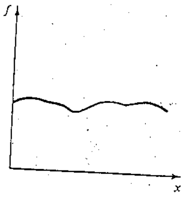
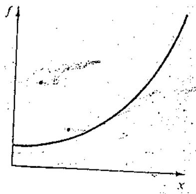
$$df = \left(\frac{df}{dx}\right) dx$$





$$dt = \left(\frac{\partial t}{\partial x}\right) dx + \left(\frac{\partial t}{\partial y}\right) dy + \left(\frac{\partial t}{\partial z}\right) dz.$$

$$\begin{aligned} dt &= \left(\frac{\partial t}{\partial x}\hat{i} + \frac{\partial t}{\partial y}\hat{j} + \frac{\partial t}{\partial z}\hat{k}\right) \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}) \\ &= (\nabla t) \cdot (d\mathbf{l}). \end{aligned}$$

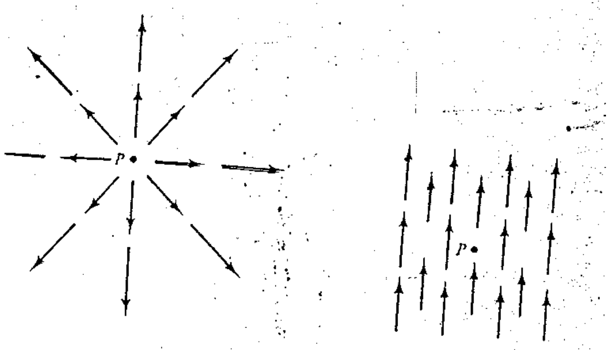
"The gradient ∇t points in the direction of maximum increase of the function t."

Moreover,

The magnitude $|\nabla t|$ gives the slope (rate of increase) along this maximal direction.

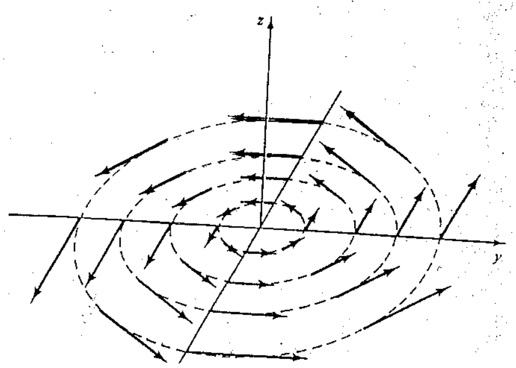
Example 3

$$\nabla \cdot \mathbf{v} = \left(\hat{\imath} \frac{\partial}{\partial x} + \hat{\jmath} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \cdot (\hat{\imath} v_x + \hat{\jmath} v_y + \hat{k} v_z)$$
$$= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}.$$



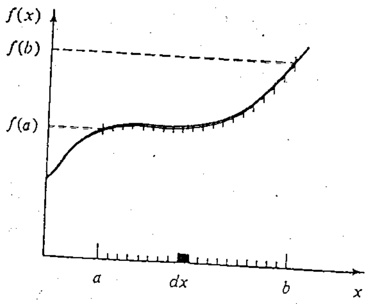
$$\nabla \times \mathbf{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

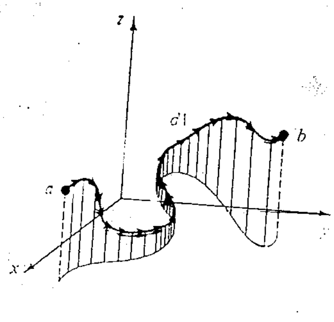
$$= \hat{\imath}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \hat{\jmath}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{k}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right).$$



$$\int_a^b \left(\frac{df}{dx}\right) dx = f(b) - f(a).$$

$$\int_a^b F(x)\, dx = f(b) - f(a),$$



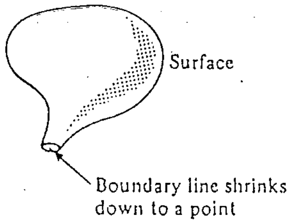


$$\int_a^b (\nabla t) \cdot dl = t(b) - t(a).$$

$$\int_{\text{volume}} (\nabla \cdot \mathbf{v})\, d\tau = \int_{\text{surface}} \mathbf{v} \cdot d\mathbf{a}.$$

$$\int (\text{all faucets in the volume}) = \int (\text{flow out through the surface}).$$

$$\int_{\text{surface}} (\nabla \times \mathbf{v}) \cdot d\mathbf{a} = \oint_{\substack{\text{boundary}\\ \text{line}}} \mathbf{v} \cdot d\mathbf{l}.$$



$$\int (\nabla \times \bar{v})\, d\bar{a} = \oint \bar{v}\, d\bar{l}$$