	A	noA	Total
B	0.4	0.2	

Completa la següent taula de contingència. A partir de la taula, confecciona un diagrama d'arbre i determina P(B/A), P(noB/A), P(B/noA) i P(noB/noA) (Matemáticas 4t ESO, p. 240).

noB	0.25		
Total			1

Here, the solution is derived by relationships between probabilities, namely by: $p(A \mid B)=p(A \cap B)/p(B)$, that is, only by using probabilistic calculations and thinking.

Data expressed in absolute frequency terms

When in a conditional probability problem data are expressed in terms of absolute frequencies, they express the frequency of the objects that satisfy certain characteristics. From a mathematical point of view, frequency can be seen as a cardinal number associated to the set that represents these objects. Consequently, the quantity in a problem presented as frequencies has to be used with that meaning. Thus, p (A|B) is obtained by comparing two numbers: $p(A|B) = n(A \cap B)/n(B)$.

On the other hand, because A|B is not an event, we cannot consider a set that represents it. So, data referring to a conditional probability could never be expressed in terms of absolute frequencies. If we do so, the only meaning that one can associate to such data will be that of a cardinal number associated to an intersection event. The following example illustrates matters:

An intelligence test was administrated to a group of 500 students to assess their academic performance. The results of that test were as follows (contingency table). Let A be "having higher intelligence test" and B "having a higher academic performance" Answer the questions: a) Are A and B independent events? b) If we randomly choose a student with higher performance at school, what is the probability that he/she has higher intelligence?

En un grupo de 500 individuos se pasó un test de inteligencia y se midió su rendimiento académico. Los resultados fueron como sigue (tabla de contingencia). Considerando que A es "ser superior en inteligencia" y B es "tener rendimiento alto", averiguar: a) Si A y B son independientes. b) Si se selecciona al azar un alumno con rendimiento alto, ¿cuál es la probabilidad de que sea superior en inteligencia? (Santos Serrano, 1988, p. 248)

	Rendimiento académico	
Inteli-gencia	Alto	Bajo
Superior	200	80
Inferior	100	120

Data expressed in terms of rates

When quantities are shown in terms of rates, data are implicitly expressed in terms of probability and it is up to the solver to decide whether to translate the rates to probabilities or not. Here, we have two examples. The first one shows two rates as data and the second one shows data in percentages:

In Sikinie one man out of 12 and one woman out of 288 are affected with Daltonism. The frequencies of men and women are the same. A person is chosen at random and it is known that he/she is affected with Daltonism. What is the probability that this person is a man?

En Sikinie, un homme sur 12 et une femme sur 288 sont daltoniens. Les fréquences des deux sexes sont égales. On choisit une personne au hasard et on découvre qu'elle est daltonienne. Quelle est la probabilité pour que ce sois un homme? (Engel 1975, p. 270)

Experience shows that during the process of making circuits for radio transistors 5% of them faulty. A device that is used to find out which are faulty, detects 90% of the faulty ones, but also qualifies 2% of faulty circuits as correct. What is the probability that a circuit is correct if the device says that it is faulty? What is the probability that a faulty circuit is qualified as correct?

En el proceso de fabricación de circuitos impresos para radio transistores se obtiene, según demuestra la experiencia de cierto fabricante, un 5% de circuitos defectuosos. Un dispositivo para comprobar los defectuosos detecta el 90% de ellos, pero también califica como defectuosos al 2% de los correctos. ¿Cuál es la probabilidad de que sea correcto un circuito al que el dispositivo califica como defectuoso?¿Cuál es la probabilidad de que sea defectuoso un circuito calificado de correcto? (Grupo Cero 1982, p. 170)

Data expressed in combined terms

There are certain conditional probability problems in textbooks where not all data are expressed by the same type, like in the examples above, but by combining more than one.We find data presented both in terms of probability and percentages, probability and rates, or rates and frequencies. In the following problem, for example, data as percentages combines more than one sense.

In a high school class, the percentage of students that succeeded in History (A) was 60%. In Mathematics (B) it was 55%. Knowing p(A|B)=70%, what is the probability that a student chosen at random did not succeed in either topic?

En un curso el porcentaje de aprobados en Historia (A) es 60 %. Para Matemáticas (B) es del 55 %. Sabiendo que p(B/A) = 70 %, ¿cuál es la probabilidad de que, escogido al azar un alumno, resulte no haber aprobado ninguna de las dos asignaturas? (Santos Serrano, 1988, p. 248)

The examples show how data in conditional probability problems are not always expressed in probability terms or with the same type. When this occurs, the solver should have the ability to interpret them according to or different from the desired meaning in the problem. Depending on how the solver actually interprets the data the solving process may imply either numerical or probabilistic thinking.

The empirical study

One of the objectives of our study was to explore how students solved conditional probability problems when data in problems satisfied specific criteria with resprect to their structure and nature. Mainly, we were interested in exploring what kind of thinking –arithmetical or probabilistic– students used in solving the problems, in relation to the structure and nature of the presented data.

The test

We prepared a collection of sixteen conditional probability problems with similar data structure, varying the nature of the presented data and the context. All problems had three pieces of data explicitly mentioned in their formulation. For each constellation of context, we designed a pair of problems, one contained the quantities in terms of percentages, the sibling in probability terms.For example, in problem 1, one can interpret data as follows

$p(A\cap B)=30\%$, $p(A\cap \overline{B})=30\%$ and $p(B\mid \overline{A})=40\%$

In its isomorphic problem, no. 9, data are explicitly mentioned as probabilities

$p(A\cap B)=0.3$, $p(A\cap \overline{B})=0.3$, $p(B\mid \overline{A})=0.4$.

Both problems, however, share the same question —calculate p(B)— and the same context. The collection of problems contained problems from 1 to 8 with data in terms of percentages and problems 9 to 16, with data explicitly expressed in terms of probability:1-9, 2-10, 3-11, 4-12, 5-13, 6-14, 7-15, and 8-16; see the appendix. Only a Spanish version is given as we think that a translation cannot fully preserve meaning, and semantic and semiotic factors are influential for the problem perception.

Considering time limitations, we asked each student to solve a total of four problems - two from the ”frequency type problems and two with probability format of the presented quantities.

The students

The test was administered during student's regular class time. The sample of students that took part in the study was a total of 166 students distributed as follows as follows over school levels and ages:

School	U-FM	HS2-TS	HS2-SS	HS1-TS	HS1-SS	CS4
Age	20	17-18		16-17		15-16
#	10	38	16	38	37	27

U-FM: University students at the Math's College, studying "Didactics of Mathematics"
HS2-TS, SS: 2nd year high school students specializing in TS – technical subjects, or SS – social sciences, which means different competence at mathematics
HS1-TS, SS: 1st year high school students
CS4: 4th year compulsory students

Analysis of results

The results in the tables below are organized according to the following variables: nature of data, number of students who attempted to solve each problem; number of students who succeeded in solving each problem including its distribution over the various school levels; the number of students that did not answer the specific problem; and depending on the reasoning used in problem solving, the distribution of the number of students that succeeded with probability assignment or probability calculation strategies.

Tables 1 and 2 display the number of students who succeeded in solving and also the number of students who did not attempt to solve the problems or similar - expressed as blanks. Information about students who did not complete successfully the problems is not reported here. In the process of solving we could observe mistakes and misunderstandings of different nature.

	Counts	Frequencies type problems								Probabilistic type problems							
	Problem	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16
	Sample	34	33	33	34	66	33	67	34	34	33	33	34	33	33	33	33
	Succeeded	4	6	1	8	20	2	4	0	0	2	0	6	3	2	2	0
Level	UFM	1	0	0	2	1	2	4	0	0	1	0	2	1	1	2	0
	HS2-TS	0	3	0	2	6	0	0	0	0	1	0	1	2	0	0	0
	HS2-SS	0	0	0	3	2	0	0	0	0	0	0	1	0	0	0	0
	HS1-TS	1	1	0	0	8	0	0	0	0	0	0	0	0	1	0	0
	HS2-SS	2	2	1	1	3	0	0	0	0	0	0	1	0	0	0	0
	CS4	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	Blank	5	11	4	2	11	4	12	9	12	4	13	12	11	6	18	19
Strategy	Assignment	4	6	1	7	20	0	1	0	0	0	0	1	0	1	0	0
	Calculation	0	0	0	1*	0	2	3	0	0	2	0	5	3	1*	2	0

Table 1: Number of students succeeded in solving by school level and type of reasoning used – * UFM

	%	Frequencies type problems								Probabilistic type problems							
	Problem	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16
	Succeeded	12	18	3	24	30	6	6	0	6	0	18	9	6	6	0	0
	Blank	15	33	12	6	16.6	12	18	27	12	40	35	33	18	55	58	19
Strategy	Assignment	100	100	100	87	100	0	25	0	0	0	16.7	0	50	0	0	0
	Calculation	0	0	0	13*	0	100	75	0	100	0	83.3	100	50*	100	0	0

Table 2 Percentages of students succeeded, blank and, type of thinking of successful students – * UFM

Tables 1 and 2 display the same information, with Table 1 in absolute frequencies while Table 2 gives the results in percentages. It should be noted that for problems 9 to 16, where data are expressed in terms of probability, the percentages of correct solutions is lower than for the first 8 ones. On the other hand, we would like to point out that when data are shown in terms of probability, the number of students trying to solve the problem is much smaller[1] compared to frequency type problems;with the

[1] According to the official curriculum of students for 1st and 2nd year at high school studying social science-humanities option, some conditional probability knowledge is provided. However, this does not always happen. We cannot assure

exception of P2 where the high percentage of "blanks" stands out. This confirms that nature of data is an influential factor in the problem's solution processes. The summary table below gathers those columns of Table 2 that give evidence of the solution process of the problems, the nature of their data and those that have been successfully completed:

	%	Frequencies type problems								Probabilistic type problems							
	Problem	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16
Strategy	Assignment	100	100	100	87	100	0	25	0	0	0	16.7	0	50	0	0	0
	Calculation	0	0	0	13*	0	100	75	0	100	0	83.3	100	50*	100	0	0

Table 3: Percentages of the types of solutions for those problems successfully completed – * UFM

One of the features to be seen from the Table 3 is that in some cases it contradicts the assertion that data expressed in percentages favours the solution of the problem by assignment. We can see, for example, that in problems 6 and 7; the majority of those students that successful solved the problems used probability calculations. However, all the students in this sample belong to the Math's College, who have more education in the theory of probability.

We can also notice that in some cases, when data are expressed in terms of probability, some students translate those terms into percentages and solve the problem by using arithmetic thinking and assigning probability at the end of the solution process. This is the case in problem 12. One student from the HS1-SS group translated data expressed in terms of probability into percentages. When solving problem 14, another student, from the HS1-TS group, used the same process of translation.

The eight problems with data presented in percentages and the number of students that succeeded in solving them can be seen from Table 4: The results of problem 8 are coherent with the enunciation of the problem and the competence level of the students. The two students from the Math's College, theoretically provided with a good knowledge of the theory of probability, tried to solve it by using wrong formulas. The high percentage of answers left "blank" is basically due to the way these data are presented: p (B|A)=70%.

that students belonging to these courses were knowledgeable in the subject. Consequently, problems with data expressed in terms of probability are not attempted to be solved very frequently.

%	Frequencies type problems							
Problem	P1	P2	P3	P4	P5	P6	P7	P8
Succeeded	12	18	3	24	30	6	6	0
Blank	15	33	12	6	16.6	12	18	27

Table 4: Percentages of problems that give evidence of the success or failure of the solution.

With reference to problem 2, also with a very high percentage left "blank"– 33% – correspond to 8 blank answers and 3 with an unfinished solution. We do not understand the reason why this happens because the enunciation is similar to problems 1 and 3, where the percentages with a blank response are not so relevant. Moreover, if we observe the results of its isomorphic problem no 9, there are only 12% of left blanks.

Conclusions

We suspect that apart from the nature of the data, there are some other factors that have a direct effect on the way students approach conditional probability problems. These factors are not necessarily related to knowledge of relationships between probabilities. The nature of probability problems in textbooks is influential in the classification of the problems: problems of probability assignment or problems of probability calculation. The problems successfully solved by the students who took part in this research could be classified as problems of probability assignment. Most of these students did not understand data as probabilities and consequently would never use the relations between probabilities to calculate the probabilities requested in the problems. Data in these problems are presented in percentages and students solve them by using numerical thinking and final assignment of a probability. However, a few students attempted to solve problems with data presented in terms of probability. In this case, the students mostly approached the problems by using probability calculation rules.

Numerical data shown in selected probability problems, acquire some meaning to the students when they are expressed in terms of percentages. When quantities have specific meaning for students, they can produce new quantities that can also be relevant for the solution of the problem, thus facilitating the problem solving process. However, when quantities are expressed in terms of probabilities they do not make feasible that production of new quantities, mainly if one is not competent with relationships between probabilities or with formulas. Consequently, we will not be saying anything new (Ojeda, 1996) if we continue believing in the solution of probability problems where data suggest a focus on probability as a frequency before this is shown in a formal way. This applies not only to solving probability problems, but also to conditional probability. If conditional probability problems were focused in the way we suggested, it would allow their inclusion in arithmetic lessons or the use of rates and proportions for their solution, as a prior step to teaching rules or formulas.

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Appendix:

The test problems to the right are equivalent to the left ones in the same line with respect to context.

Frequency type problems	Probability type problems
P1: De todos los alumnos del instituto, un 30% practican baloncesto y fútbol y un 30% practican el baloncesto y no practican el fútbol. Sabemos que de los alumnos que no practican baloncesto un 40% hacen fútbol. Calcula la probabilidad de practicar fútbol.	P9: En un instituto, la probabilidad de practicar baloncesto y fútbol es 0'3 y la probabilidad de practicar el baloncesto y no practicar el fútbol es 0'3. Sabemos que la probabilidad de que elegido un alumno de los que no practica baloncesto, éste practique fútbol es 0'4. Calcula la probabilidad de practicar fútbol.
P2: Un 30% de los huéspedes de un hotel practican el tenis y el golf y un 30% practican el tenis y no practican el golf. Además conocemos que de los huéspedes que no practican tenis un 40% practican golf. Calcula la probabilidad de que elegido un huésped al azar no practique ni tenis ni golf	P10: En un hotel, la probabilidad de que elegido un huésped al azar éste practique el tenis y el golf es 0'3 y la probabilidad de que practique el tenis y no practique el golf es 0'3. Además conocemos que la probabilidad de que elegido un huésped de los que no practican tenis éste practique golf es 0'4. Calcula la probabilidad de que elegido un huésped al azar no practique ni tenis ni golf.
P3: En una academia de idiomas un 30% de los alumnos estudian inglés y francés y un 30% estudian inglés y no estudian francés. Además, de los alumnos que no estudian inglés, un 40% estudian francés. Calcula la probabilidad de que estudie inglés elegido un alumno que estudia francés.	P11: En una academia de idiomas, elegido un estudiante al azar la probabilidad de que estudie inglés y francés es 0'3 y de que estudie inglés y no estudie francés es 0'3. Además, elegido un alumno de los que no estudian inglés, la probabilidad de que estudie francés es de 0'4. Calcula la probabilidad de que estudie inglés elegido un alumno que estudia francés.
P4: En una empresa el 55% de los trabajadores son mujeres. De las mujeres, el 20% se dedican a las tareas administrativas, y de todos los trabajadores, el 11'25% son hombres y administrativos. Calcula la probabilidad de ser mujer y no realizar tareas administrativas	P12: De los trabajadores de una empresa, la probabilidad de ser mujer es de 0'55. De las mujeres, la probabilidad de dedicarse a las tareas administrativas es de 0'2, y elegido un trabajador al azar, la probabilidad de ser hombre y administrativo es 0'1125. Calcula la probabilidad de ser mujer y no realizar tareas administrativas.
P5: En una universidad el 55% de los estudiantes son mujeres. De éstas, el 20% estudian carreras de letras, y de todos los estudiantes, el 11'25% son hombres y estudian carreras de letras. Calcula la probabilidad de que elegido un estudiante al azar (hombre o mujer) estudie carrera de letras	P13: En una universidad, elegido un estudiante al azar, la probabilidad de que sea mujer es 0'55. De éstas, la probabilidad de que estudien carreras de letras es de 0'2, y elegido un estudiante al azar, la probabilidad de ser hombre y estudiar carrera de letras es de 0'1125. Calcula la probabilidad de que elegido un estudiante al azar (hombre o mujer) estudie carrera de letras.
P6: En un campamento de verano el 55% de los integrantes son niñas. De las niñas, el 20% realizan actividades acuáticas, y de todos los integrantes, el 11'25% son niños y realizan actividades acuáticas. Calcula la probabilidad de que eligiendo un integrante que realiza actividades acuáticas, éste sea niña.	P14: La probabilidad de que los integrantes de un campamento de verano sean niñas es de 0'55. De las niñas, la probabilidad de realizar actividades acuáticas es de 0'2, y elegido un integrante al azar, la probabilidad de ser niño y realizar actividades acuáticas es de 0'1125. Calcula la probabilidad de que eligiendo un integrante que realiza actividades acuáticas, éste sea niña.
P7: Un 60% de los alumnos de un colegio aprobaron filosofía y un 70% matemáticas. Además, un 80% de los alumnos que aprobaron matemáticas, aprobaron también filosofía. Si Juan aprobó filosofía, ¿qué probabilidad tiene de haber aprobado también matemáticas? (Grupo Erema, 2002, p. 26, adaptado para la prueba)	P15: En un colegio, la probabilidad de aprobar filosofía es de 0'6 y la de aprobar matemáticas es de 0'7. Además, elegido un alumno de los que aprobaron matemáticas, la probabilidad de que aprobara filosofía es de 0'8. Si Juan aprobó filosofía, ¿qué probabilidad tiene de haber aprobado también matemáticas?
P8: En un curso el porcentaje de aprobados en Historia (A) es 60 %. Para Matemáticas (B) es del 55 %. Sabiendo que $p(B/A) = 70$ %, ¿cuál es la probabilidad de que, escogido al azar un alumno, resulte no haber aprobado ninguna de las dos asignaturas? (Santos Serrano, 1988, p. 248, adaptado para la prueba)	P16: En un curso la probabilidad de aprobar Historia (A) es 0'6 y la de aprobar Matemáticas (B) es 0'5. Sabiendo que $p(B/A) = 0.7$, ¿cuál es la probabilidad de que, escogido al azar un alumno, resulte no haber aprobado ninguna de las dos asignaturas?