A. **Relaciones vectoriales.**

A.1. Vectores unitarios.

$\vec{u}_x, \vec{u}_y, \vec{u}_z$	–	coordenadas rectangulares	(constantes)
$\vec{u}_\rho, \vec{u}_\varphi, \vec{u}_z$	–	coordenadas cilíndricas	(no constantes, salvo $\vec{u}_z$)
$\vec{u}_r, \vec{u}_\theta, \vec{u}_\varphi$	–	coordenadas esféricas	(no constantes)

A.2. Transformaciones de coordenadas.

$$\begin{array}{l}
x = \rho\cos\varphi = r \text{ sen } \theta\cos\varphi \\
y = \rho \text{ sen } \varphi = r \text{ sen } \theta \text{ sen } \varphi \\
z = r\cos\theta \\
\rho = \sqrt{x^2+y^2} = r \text{ sen } \theta \\
\varphi = \tan^{-1}(y/x) \\
r = \sqrt{x^2+y^2+z^2} = \sqrt{\rho^2+z^2} \\
\theta = \tan^{-1}(\sqrt{x^2+y^2}/z) = \tan^{-1}(\rho/z)
\end{array}$$

A.3. Transformaciones de las componentes coordenadas.

$$\begin{array}{rcl}
A_x & = & A_\rho\cos\varphi - A_\varphi \text{ sen } \varphi \\
 & = & A_r \text{ sen } \theta\cos\varphi + A_\theta\cos\theta\cos\varphi - A_\varphi \text{ sen } \varphi \\
A_y & = & A_\rho \text{ sen } \varphi + A_\varphi\cos\varphi \\
 & = & A_r \text{ sen } \theta\sin\varphi + A_\theta\cos\theta \text{ sen } \varphi + A_\varphi\cos\varphi \\
A_z & = & A_r\cos\theta - A_\theta \text{ sen } \theta \\
A_\rho & = & A_x\cos\varphi + A_y \text{ sen } \varphi \\
 & = & A_r \text{ sen } \theta + A_\theta\cos\theta \\
A_\varphi & = & -A_x \text{ sen } \varphi + A_y\cos\varphi \\
A_r & = & A_x \text{ sen } \theta\cos\varphi + A_y \text{ sen } \theta \text{ sen } \varphi + A_z\cos\theta \\
 & = & A_\rho \text{ sen } \theta + A_z\cos\theta \\
A_\theta & = & A_x\cos\theta\cos\varphi + A_y\cos\theta \text{ sen } \varphi - A_z \text{ sen } \theta \\
 & = & A_\rho\cos\theta - A_z \text{ sen } \theta
\end{array}$$

A.4. Elementos diferenciales de longitud.

$$d\vec{l} = \begin{cases} \vec{u}_x dx + \vec{u}_y dy + \vec{u}_z dz \\ \vec{u}_\rho d\rho + \vec{u}_\varphi \rho d\varphi + \vec{u}_z dz \\ \vec{u}_r dr + \vec{u}_\theta r d\theta + \vec{u}_\varphi r \text{ sen } \theta d\varphi \end{cases}$$

A.5. Elementos diferenciales de superficie.

$$d\vec{s} = \begin{cases} \vec{u}_x dy\, dz + \vec{u}_y dx\, dz + \vec{u}_z dx\, dy \\ \vec{u}_\rho \rho\, d\varphi\, dz + \vec{u}_\varphi d\rho\, dz + \vec{u}_z \rho\, d\rho\, d\varphi \\ \vec{u}_r r^2 \text{ sen } \theta\, d\theta\, d\varphi + \vec{u}_\theta r \text{ sen } \theta\, dr\, d\varphi + \vec{u}_\varphi r\, dr\, d\theta \end{cases}$$

A.6. Elementos diferenciales de volumen.

$$dv = \begin{cases} dx\, dy\, dz \\ \rho\, d\rho\, d\varphi\, dz \\ r^2\, dr \text{ sen } \theta\, d\theta\, d\varphi \end{cases}$$

A.7. Operaciones vectoriales – coordenadas rectangulares.

$$\vec{\nabla}\alpha = \vec{u}_x\frac{\partial\alpha}{\partial x} + \vec{u}_y\frac{\partial\alpha}{\partial y} + \vec{u}_z\frac{\partial\alpha}{\partial z}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A} = \vec{u}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \vec{u}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \vec{u}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2\alpha = \frac{\partial^2\alpha}{\partial x^2} + \frac{\partial^2\alpha}{\partial y^2} + \frac{\partial^2\alpha}{\partial z^2} \equiv \vec{\nabla}\cdot\vec{\nabla}\alpha$$

$$\nabla^2\vec{A} = \vec{u}_x\nabla^2 A_x + \vec{u}_y\nabla^2 A_y + \vec{u}_z\nabla^2 A_z \equiv \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \vec{\nabla}\times(\vec{\nabla}\times\vec{A})$$

A.8. Operaciones vectoriales – coordenadas cilíndricas.

$$\vec{\nabla}\alpha = \vec{u}_\rho\frac{\partial\alpha}{\partial\rho} + \vec{u}_\varphi\frac{1}{\rho}\frac{\partial\alpha}{\partial\varphi} + \vec{u}_z\frac{\partial\alpha}{\partial z}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{\rho}\frac{\partial\rho A_\rho}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A} = \vec{u}_\rho\left(\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right) + \vec{u}_\varphi\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right) + \vec{u}_z\frac{1}{\rho}\left(\frac{\partial\rho A_\varphi}{\partial\rho} - \frac{\partial A_\rho}{\partial\varphi}\right)$$

$$\nabla^2\alpha = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\alpha}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\alpha}{\partial\varphi^2} + \frac{\partial^2\alpha}{\partial z^2}$$

A.9. Operaciones vectoriales – coordenadas esféricas.

$$\begin{aligned}
\vec{\nabla}\alpha &= \vec{u}_r\frac{\partial\alpha}{\partial r} + \vec{u}_\theta\frac{1}{r}\frac{\partial\alpha}{\partial\theta} + \vec{u}_\varphi\frac{1}{r\text{ sen }\theta}\frac{\partial\alpha}{\partial\varphi} \\
\vec{\nabla}\cdot\vec{A} &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\text{ sen }\theta}\frac{\partial}{\partial\theta}(\text{ sen }\theta A_\theta) + \frac{1}{r\text{ sen }\theta}\frac{\partial A_\varphi}{\partial\varphi} \\
\vec{\nabla}\times\vec{A} &= \vec{u}_r\frac{1}{r\text{ sen }\theta}\left(\frac{\partial}{\partial\theta}(A_\varphi\text{ sen }\theta) - \frac{\partial A_\theta}{\partial\varphi}\right) + \vec{u}_\theta\frac{1}{r}\left(\frac{1}{\text{sen }\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial(rA_\varphi)}{\partial r}\right) \\
&+ \vec{u}_\varphi\frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right) \\
\nabla^2\alpha &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\alpha}{\partial r}\right) + \frac{1}{r^2\text{ sen }\theta}\frac{\partial}{\partial\theta}\left(\text{ sen }\theta\frac{\partial\alpha}{\partial\theta}\right) + \frac{1}{r^2\text{ sen}^2\theta}\frac{\partial^2\alpha}{\partial\varphi^2} \\
\nabla^2\vec{A} &= \vec{u}_r\left[\nabla^2 A_r - \frac{2}{r^2}\left(A_r + \cot\theta A_\theta + \text{ cosec }\theta\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_\theta}{\partial\theta}\right)\right] \\
&+ \vec{u}_\theta\left[\nabla^2 A_\theta - \frac{1}{r^2}\left(\text{ cosec }^2\theta A_\theta - 2\frac{\partial A_r}{\partial\theta} + 2\cot\theta\text{ cosec }\theta\frac{\partial A_\varphi}{\partial\varphi}\right)\right] \\
&+ \vec{u}_\varphi\left[\nabla^2 A_\varphi - \frac{1}{r^2}\left(\text{ cosec }^2\theta A_\varphi - 2\text{ cosec }\theta\frac{\partial A_r}{\partial\varphi} - 2\cot\theta\text{ cosec }\theta\frac{\partial A_\theta}{\partial\varphi}\right)\right]
\end{aligned}$$

A.10. Operaciones vectoriales – diferenciación.

$$
\begin{aligned}
\vec{\nabla}(\alpha+\beta) &= \vec{\nabla}\alpha+\vec{\nabla}\beta \\
\vec{\nabla}\cdot(\vec{A}+\vec{B}) &= \vec{\nabla}\cdot\vec{A}+\vec{\nabla}\cdot\vec{B} \\
\vec{\nabla}\times(\vec{A}+\vec{B}) &= \vec{\nabla}\times\vec{A}+\vec{\nabla}\times\vec{B} \\
\vec{\nabla}(\alpha\beta) &= \alpha\vec{\nabla}\beta+\beta\vec{\nabla}\alpha \\
\vec{\nabla}\cdot(\alpha\vec{A}) &= \alpha\vec{\nabla}\cdot\vec{A}+\vec{A}\cdot\vec{\nabla}\alpha \\
\vec{\nabla}\times(\alpha\vec{A}) &= \alpha(\vec{\nabla}\times\vec{A})-\vec{A}\times\vec{\nabla}\alpha \\
\vec{\nabla}\cdot(\vec{A}\times\vec{B}) &= \vec{B}\cdot(\vec{\nabla}\times\vec{A})-\vec{A}\cdot(\vec{\nabla}\times\vec{B}) \\
\nabla^2\vec{A} &= \vec{\nabla}(\vec{\nabla}\cdot\vec{A})-\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) \\
\vec{\nabla}\times(\alpha\vec{\nabla}\beta) &= \vec{\nabla}\alpha\times\vec{\nabla}\beta \\
\vec{\nabla}\times\vec{\nabla}\alpha &= 0 \\
\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) &= 0 \\
\vec{\nabla}\times(\vec{A}\times\vec{B}) &= \vec{A}(\vec{\nabla}\cdot\vec{B})+(\vec{B}\cdot\vec{\nabla})\vec{A}-\vec{B}(\vec{\nabla}\cdot\vec{A})-(\vec{A}\cdot\vec{\nabla})\vec{B}
\end{aligned}
$$

A.11. Operaciones vectoriales – integración.

$$
\begin{aligned}
\iiint_V \vec{\nabla}\cdot\vec{A}d^3r &= \oiint_S \vec{A}\cdot d\vec{s} \\
\int\int_S \vec{\nabla}\times\vec{A}\cdot d\vec{s} &= \oint_l \vec{A}\cdot d\vec{l} \\
\iiint_V \vec{\nabla}\times\vec{A}d^3r &= -\oiint_S \vec{A}\times d\vec{s} \\
\iiint_V \vec{\nabla}\alpha d^3r &= \oiint_S \alpha d\vec{s} \\
\int\int_S \vec{u}_n\times\vec{\nabla}\alpha ds &= \oint_l \alpha d\vec{l}
\end{aligned}
$$

B. **Fórmulas útiles**

B.1. Integrales de uso más frecuente.

$$
\int\frac{dx}{(a^2+x^2)^{3/2}}=\frac{x}{a^2\sqrt{a^2+x^2}} \qquad ; \qquad \int\frac{xdx}{(a^2+x^2)^{3/2}}=\frac{-1}{\sqrt{a^2+x^2}}
$$

$$
\int\frac{xdx}{\sqrt{a^2+x^2}}=\sqrt{a^2+x^2} \qquad ; \qquad \int\frac{dx}{\sqrt{a^2+x^2}}=\ln\left(x+\sqrt{a^2+x^2}\right)
$$

$$\int \frac{x^2 dx}{(a^2+x^2)^{3/2}} = \frac{-x}{\sqrt{a^2+x^2}} + \ln\left(x + \sqrt{a^2+x^2}\right) \qquad ; \qquad \int \frac{a dx}{a^2+x^2} = \tan^{-1}\frac{x}{a}$$

B.2. Desarrollos más utilizados.

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \dots$$

$$\ln\left(x + \sqrt{a^2+x^2}\right) = \ln a + \frac{x}{a} - \frac{1}{3!}\frac{x^3}{a^3} + \dots$$