Newtonian and relativistic location systems

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On Newtonian frames

Bartolomé Coll, Joan Josep Ferrando and Juan Antonio Morales

- The Newtonian space-time causal structure allows us to classify frames and coordinate systems in four causal classes.

	t e e e	**t t e e**	**t t t e**	**t t t t**
e e e e (TTTT)		**TTTTTE**	**TTTTTT**	**TTTTTT**
t e e e (TTTE)	**TTTEEE**			

Simple description: Newtonian emission coordinates

Non standard Newtonian synchronizations

Location systems

- A location system is a physical realization of a coordinate system.

- In dimension $n = 4$, whatever be the complete description of a coordinate system, it may be equivalently determined by suitable:

4 one-parameter families of coordinate 3-surfaces

6 families of coordinate 2-surfaces

4 congruences of coordinate lines

So, a location system must include the protocols for the physical construction of one of the geometric elements (lines, surfaces and hypersurfaces) of the coordinate system that it physically realizes.

For example,

timelike lines may be realized by means of clocks,

null lines by laser pulses,

spacelike lines by synchronized inextensible threads,

timelike surfaces by the history of threads or by lasers beams,

null hypersurfaces by light-front signals, ...

Interest of the causal properties of location systems

- In post-Newtonian developments, it is convenient to choose coordinate systems such that their causal properties be the same for the relativistically corrected metric structure as well as for the starting Newtonian one.

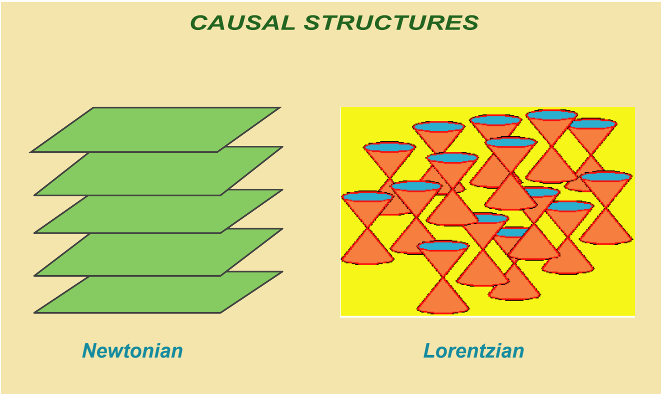
  This convenient choice of analogous causal properties is usually made by

  - taking the starting Newtonian coordinate system to be the standard one, and
  - considering weak gravitational fields that are unable to change, with the lower order perturbed relativistic values of the metric, these causal properties.

- But new problems, concerning black holes, binary systems, gravitational waves, positioning systems, ... could induce to start from other Newtonian coordinate systems, best adapted to these problems.

Lecture planning

Here, we shall compare the incidences of the Newtonian and Lorentzian space-time structures on the construction of location systems.



1. Vocabulary.
2. Causal classification of frames and coordinate systems.
3. Emission positioning systems.
4. Timelike synchronizations.

Coordinate parameters and gradient coordinates

- There are two natural variations associated with a given coordinate x^α,

$$\partial_\alpha \qquad \text{and} \qquad dx^\alpha$$

coordinate lines — coordinate hypersurfaces

and such variations have, in general, different causal orientations.

- We say that a coordinate x^α is a
  - t, l, e coordinate parameter when ∂_α is respectively t, l, e,
  - t, l, e gradient coordinate when dx^α is respectively t, l, e.

Spacelike and timelike synchronizations

- If a coordinate t is $\begin{cases} \text{a timelike } \textit{coordinate paramenter} \text{ and} \\ \text{a timelike } \textit{gradient coordinate} \end{cases}$

  we say that it defines a spacelike *synchronization*.

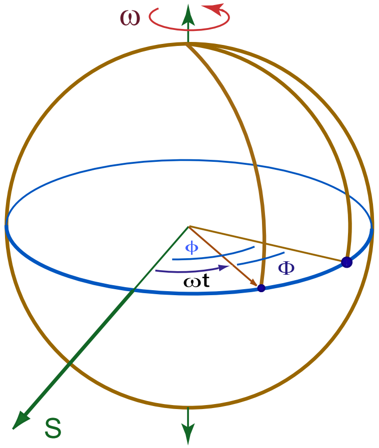
  - Example: the absolute Newtonian time.

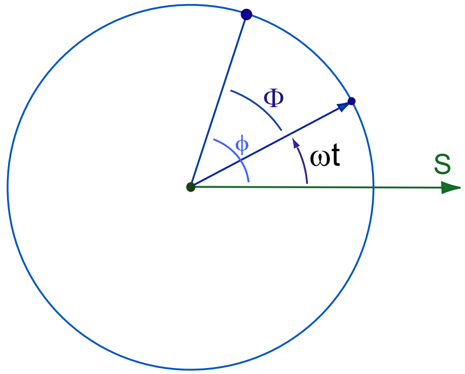
- If a coordinate t is $\begin{cases} \text{a timelike } \textit{coordinate paramenter} \text{ and} \\ \text{a spacelike } \textit{gradient coordinate} \end{cases}$

  we say that it defines a timelike *synchronization*.

  - Example: the local Solar time.

Solar time and solar synchronization





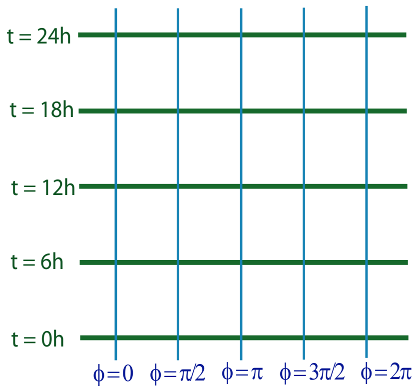
$\{t, r, \theta, \phi\}$ geocentric inertial spherical coordinates.

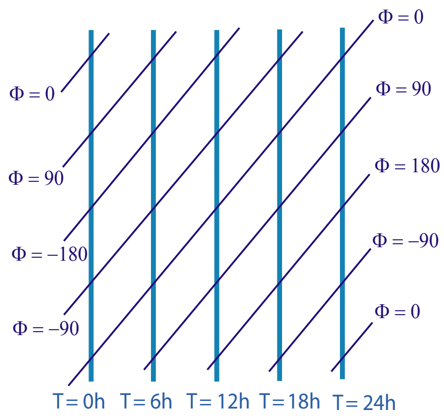
$$\color{magenta}{T} = t + \frac{\Phi}{\omega} = \color{magenta}{\frac{\phi}{\omega}}$$

$$\color{magenta}{\Phi = \phi - \omega t}$$

$$r = R_\oplus, \quad \theta = 0$$

$\color{blue}{\{T, r, \theta, \Phi\}}$ local solar time spherical coordinates.





T is $\begin{cases} \text{a } \color{blue}{\text{timelike}} \textit{ coordinate parameter} \text{ and} \\ \text{a } \color{green}{\text{spacelike}} \textit{ gradient coordinate}, \ \color{green}{dT \wedge dt \neq 0}. \end{cases}$

Causal class: definition

The causal signature of a frame $\{\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3, \mathrm{v}_4\}$ is defined by a set of 14 causal orientations:

$$\{\mathrm{c}_1\mathrm{c}_2\mathrm{c}_3\mathrm{c}_4, \mathrm{C}_{12}\mathrm{C}_{13}\mathrm{C}_{14}\mathrm{C}_{23}\mathrm{C}_{24}\mathrm{C}_{34}, c_1\, c_2\, c_3\, c_4\}$$

- c_i is the causal orientation of the vector v_i,
- C_ij $(\mathrm{i} \neq \mathrm{j})$ is the causal orientation of the 2-plane $\{\mathrm{v}_\mathrm{i}\mathrm{v}_\mathrm{j}\}$, and
- c_i is the causal orientation of the covector θ^i of the dual coframe.

The causal class of a frame is the set of all the frames that have the same causal signature.

$$Causal\ class\ of\ a\ \begin{cases} \text{frame} \\ \text{coordinate system} \end{cases}$$

Causal homogeneity

- When the causal orientations of all the geometric elements of a coordinate system are uniform on a given space-time region we say that the region under consideration is a causal homogeneous region for the coordinate system in question.

- The point of interest here is that every protocol physically realizes coordinate lines, coordinate surfaces or coordinate hypersurfaces of specific causal orientations allowing to analyze the different causal homogeneous regions of the constructed coordinate system.

The causal classification of frames and coordinates

- There exist four causal classes of Newtonian frames:

	teee	ttee	ttte	tttt
eeee (TTTT)		TTTTTE	TTTTTT	TTTTTT
teee (TTTE)	TTTEEE			

$\{\text{teee}, \text{TTTEEE}, \textit{teee}\}$

$\{\text{ttee}, \text{TTTTTE}, \textit{eeee}\}$

$\{\text{ttte}, \text{TTTTTT}, \textit{eeee}\}$

$\{\text{tttt}, \text{TTTTTT}, \textit{eeee}\}$

The causal classification of frames and coordinates

- Note that in the Newtonian causal structure four spacelike vectors are necessarily dependent. This contrasts with the Lorentzian situation, where bases having 4 spacelike vectors exist.

- Concerning spacelike vectors, the main difference between Newtonian and relativistic causal structures comes from the essential property that in a Lorentzian metric two spacelike vectors generates a 2-plane that may be spacelike, null or timelike.

- Then, how many causal classes of Lorentzian frames (and then how many causally different realizations of coordinate systems) exist in a relativistic space-time domain?

There exist 199 causal classes of Lorentzian frames

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eeee	EEEEEE LEEEEE TEEEEE LLEEEE TLEEEE TTEEEE LLLEEE TLLEEE TTLEEE TTTEEE LLLLEE TLLLEE TTLLEE TTTLEE TTTTEE LLLLLE TLLLLE TTLLLE TTTLLE TTTTLE TTTTTE LLLLLL TLLLLL TTLLLL TTTLLL TTTTLL TTTTTL TTTTTT	TTLEEE TTTEEE TTLLEE TTLELE TTTLEE TTLTEE TTLETL TTTTEE TTLLLE TTLELL TTTLLE TTLTLE TTLLTE TTLETL TTTTLE TTLTTE TTLETT TTTTTE TTLLLL TTTLLL TTLTLL TTLLTL TTTTLL TTLTTL TTLLTT TTTTTL TTLTTT TTTTTT		TTTEEE TTTLEE TTTTEE TTTLLE TTTTLE TTTTTE TTTLLL TTTTLL TTTTTL TTTTTT	TTLTLE TTLLTE TTTTLE TTTTTE TTLTLL TTLLTL TTTTLL TTLLTT TTLTLT TTTTTL TTTTLT TTTTTT	TTTTLE TTTTTE TTTTLL TTTTTL TTTTLT TTTTTT	TTTTTE TTTTTL TTTTTT	TTLTLL TTTTLL TTTTTL TTTTTT	TTTTLL TTTTTL TTTTTT	TTTTTL TTTTTT	TTTTTT	TTTTTT	TTTTTT	TTTTTT	TTTTTT	TTTTTT
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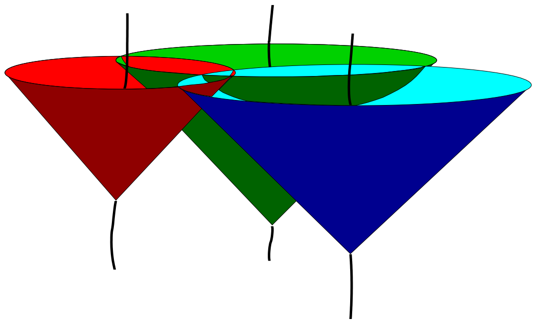
B. Coll and J. A. Morales, Int. Jour. Theor. Phys. **31**, 1045–1062 (1992).

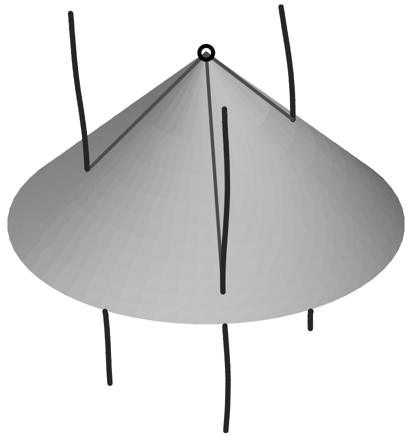
Newtonian positioning and emission coordinates

Suppose four clocks $\kappa^A(t)$ $(A = 1, 2, 3, 4)$ broadcasting their times.



Such emitters fill the space-time with four one-parameter families of cones $t^A = \text{constant}$.

- The past (sound, light) cone of every event cuts the emitter world lines at $\kappa^A(t^A)$.
- Then, the set $\{t^A\}$ constitutes the four emission coordinates of the event.



(3-dimensional pictures)

Newtonian emission coordinates. Four emitters at rest

Here we will consider the simple case of four emitters at rest with respect to an inertial non-dispersive medium. In a standard coordinate system $\{t, x^i\} = \{t, \vec{r}\}$, the emitter world-lines are expressed:

$$\kappa^A(t) = (t, \vec{c}^A) \, .$$

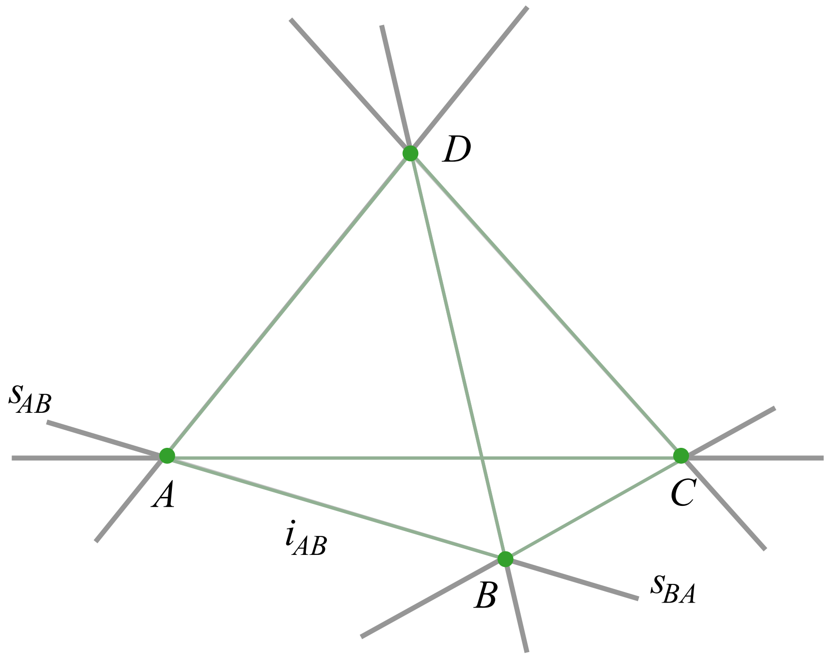
Then, the signal emitted by the clock κ^A at the instant t^A at velocity v describes in the space-time a **cone** of equation

$$v(t - t^A) = \left|\vec{r} - \vec{c}^A\right| ,$$

so that the emission coordinates $\{t^A\}$ are related to the inertial ones $\{t, \vec{r}\}$ by

$$t^A = t - \frac{1}{v}\left|\vec{r} - \vec{c}^A\right| .$$

Newtonian emission coordinates



Four emitters at rest

At the events where the Jacobian is not degenerate, the coordinate lines of the emission coordinates are of the type:

- $\{\mathrm{t\,t\,t\,t}\}$ *(generically)*,
- $\{\mathrm{t\,t\,t\,e}\}$ *(generically)* on the events of the timelike 3-planes containing three emitters, and
- $\{\mathrm{t\,t\,e\,e}\}$ on the events of the timelike strips generated by every pair of clocks.

- In Newtonian space-time, the emission coordinate system generated by a positioning system is never causally homogeneous, but always presents three regions corresponding to the non standard three causal classes.

Minkowski space-time

Emission coordinates

- Now, every emitter κ is supposed to continuously broadcast, in an inertial non-dispersive medium, their proper time τ^A by means of sound or light signals that propagate in the medium at constant velocity $v \leq 1$.

- For simplicity, the four emitters will be consider at rest with respect to the medium referred to a standard coordinate system $\{t, x^i\} = \{t, \vec{r}\}$. Then, the inertial time t is also the proper time of the four emitters and their world-lines take the expression $\kappa^A(t) = (t, \vec{c}^A)$.

- Then, the emission coordinates $\{t^A\}$ are related to the inertial ones $\{t, \vec{r}\}$ by

$$t^A = t - \frac{1}{v} \left| \vec{r} - \vec{c}^A \right| .$$

Light emission coordinates

Let us first consider the (light) case $v = 1$. Here, we have $(dt^A)^2 = 0$ so that

- The coframe of the relativistic emission coordinate systems with $v = 1$ is of causal type $\{l\,l\,l\,l\}$.
- All the relativistic positioning systems with light signals define in their whole domains a sole causal class, of causal signature

$$\{\mathrm{e\,e\,e\,e}\,,\mathrm{E\,E\,E\,E\,E\,E},\, l\,l\,l\,l\}$$

- This result, obtained for an inertial homogeneous medium and four static clocks, may be shown true also for arbitrary clocks in general space-times.

Sound emission coordinates

▷ Let us now consider the (sound) case $v < 1$. Then, the causal classes of the emission coordinate systems are of the form:

$$\{\mathrm{c}_1\,\mathrm{c}_2\,\mathrm{c}_3\,\mathrm{c}_4,\ \mathrm{C}_{12}\,\mathrm{C}_{13}\,\mathrm{C}_{14}\,\mathrm{C}_{23}\,\mathrm{C}_{24}\,\mathrm{C}_{34},\ e\,e\,e\,e\}$$

where the causal orientations, c_A, C_AB depend on the cosines μ_{AB} of the angles between the signals coming from the emitters A and B as:

$$\mathrm{c}_\mathrm{A} = \begin{cases} \mathrm{t} & \dfrac{\Lambda_A}{\Delta_A} < \dfrac{1-v^2}{v^2} \\ \mathrm{l} & \dfrac{\Lambda_A}{\Delta_A} = \dfrac{1-v^2}{v^2} \\ \mathrm{e} & \dfrac{\Lambda_A}{\Delta_A} > \dfrac{1-v^2}{v^2} \end{cases} \qquad \mathrm{C}_\mathrm{AB} = \begin{cases} \mathrm{T} & \mu_{CD} > 2v^2 - 1 \\ \mathrm{L} & \mu_{CD} = 2v^2 - 1 \\ \mathrm{E} & \mu_{CD} < 2v^2 - 1 \end{cases}$$

with $C, D \neq A, B$, and where

$$\Delta_D \equiv 1 + 2\mu_{AB}\mu_{BC}\mu_{CA} - (\mu_{AB}^2 + \mu_{BC}^2 + \mu_{CA}^2)$$

$$\Lambda_D \equiv 2(1-\mu_{AB})(1-\mu_{BC})(1-\mu_{CA}).$$

▷ Depending on $\begin{cases}\text{the different configurations of the emitters and / or of} \\ \text{the different values of the velocity } v < 1,\end{cases}$

the sound-emission coordinate systems may present space-time regions of 102 different causal classes.

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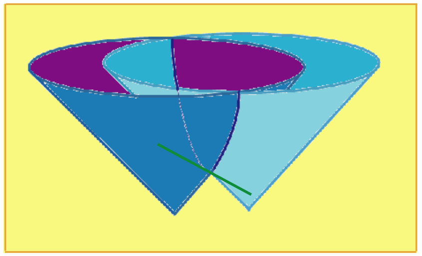
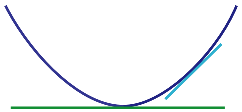
The 1+102 causal classes of coordinate signals.

Coordinate lines of the emission coordinates

- In the Newtonian as well as in the relativistic situation, the coordinate lines of emission coordinates are hyperbolas. Nevertheless, their causal types differ:





In the Newtonian case every hyperbola is everywhere time-like up to at its base point, where it is spacelike.

In the relativistic case:

- when $v < 1$ the corresponding spacelike point becomes enlarged to a whole spacelike domain, bounded by two lightlike points, the rest of the branches being timelike, and
- when $v = 1$ the hyperbolas are spacelike everywhere.

- Of course, this is at the basis of the richness (the above mentioned 103 causal classes) of the signal-based positioning systems.

The role played by the synchronizations

- The example of the solar synchronization previously considered,

$$T = t + \frac{\Phi}{\omega}$$

suggests us that we will be able to generate all the Newtonian causal classes using the linear synchronization group,

$$X^0 = x^0 + a_i x^i \ , \quad X^i = x^i \ .$$

- The natural frame and coframe of the new system $\{X^\alpha\}$ are given by

$$\partial_{X^0} = \partial_{x^0} \ , \qquad \partial_{X^i} = -a_i \partial_{x^0} + \partial_{x^i} \ ,$$

$$dX^0 = dx^0 + a_i dx^i \ , \qquad dX^i = dx^i \ .$$

Newtonian causal classes and non-standard synchronization

- In the Newtonian space-time, starting from a standard coordinate system $\{x^0, x^i\}$ of causal type $\{\mathrm{t\,e\,e\,e}\}$, the linear synchronization transformations

$$X^0 = x^0 + a_i x^i \ , \quad X^i = x^i$$

define a coordinate system $\{X^\alpha\}$ whose causal class is

$$\{\mathrm{t\,t\,e\,e},\ \mathrm{T\,T\,T\,T\,T\,E},\ e\,e\,e\,e\} \qquad \text{if} \qquad \exists!\ i,\ a_i \neq 0$$

$$\{\mathrm{t\,t\,t\,e},\ \mathrm{T\,T\,T\,T\,T\,T},\ e\,e\,e\,e\} \qquad \text{if} \qquad \exists!\ i,\ a_i = 0$$

$$\{\mathrm{t\,t\,t\,t},\ \mathrm{T\,T\,T\,T\,T\,T},\ e\,e\,e\,e\} \qquad \text{if} \qquad \forall i,\ a_i \neq 0$$

- Then, the different causal classes have been obtained by simple, *pure*, changes of synchronization of the *same* system of clocks, excluding any other change of coordinates or of observers.

Minkowski space-time

The Linear Synchronization Group

- Let us consider, in Minkowski space-time, the linear synchronization group acting on an inertial laboratory referred to a standard coordinate system $\{x^0, x^i\}$.

- It follows, by direct scalar products of the above expressions

$$\partial_{X^0} = \partial_{x^0} \ , \qquad \partial_{X^i} = -a_i \partial_{x^0} + \partial_{x^i} \ ,$$
$$dX^0 = dx^0 + a_i dx^i \ , \qquad dX^i = dx^i \ .$$

that the covariant and contravariant components, $g_{\alpha\beta}$ and $g^{\alpha\beta}$ respectively, of the metric η in the new system $\{X^\alpha\}$ are:

$$g_{\alpha\beta} = \begin{pmatrix} -1 & \vec{a} \\ \vec{a} & I - \vec{a} \otimes \vec{a} \end{pmatrix} , \qquad g^{\alpha\beta} = \begin{pmatrix} -1 + \vec{a}^{\,2} & \vec{a} \\ \vec{a} & I \end{pmatrix} .$$

where $\vec{a} \equiv (a_1, a_2, a_3)$ and I is the 3×3 identity matrix.

Relativistic causal classes and non-standard synchronization

- All the causal classes obtained by a linear synchronization transformation have a causal signature of the form:

$$\{\mathrm{t}\,\mathrm{c}_1\,\mathrm{c}_2\,\mathrm{c}_3,\ \mathrm{T}\,\mathrm{T}\,\mathrm{T}\,\mathrm{C}_{12}\,\mathrm{C}_{13}\,\mathrm{C}_{23},\ c_0\, e\, e\, e\}$$

where the seven non-fixed causal orientations, $\mathrm{c}_1, \mathrm{c}_2, \mathrm{c}_3, \mathrm{C}_{12}, \mathrm{C}_{13}, \mathrm{C}_{23}, c_0$ depend on the a_i parameters as follows:

$$\mathrm{c_i} = \begin{cases} \mathrm{t} & |a_i| > 1 \\ \mathrm{l} & |a_i| = 1 \\ \mathrm{e} & |a_i| < 1 \end{cases} \qquad \mathrm{C_{ij}} = \begin{cases} \mathrm{T} & a_i^2 + a_j^2 > 1 \\ \mathrm{L} & a_i^2 + a_j^2 = 1 \\ \mathrm{E} & a_i^2 + a_j^2 < 1 \end{cases} \qquad c_0 = \begin{cases} t & |\vec{a}| < 1 \\ l & |\vec{a}| = 1 \\ e & |\vec{a}| > 1 \end{cases}$$

▷ The number of different causal classes that may be generated by a linear synchronization transformation is 29, in contrast with the only 4 Newtonian ones.

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Newtonian analogues

- The Lorentzian causal classes of same causal signature that the four Newtonian ones correspond to the following values of the parameters a_i:

$$\{\mathrm{t\,t\,t\,t},\ \mathrm{T\,T\,T\,T\,T\,T},\ e\,e\,e\,e\} \quad \text{if} \quad \forall\ i, \qquad |a_i| > 1$$

$$\{\mathrm{t\,t\,t\,e},\ \mathrm{T\,T\,T\,T\,T\,T},\ e\,e\,e\,e\} \quad \text{if} \quad \begin{cases} \exists!\ i, & |a_i| < 1 \\ \forall\ j \neq i, & |a_j| > 1 \end{cases}$$

$$\{\mathrm{t\,t\,e\,e},\ \mathrm{T\,T\,T\,T\,T\,E},\ e\,e\,e\,e\} \quad \text{if} \quad \begin{cases} \exists!\ i, & |a_i| > 1 \\ j, k \neq i, & a_j^2 + a_k^2 < 1 \end{cases}$$

$$\{\mathrm{t\,e\,e\,e},\ \mathrm{T\,T\,T\,E\,E\,E},\ t\,e\,e\,e\} \quad \text{if} \quad \forall\ i, \qquad |a_i| < 1$$

Schwarzschild space-time

Painlevé-Gullstrand-Lemaître coordinates $\{T, r, \theta, \phi\}$

Painlevé (1921), Gullstrand (1922) and Lemaître (1933) expressed the Schwarzschild solution without divergence at $r = 2m$,

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dT^2 + 2\sqrt{\frac{2m}{r}}\, dT\, dr + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

- The coordinate basis $\{\partial_T, \partial_r, \partial_\theta, \partial_\phi\}$ belong to the causal class

$$\{\mathrm{t\,e\,e\,e},\ \mathrm{T\,T\,T\,E\,E\,E},\ t\,e\,e\,e\} \quad \text{if} \quad r > 2m$$

$$\{\mathrm{l\,e\,e\,e},\ \mathrm{T\,L\,L\,E\,E\,E},\ t\,l\,e\,e\} \quad \text{if} \quad r = 2m$$

$$\{\mathrm{e\,e\,e\,e},\ \mathrm{T\,E\,E\,E\,E\,E},\ t\,t\,e\,e\} \quad \text{if} \quad r < 2m$$

- T is a time-like gradient coordinate

Schwarzschild space-time

The Painlevé-Gullstrand-Lemaître T-coordinate

$$dT = dt + \frac{\sqrt{\frac{2m}{r}}}{1 - \frac{2m}{r}}\, dr, \qquad T = t + f(r)$$

$$f(r) = 2\sqrt{2mr} + 2m \ln \frac{\sqrt{r} - \sqrt{2m}}{\sqrt{r} + \sqrt{2m}}$$

- The relation between Schwarzschild time t and the T-coordinate used by Painlevé-Gullstrand-Lemaître is obtained as a non-linear synchronization transformation over the congruence of a static observer ∂_t.

- T is the proper time of a freely falling observer whose initial velocity at $r = \infty$ is zero with respect to a static observer.

Last comments

- In this talk we have pointed out that the causal space-time structure has an important incidence in the comprehension of location systems.

- Nevertheless, in order to better understand the role that location systems as physical objects, or coordinate systems as mathematical objects, play in the conception and analysis of experimental situations, a lot of work remains to be done, the present one being only one of the first little pieces.

- Here, my intention has been to show that the interest of the causal classification of frames is not only taxonomic. Among the 198 admissible *cuts* of the space-time others than the very usual space $\oplus$ time decomposition, a lot of them admit simple physical realizations (from synchronization transformations and/or emission coordinates).

- In going from Newtonian to relativistic physics, the causal classification of frames is the starting point to analyze location systems.

The causal classification of frames and coordinates

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There exist four causal classes of Newtonian frames.

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There exist 199 causal classes of Lorentzian frames.