Creatable universes: a new approach

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Abstract. We are interested in the non asymptotically flat space-times for which all the momenta (energy, 3-momentum and angular 4-momenta) are conserved in time. We call *universes* such space-times. Starting from the Weinberg definition of the momenta associated to a spacelike 3-surface, we give a coordinate prescription to properly define the energy of a universe. The prescription includes the vanishing of linear and angular 3-momenta. This result allows us to consider the case of universes with vanishing 4-momenta (*creatable universes*) in a consistent way.

1. Introduction

People have speculated about the possibility that our Universe came out from a vacuum quantum fluctuation [1, 2]. In this case, one could expect that the resulting universe had vanishing energy, P^0, and ever better, vanishing energy-momentum, P^α, and vanishing angular 4-momentum $J^{\alpha\beta}$.

But the problem is the dramatic dependence of these momenta on the coordinate system used, letting aside the particular case of asymptotically flat space-times where one knows how to choose the good coordinate systems, as it is well known [3, 4]. Nevertheless, if we want to deal with a space-time, V_4, which could represent our Universe, we cannot always impose this asymptotic flatness.

To circumvent this problem of lack of uniqueness in the definition of these momenta, in [5] we explain why we think that we must deal with Gauss coordinates in V_4, based on some space-like 3-surface Σ_3. Even more, we discussed why we should take coordinates which, at the same time, allow to write the corresponding instantaneous 3-space metric in a conformally flat way on the boundary, Σ_2, of Σ_3.

But, this choice of coordinates let still a lot of freedom. Then, in the present paper, we impose further that our coordinates be such that the corresponding 3-momenta, P^i and J^{ij}, vanish. To do this, we previously prove that in any *universe* (i. e. a space-time where, once the convenient coordinate system has been choose, the defined 4-momenta remain constant in time) we always can change Σ_3, without changing Σ_2, such that $P^i = J^{ij} = 0$.

Finally, in a general and consistent way, we can define a *creatable universe* as a *universe* where the 4-momenta, defined in the above coordinate systems, vanish. As we have just said, these are coordinate systems which are Gauss coordinate systems, where $P^i = J^{ij} = 0$, and where the instantaneous space metric can be written in a conformally flat way on Σ_2. We will call these coordinate systems *intrinsic* coordinate systems.

2. The good coordinate systems

As it has been stated in the Introduction, the big problem of the Universe energy and momenta definition is the lack of uniqueness: they are coordinate dependent quantities (whatever energy-momentum complex we take). In asymptotically flat space-times, there is no problem: P^α and $J^{\alpha\beta}$, are uniquely defined (in asymptotic Minkowskian coordinate systems). But, since we are interested in the energy and momenta of the Universe, we need to go beyond asymptotically flat space-times.

So, our first goal will be to reduce as much as possible the number of acceptable coordinate systems to properly define the Universe energy and momenta.

2.1. Gauss coordinates in V_4 from an space-like 3-surface, Σ_3.

We expect any possible universe to be a *universe* in the sense defined in the Introduction: a space-time whose well defined energy and momenta, i.e., P^α and $J^{\alpha\beta}$, are finite and conserved in time. Even better, we expect them to be zero for the actual Universe, since we expect it to be *creatable*.

Which coordinate time, then, to begin with? A physical universal time. That is, in order to properly define P^α and $J^{\alpha\beta}$, we need Gauss coordinates $\{x^\alpha\} = \{t, x^i\}$:

$$ds^2 = -dt^2 + g_{ij}dx^i dx^j \qquad (i,j = 1,2,3). \tag{1}$$

Since we treat all momenta, linear and angular, at the same level, we need a symmetric energy-momentum complex. We will take the Weinberg one [3]. Then, one obtains in Gauss coordinates:

- Linear 4-momentum:

$$P^0 = \frac{1}{\kappa}\int (\partial_j g_{ij} - \partial_i g)d\Sigma_{2i}\,, \quad P^i = \frac{1}{\kappa}\int (\partial_0 g\delta_{ij} - \partial_0 g_{ij})d\Sigma_{2j} \tag{2}$$

- Angular 4-momentum:

$$J^{jk} = \frac{1}{\kappa}\int (x_k \partial_0 g_{ij} - x_j \partial_0 g_{ki})d\Sigma_{2i}\,, \tag{3}$$

$$J^{0i} = P^i t - \frac{1}{\kappa}\int [(\partial_k g_{kj} - \partial_j g)x_i + g\delta_{ij} - g_{ij}]d\Sigma_{2j} \tag{4}$$

Notation: $g \equiv \delta^{ij}g_{ij}$; the 2-surface Σ_2 is the boundary of Σ_3; $d\Sigma_{2i}$ is the corresponding surface element. Indices are raised and lowered with the Minkowski tensor, and $\kappa \equiv 16\pi G$.

2.2. Conformal coordinates for the 3-space metric on the boundary Σ_2 of Σ_3.

Since P^α and $J^{\alpha\beta}$ are supposed to be conserved, we only need the metric in the neighborhood of Σ_3 ($t = 0$). Furthermore, since P^α and $J^{\alpha\beta}$ are written as surface integrals on Σ_2, or as a limit of a sequence of surface integrals, all we need is a coordinate system on Σ_2 and its immediate neighborhood. In [5], the following result was obtained:

▷ Let Σ_2 be a 2-surface of a spacelike hypersurface Σ_3. Then, there exists a Gauss coordinate system $\{t, x^i\}$, based on Σ_3 ($t = 0$), such that Σ_2 is given by $t = x^3 = 0$, and the space-time metric is written as

$$ds^2 = -dt^2 + g_{ij}(t, x^k)dx^i dx^j, \quad \text{with} \quad g_{ij}(t = x^3 = 0) = f(x^1, x^2)\delta_{ij} \tag{5}$$

That is, on Σ_2, the metric of Σ_3 may be expressed in conformally flat form.

In this new Gauss chart, at $t = 0$, the above expressions for the momenta take this simple form:

- Linear 4-momentum:

$$P^0 = -\frac{1}{\kappa} \int \partial_3 g_{aa} \; dx^1 dx^2 \,, \quad P^3 = \frac{1}{\kappa} \int \partial_0 g_{aa} \; dx^1 dx^2 \tag{6}$$

$$P^a = -\frac{1}{\kappa} \int \partial_0 g_{3a} \; dx^1 dx^2 \quad (a = 1, 2) \tag{7}$$

where $g_{aa} = g_{11} + g_{22}$.

- Angular 4-momentum:

$$J^{ij} = \frac{1}{\kappa} \int (x_j \partial_0 g_{3i} - x_i \partial_0 g_{3j}) \; dx^1 dx^2 \,, \tag{8}$$

$$J^{0a} = \frac{1}{\kappa} \int x^a \partial_3 g_{bb} \; dx^1 dx^2 \,, \tag{9}$$

$$J^{03} = -\frac{1}{\kappa} \int g_{aa} \; dx^1 dx^2 \,. \tag{10}$$

3. Universes. Intrinsic coordinates

Let us consider a *universe*: a space-time where both 4-momenta, P^α and $J^{\alpha\beta}$, have been well defined and are conserved quantities.

Then, the question is: for any *universe*, there exist Gauss coordinate systems such that both 3-momenta, P^i and J^{ij}, irrespective of the momentum origin vanish? The answer is YES, and consequently, the energy P^0 and momenta J^{0i} have an "intrinsic" meaning in these coordinate systems. More precisely, the following statement occurs:

▷ For any *universe*, a Gauss coordinate system $\{t, x^i\}$ exists such that the metric is written in the form (5) and, in addition, the lineal and angular 3-momenta of the hypersurface $t = 0$ vanish, that is

$$P^i = 0 \quad \text{and} \quad J^{ij} = 0. \tag{11}$$

We call here such a Gauss coordinate system a universe *intrinsic* coordinate system. Intrinsic coordinate systems are the appropriated ones in order to calculate the energy, P^0, and mixed momenta, J^{0i}, of a *universe*.

4. The precise definition of creatable universes

Let it be a given *universe*. Let us choose some intrinsic coordinate system in the precise sense we have just defined. Imagine that the universe is such that P^0 and J^{0i} vanish. Then, we will have in all $P^\alpha = J^{\alpha\beta} = 0$ and we will call this universe a creatable universe.

This definition drive us to do the following comment: should not this name of creatable universes be reserved for universes such that their linear and angular 4-momenta vanish irrespective of the intrinsic coordinates chosen? The answer is no, since it can be seen that even the Minkowski space, which has $P^\alpha = J^{\alpha\beta} = 0$ for some intrinsic coordinates (for Lorentzian coordinates), cannot preserve this vanishing property when we move to another arbitrary intrinsic coordinate system. The reason of this is that, in doing so, we can leave the coordinates "adapted" to the symmetries of the metric: the ones tied to the ten parameters of the Poincaré group.

Thus, given a universe which has $P^\alpha = J^{\alpha\beta} = 0$ for some intrinsic coordinate system, if there are other intrinsic coordinates where this vanishing is not preserved, we should consider that this non preservation expresses the fact that the new intrinsic coordinates are not well adapted to some basic metric symmetries. Adapted to which symmetries? To the ones which allow us to have just vanishing 4-momenta for some intrinsic coordinate system.

5. Final considerations

One of the main interests of the present paper could be to give a criterion to reject from the very beginning a very large family of space-times as candidates to represent our actual Universe. According to this criterion it could only be represented by a creatable universe. Thus, in [6], it was proved that, within the inflationary perturbed Friedmann-Lemaître-Robertson-Walker universes, only the closed case corresponds to a creatable universe.

Acknowledgments

References

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