PROBING THE
‘PRIMORDIAL DARK AGES'
with GRAVITATIONAL WAVES

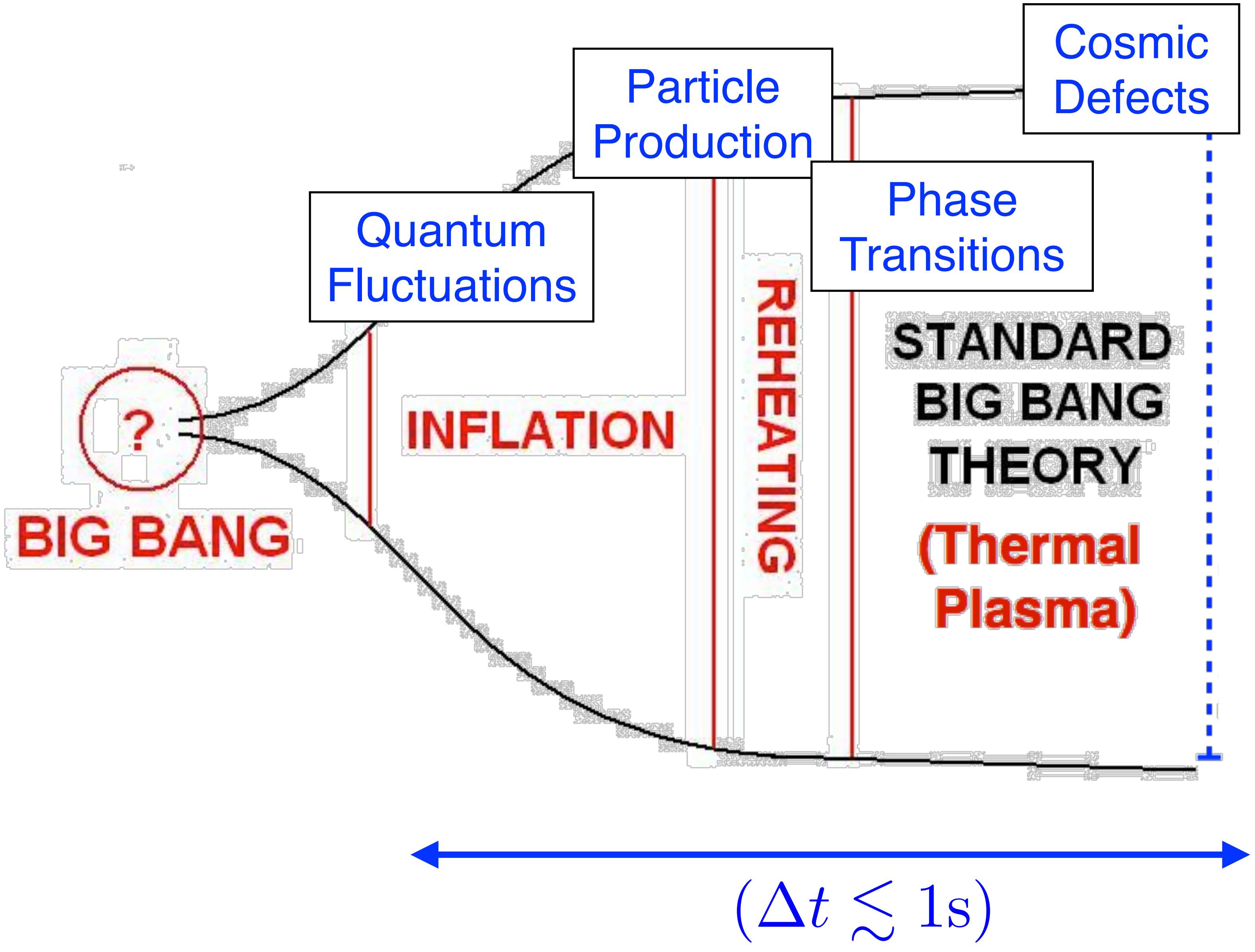
DANi FiGUEROA
🌴 IFiC, Valencia 🌴

1604.03905, 1811.04093, 1905.11960, work in progress
(+ Byrnes) (+ Tanin) (+ Opferkuch, Stefanek)

June 9-11 2021, **11th Iberian Gravitational Wave Meeting** (online)

Motivation

The Early Universe



?

BIG BANG

Quantum Fluctuations

INFLATION

Particle Production

REHEATING

Phase Transitions

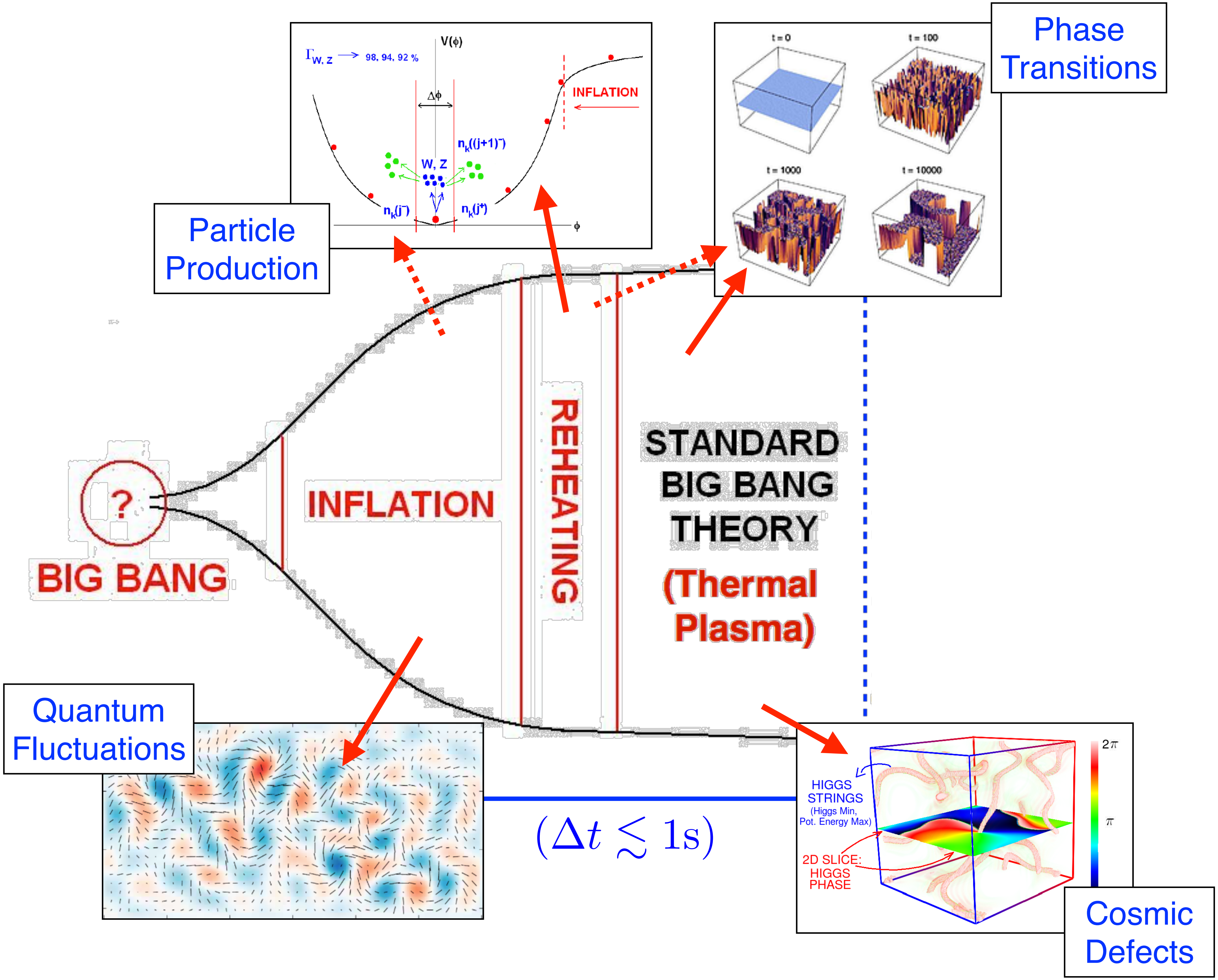
Cosmic Defects

STANDARD BIG BANG THEORY

(Thermal Plasma)

$(\Delta t \lesssim 1\mathrm{s})$

The Early Universe



Particle Production

Phase Transitions

? BIG BANG

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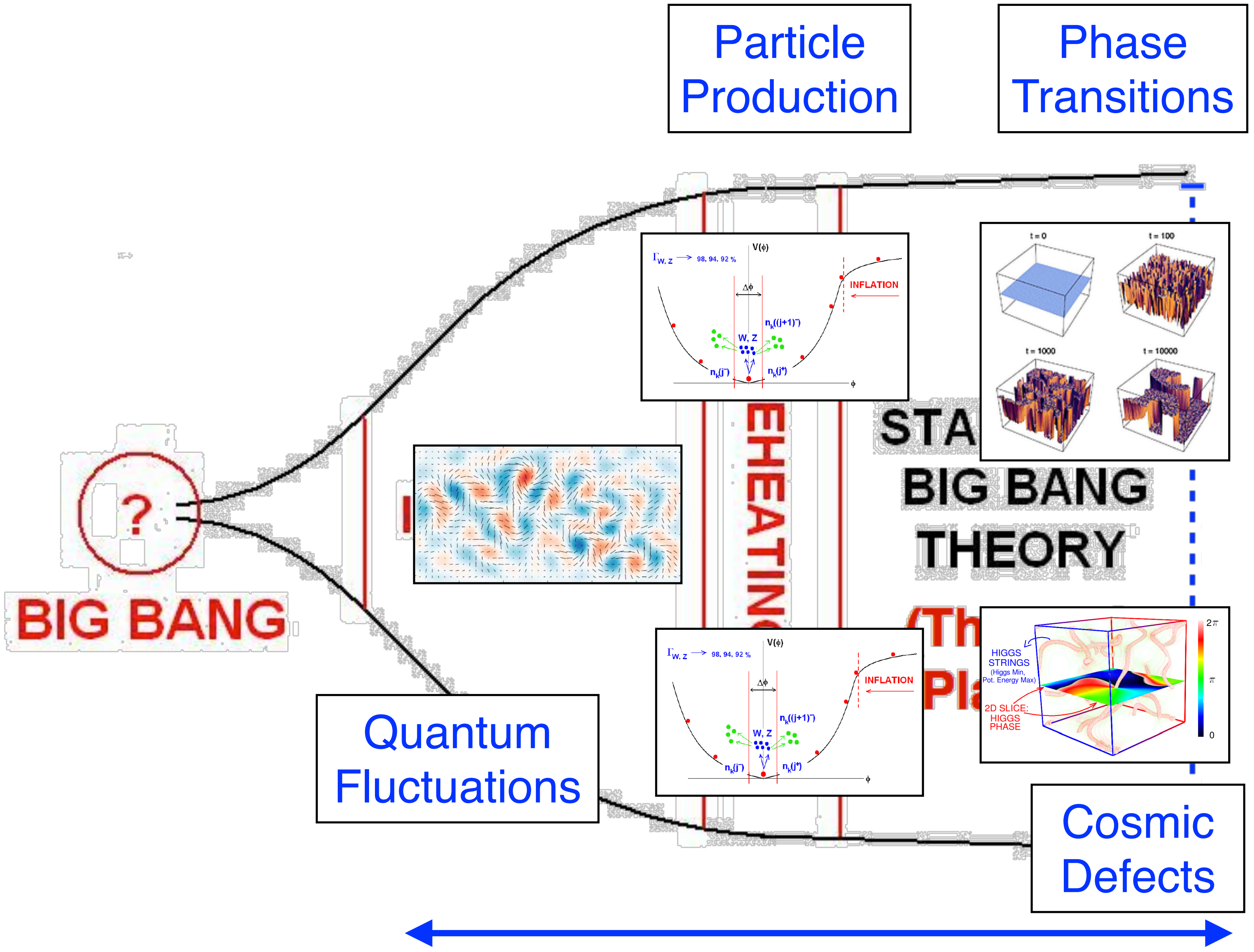


Particle Production

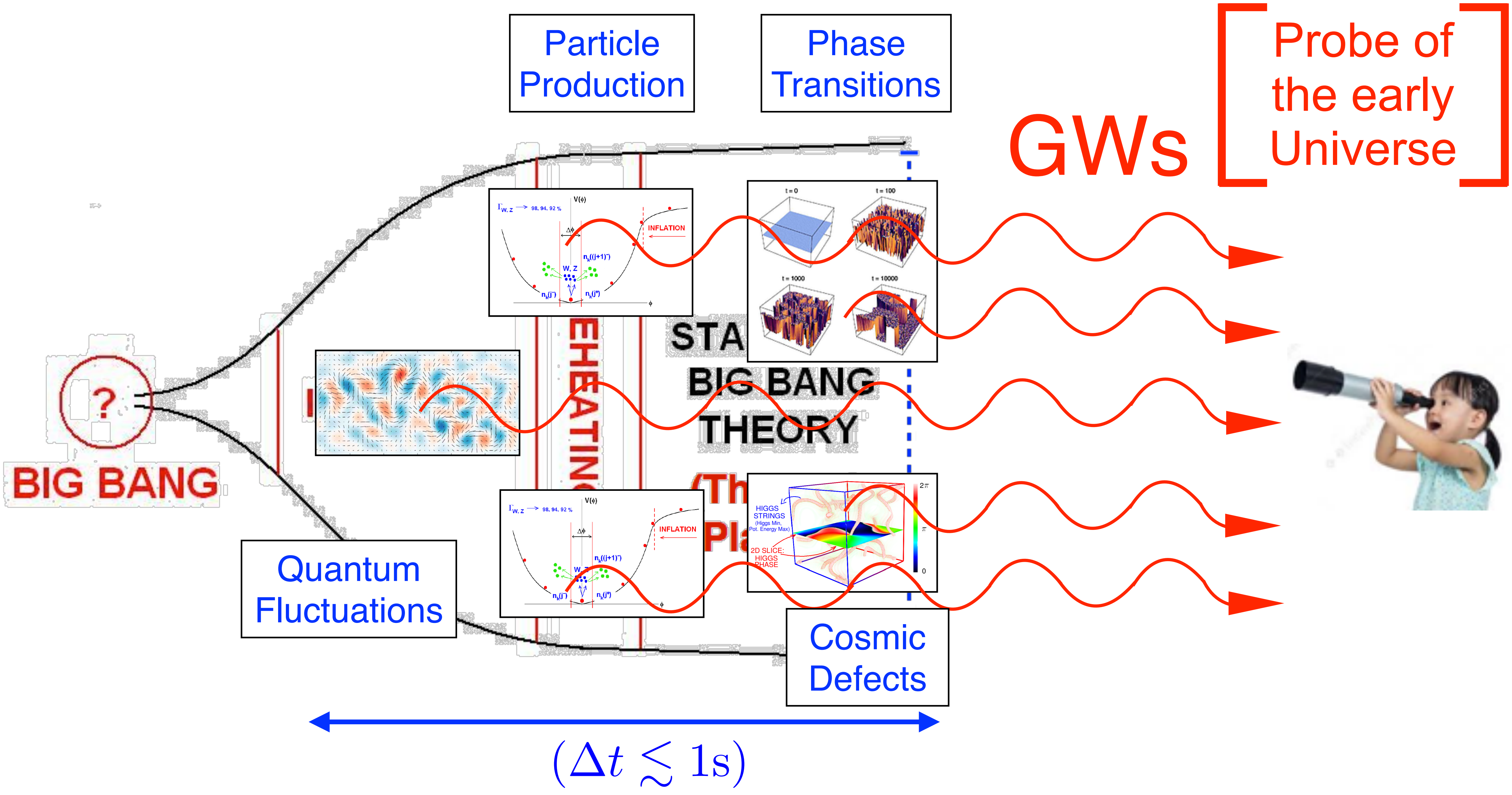
Phase Transitions

Quantum Fluctuations

Cosmic Defects

$(\Delta t \lesssim 1\mathrm{s})$

The Early Universe



Particle Production

Phase Transitions

GWs

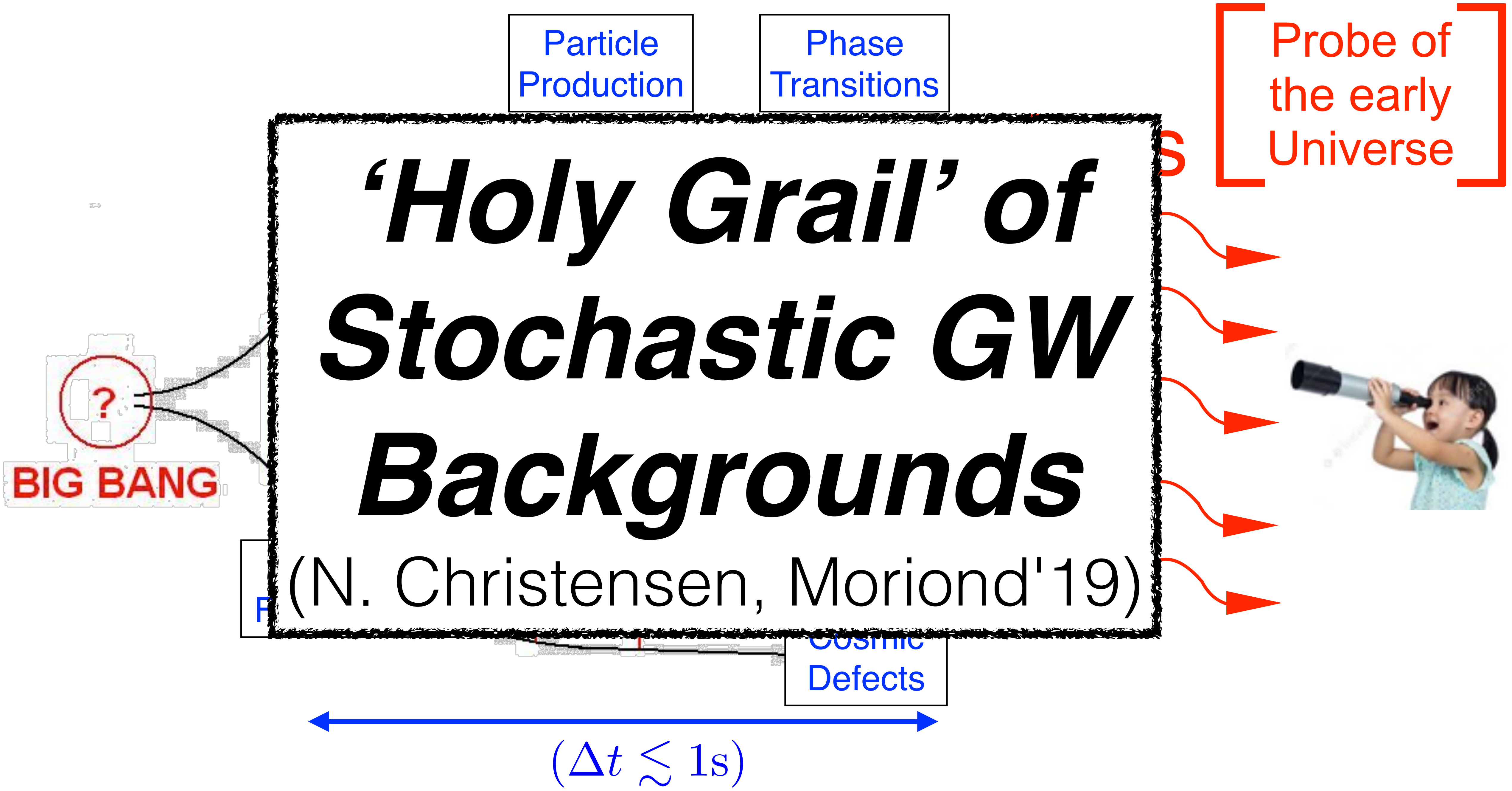
Probe of the early Universe

Quantum Fluctuations

Cosmic Defects

$(\Delta t \lesssim 1\mathrm{s})$

The Early Universe



Particle Production

Phase Transitions

Probe of the early Universe

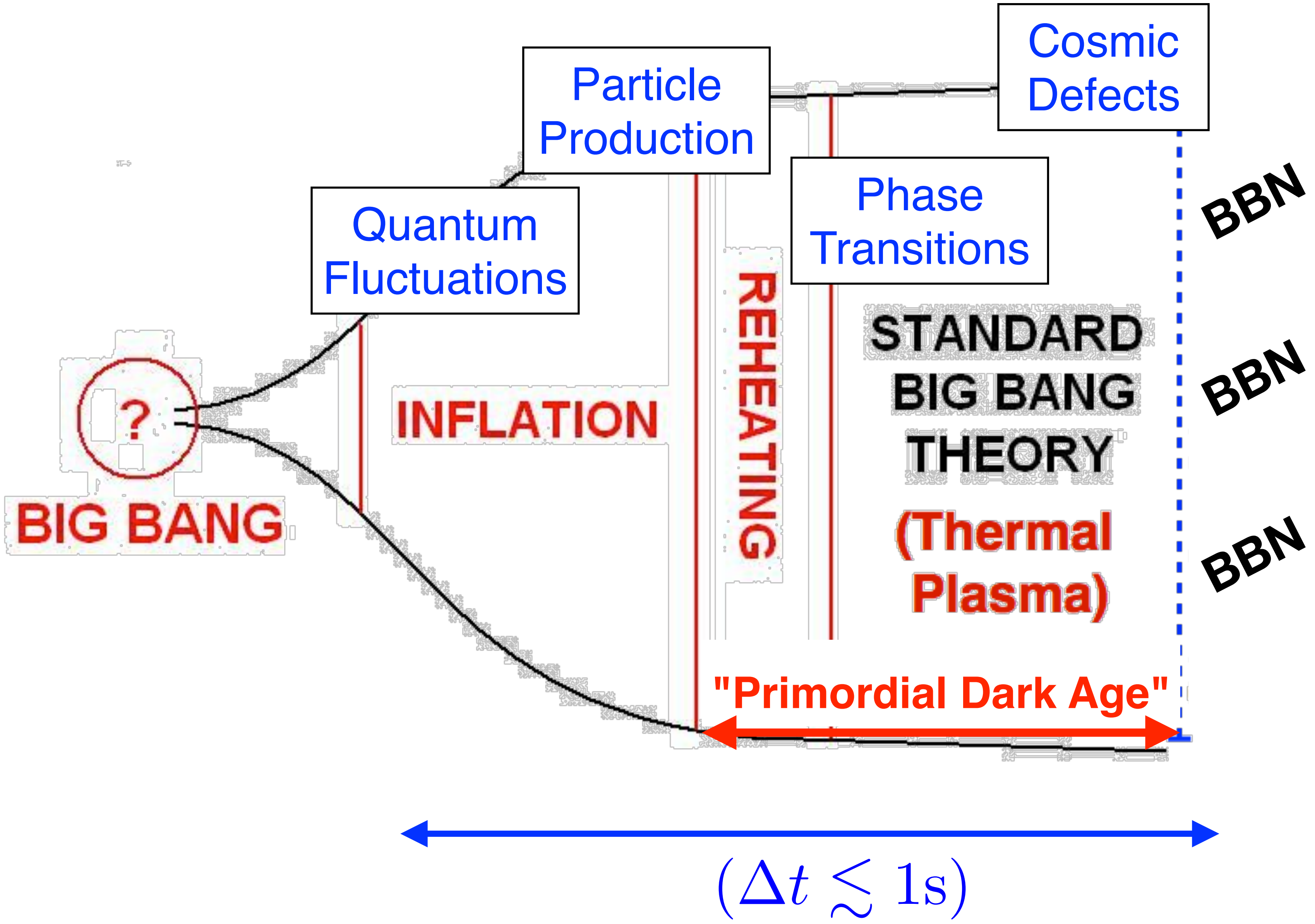
'Holy Grail' of Stochastic GW Backgrounds

(N. Christensen, Moriond'19)

Cosmic Defects

$(\Delta t \lesssim 1\mathrm{s})$

The Early Universe

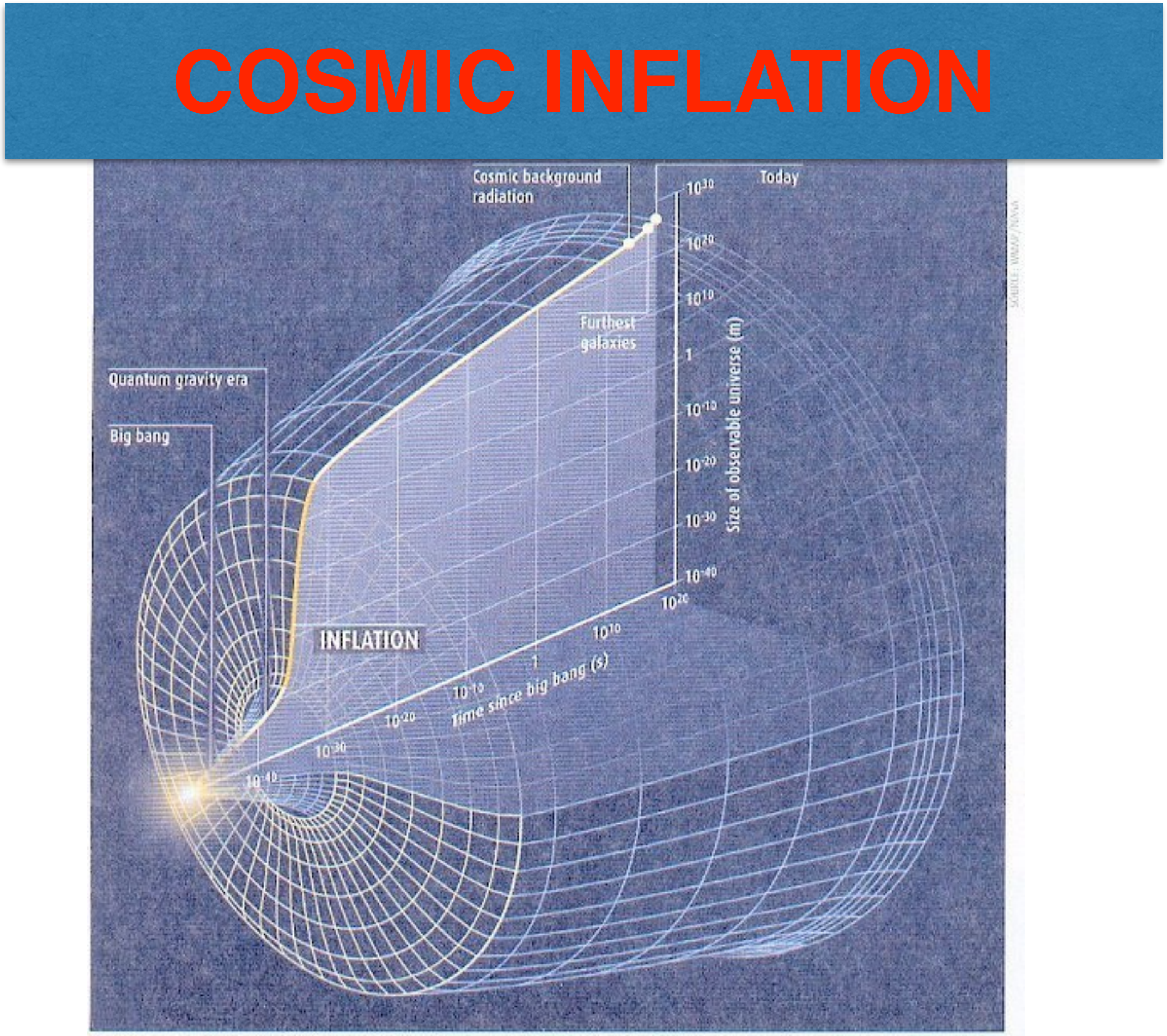


Let’s Start !

Part 1

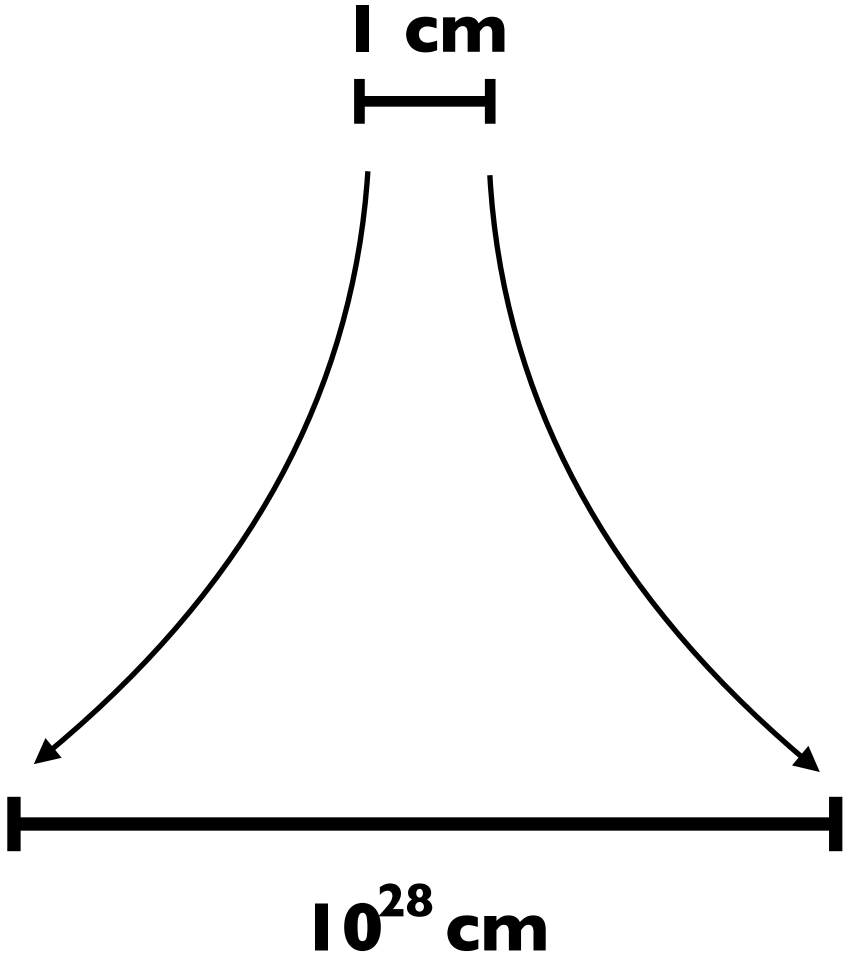
GRAVITATIONAL WAVES from INFLATION

Inflation (basics)



Required for Consistency of the Big Bang theory

$$\boxed{a \sim e^{H_* t} \gtrsim e^{60}}$$

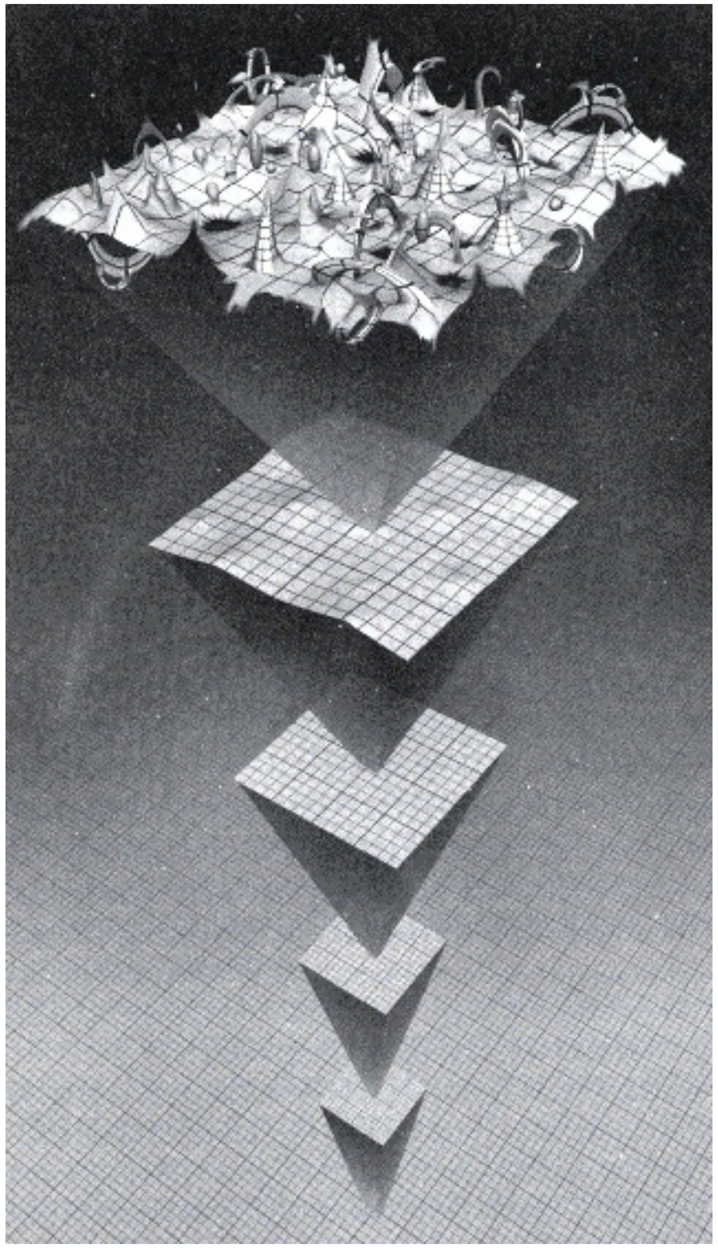


1 cm

10^{28} cm

Irreducible GW background from Inflation

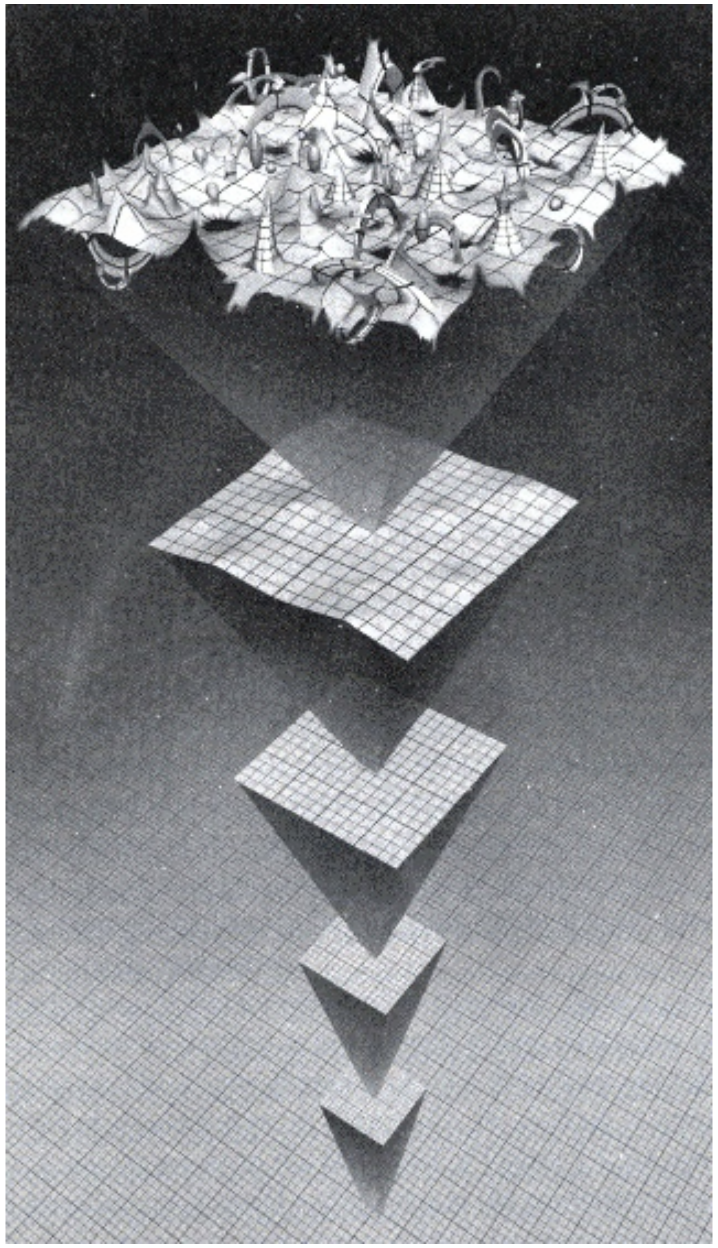
$$g_{\mu\nu} = g^{(\mathrm{B})}_{\mu\nu} + \delta g_{\mu\nu} \qquad ; \qquad [\delta g_{\mu\nu}]^{\mathrm{TT}} = h_{ij} \ , \ \left\{ \begin{array}{l} h_{ii} = 0 \\ \partial_i h_{ij} = 0 \end{array} \right.$$



Quantum Fluctuations

Irreducible GW background from Inflation

$$g_{\mu\nu} = g^{(\mathrm{B})}_{\mu\nu} + \delta g_{\mu\nu} \quad ; \quad [\delta g_{\mu\nu}]^{\mathrm{TT}} = h_{ij} \; , \; \begin{cases} h_{ii} = 0 \\ \partial_i h_{ij} = 0 \end{cases}$$



Quantum Fluctuations

$$\left\langle h_{ij}(\vec{k},t) \right\rangle = 0$$

$$\left\langle h_{ij}(\vec{k},t) h^*_{ij}(\vec{k}',t) \right\rangle \equiv (2\pi)^3 \frac{2\pi^2}{k^3} \Delta^2_h(k) \delta(\vec{k}-\vec{k}')$$

$$\boxed{\Delta^2_h(k) = \frac{2}{\pi^2} \left(\frac{H}{m_p}\right)^2 \left(\frac{k}{aH}\right)^{n_t}}$$

$$\boxed{n_t \equiv -2\epsilon}$$

energy scale

Irreducible GW background from Inflation

$$\boxed{\Omega_{\rm GW}^{(o)}(f) \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\rho_{\rm GW}}{d\log k}\right)_o = \underbrace{\frac{\Omega_{\rm Rad}^{(o)}}{24}}\Delta_{h_*}^2(k)}$$

Transfer Funct.: $T(k) \propto k^0(\mathrm{RD})$

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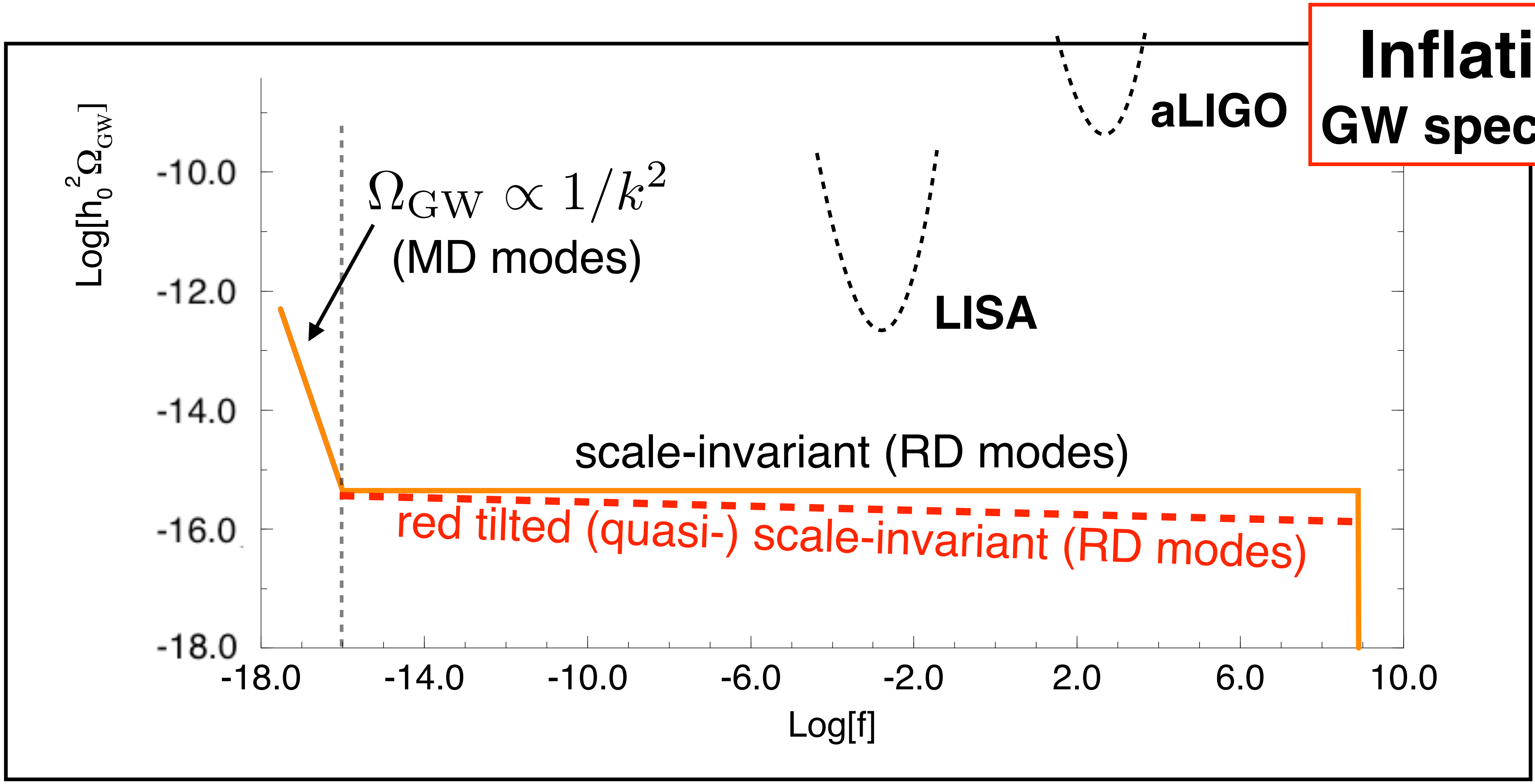
energy scale

Inflation GW spectrum



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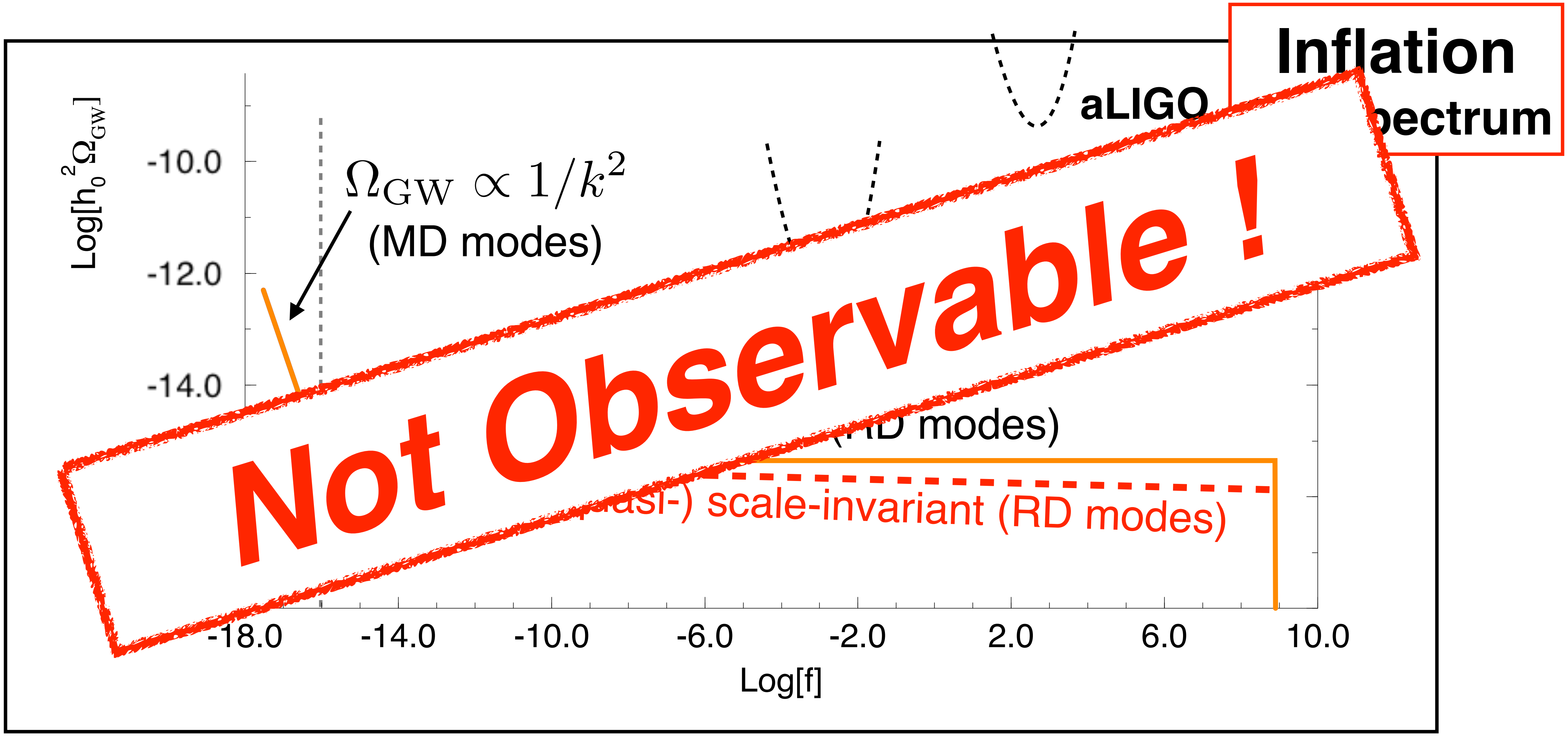

Inflation
pectrum

Not Observable !

Irreducible GW background from Inflation

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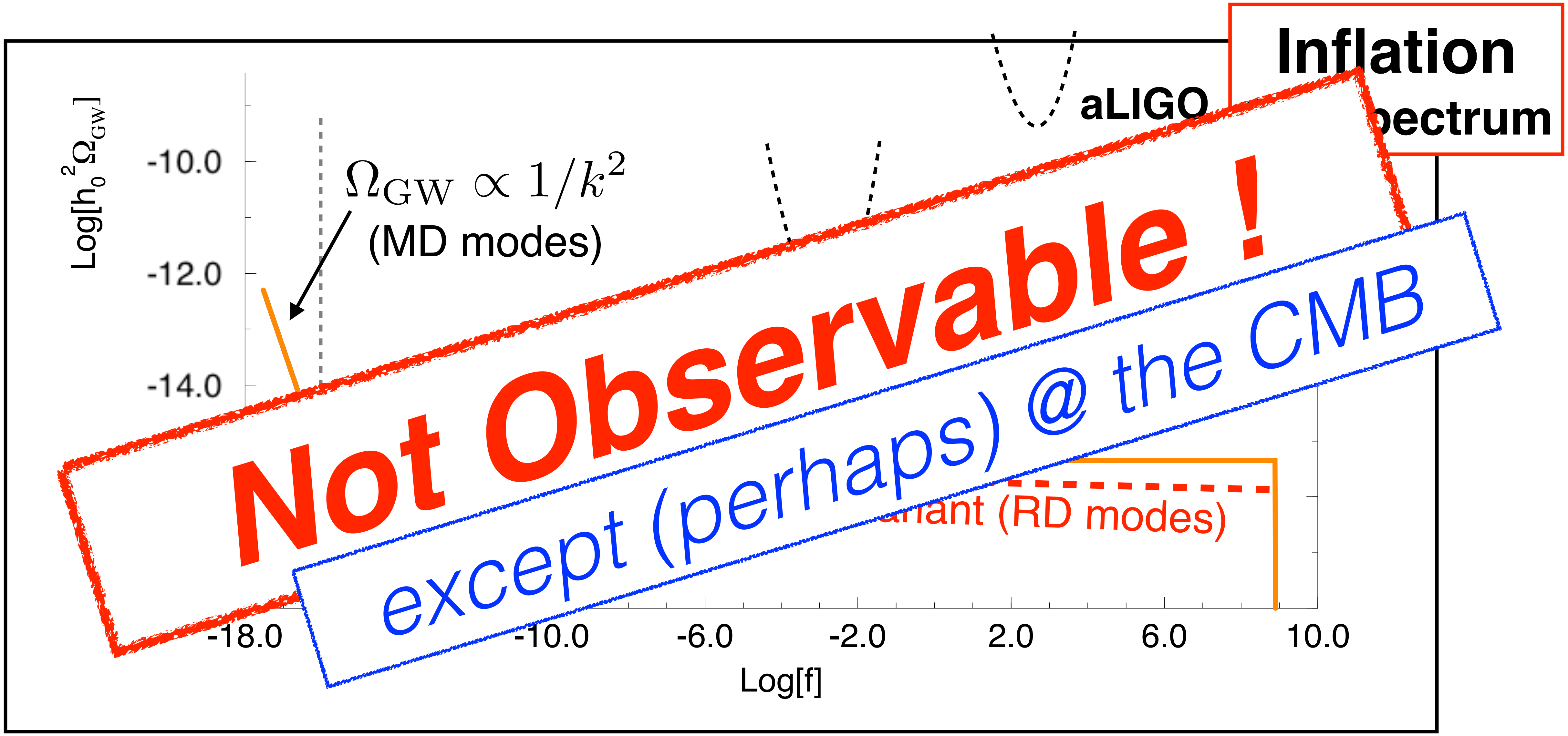

Inflation
pectrum

Not Observable !

except (perhaps) @ the CMB

Irreducible GW background from Inflation

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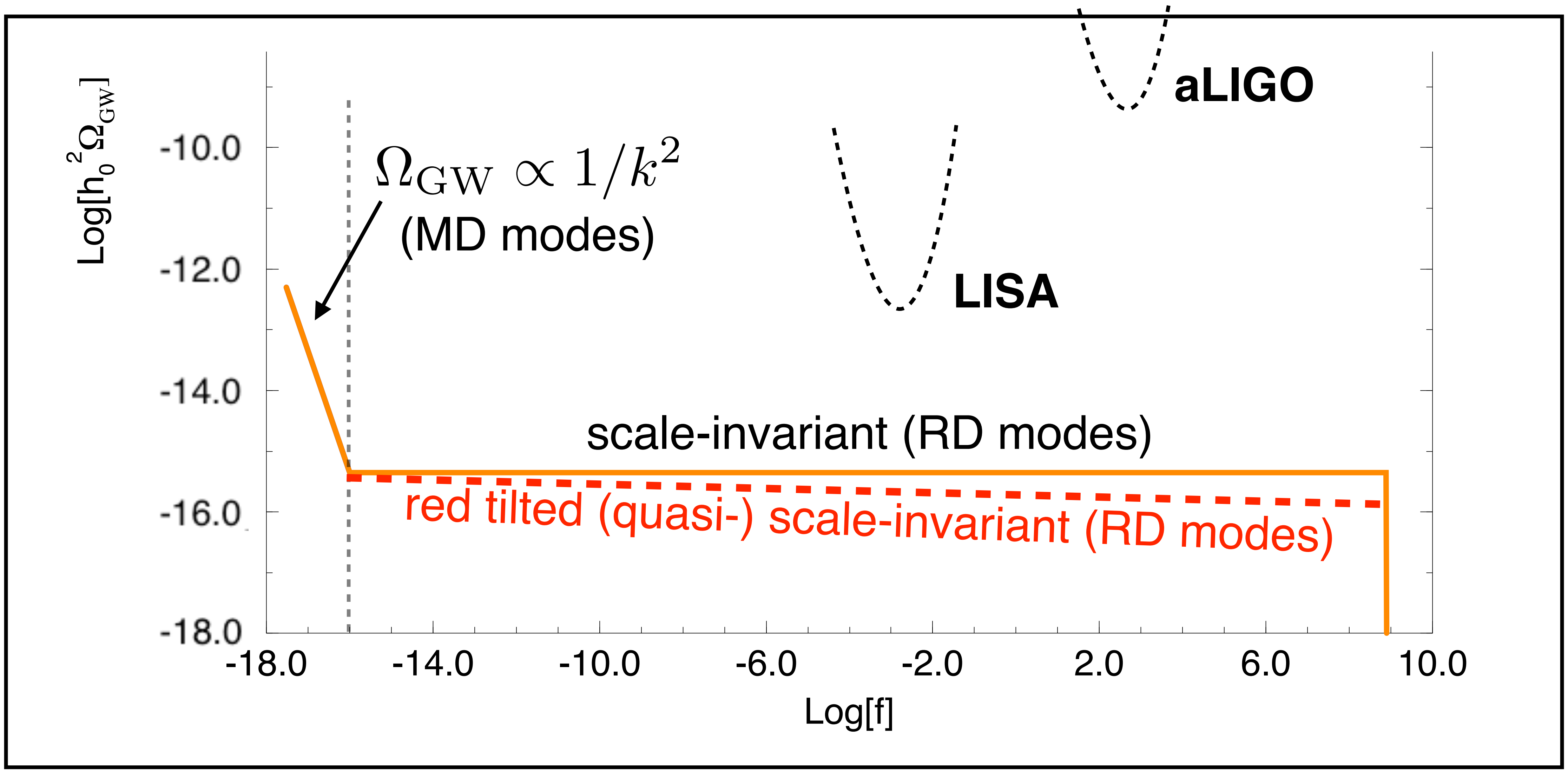

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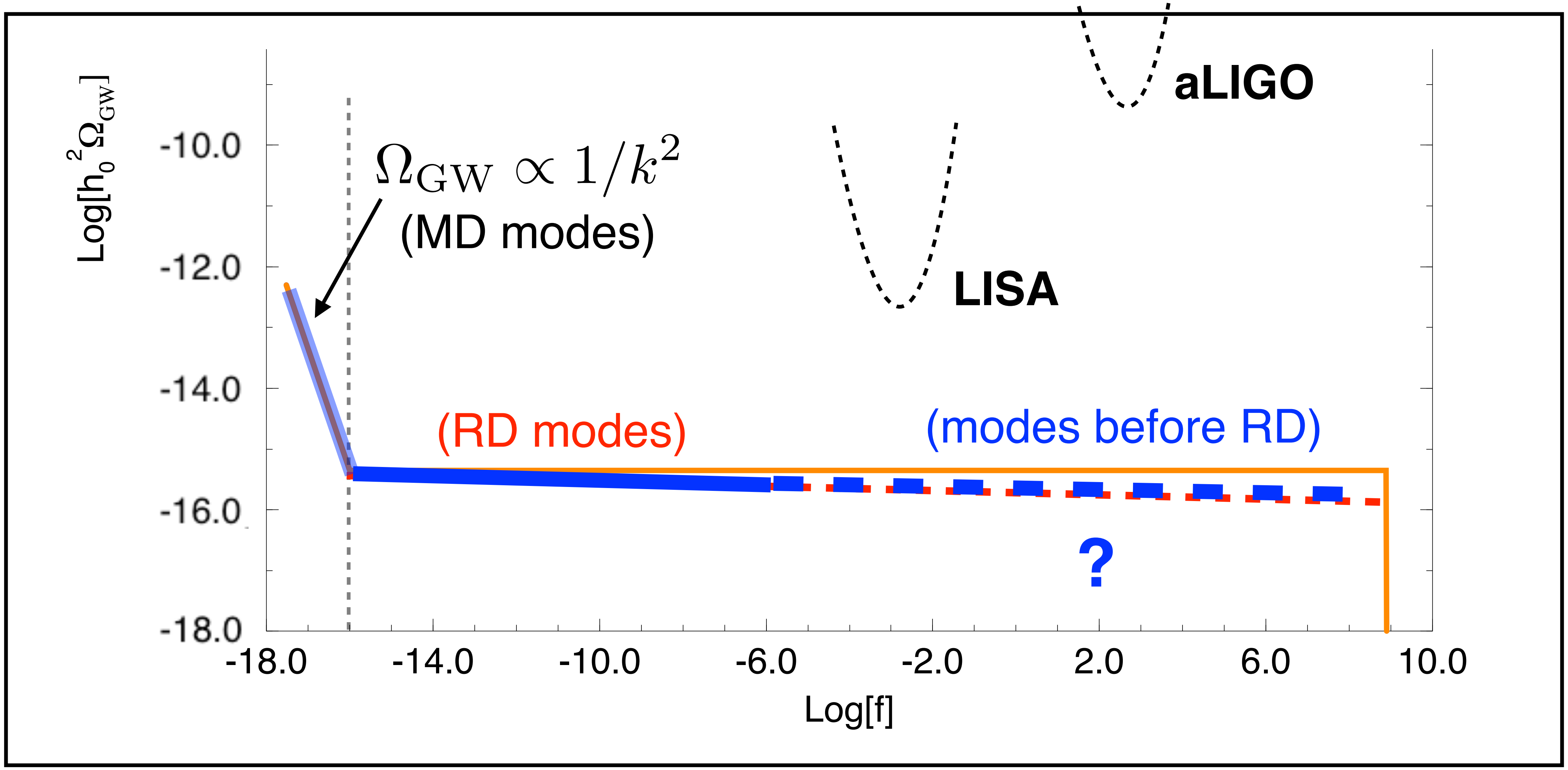
$$\boxed{n_t \equiv -2\epsilon}$$

energy scale

Transfer Funct.: $T(k) \propto k^0(\mathrm{RD})$

Period before RD: $T(k) \propto k^{2\frac{(w_s-1/3)}{(w_s+1/3)}}$



Irreducible GW background from Inflation

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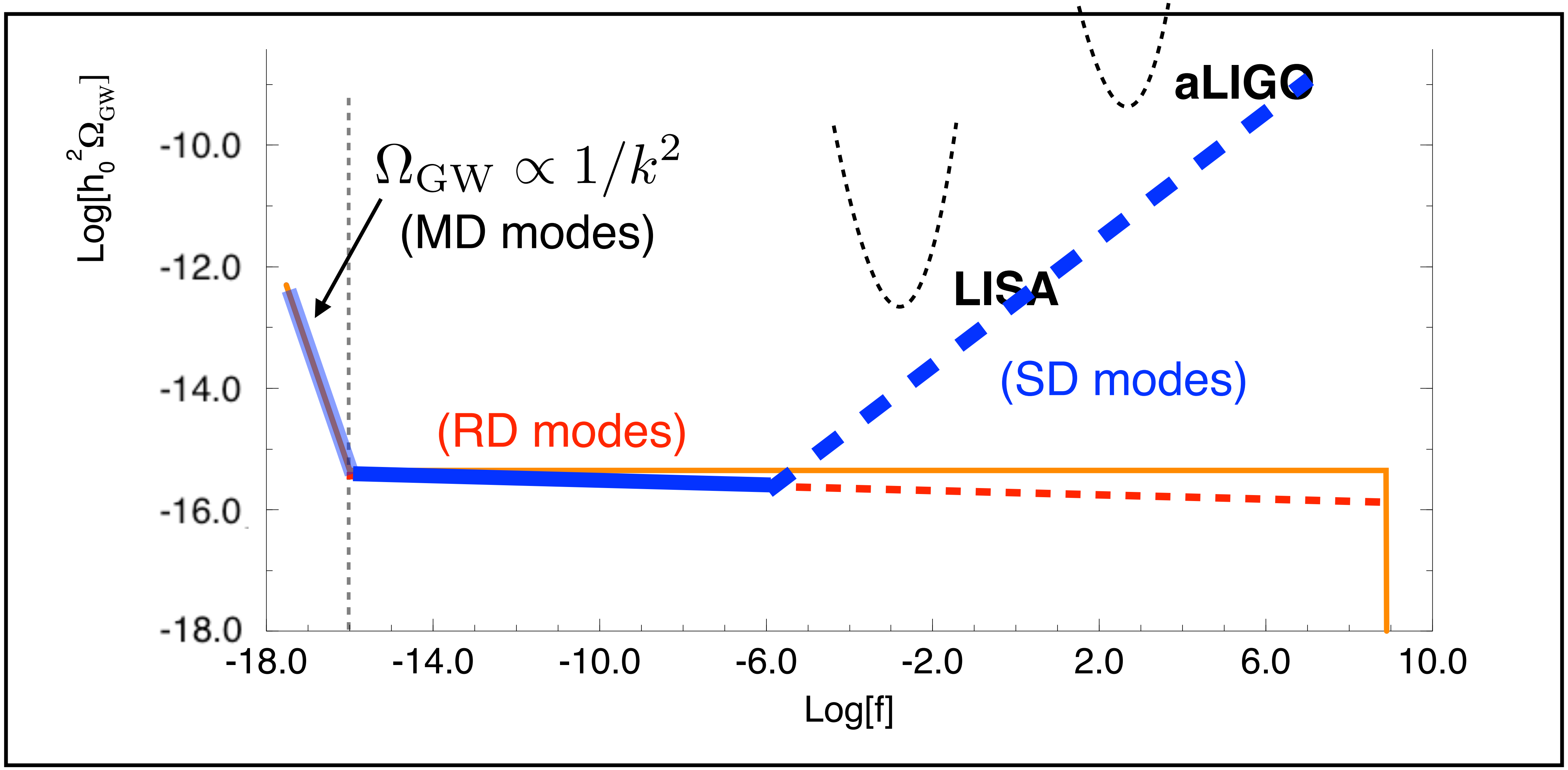
$\boxed{n_t \equiv -2\epsilon}$

energy scale

Transfer Funct.: $T(k) \propto k^0 (\mathrm{RD})$

Stiff Period: $T(k) \propto k^{2\frac{(w_s-1/3)}{(w_s+1/3)}}$ $(1/3 < \omega_s < 1)$



After a few pages computation of the Transfer function @ Stiff Domination ….

Inflationary GW background

$$\boxed{\Omega_{\rm GW}^{(0)}(f) = \underbrace{\Omega_{\rm GW}^{(0)}\Big|_{\rm plateau}}_{\textbf{Rad. Plateau}} \times \underbrace{\mathcal{W}(f/f_{\rm RD}) \times \mathcal{A}_{\rm s}\left(\frac{f}{f_{\rm RD}}\right)^{n_t(w_s)}}_{\textbf{Transfer Funct. Stiff Period}}}\,, \quad \boxed{\Omega_{\rm GW}^{(0)}\Big|_{\rm plateau} \simeq 2\cdot 10^{-16}\left(\frac{H_*}{H_{\rm max}}\right)^2}$$

Window x *power-law*

Inflationary GW background

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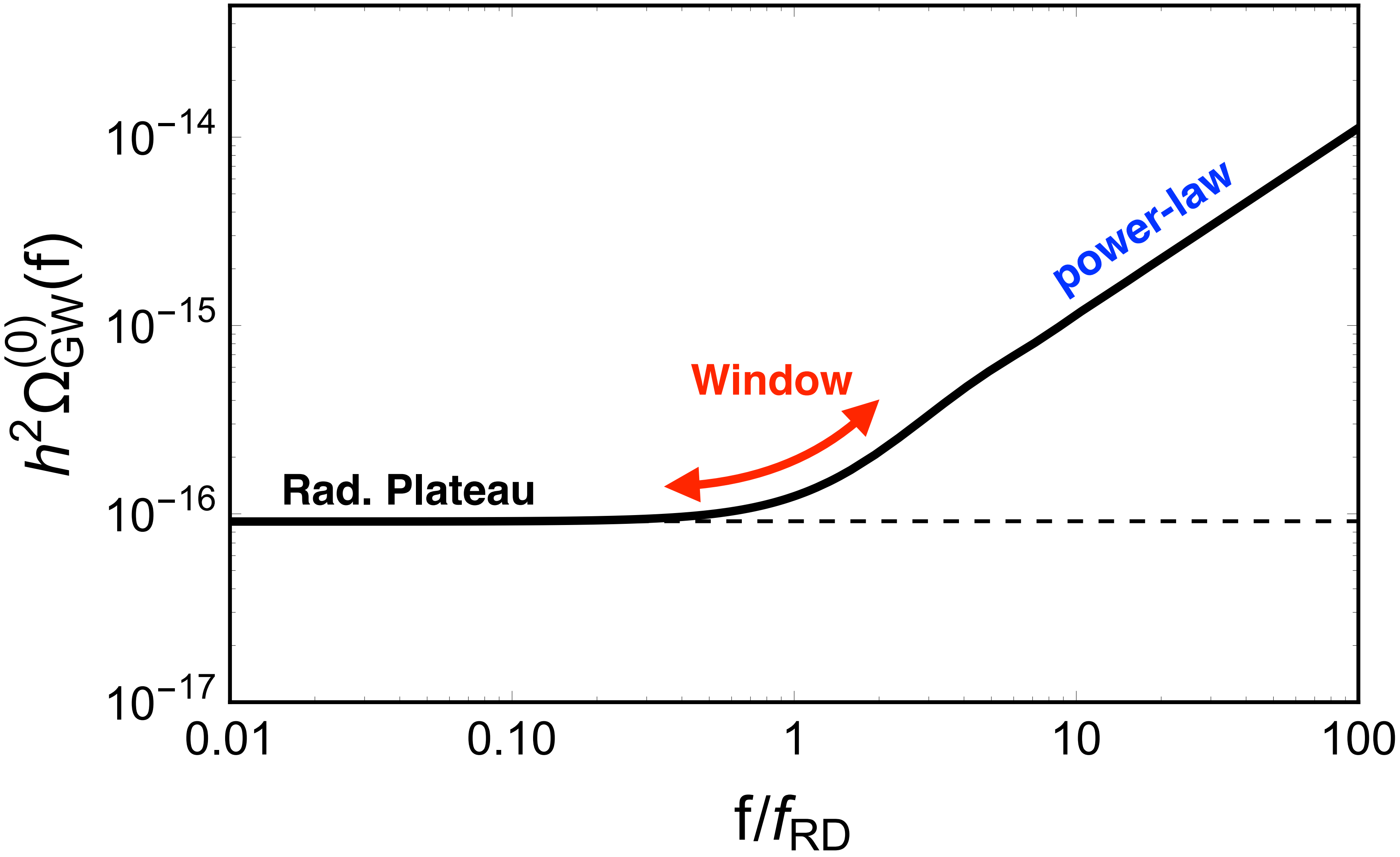

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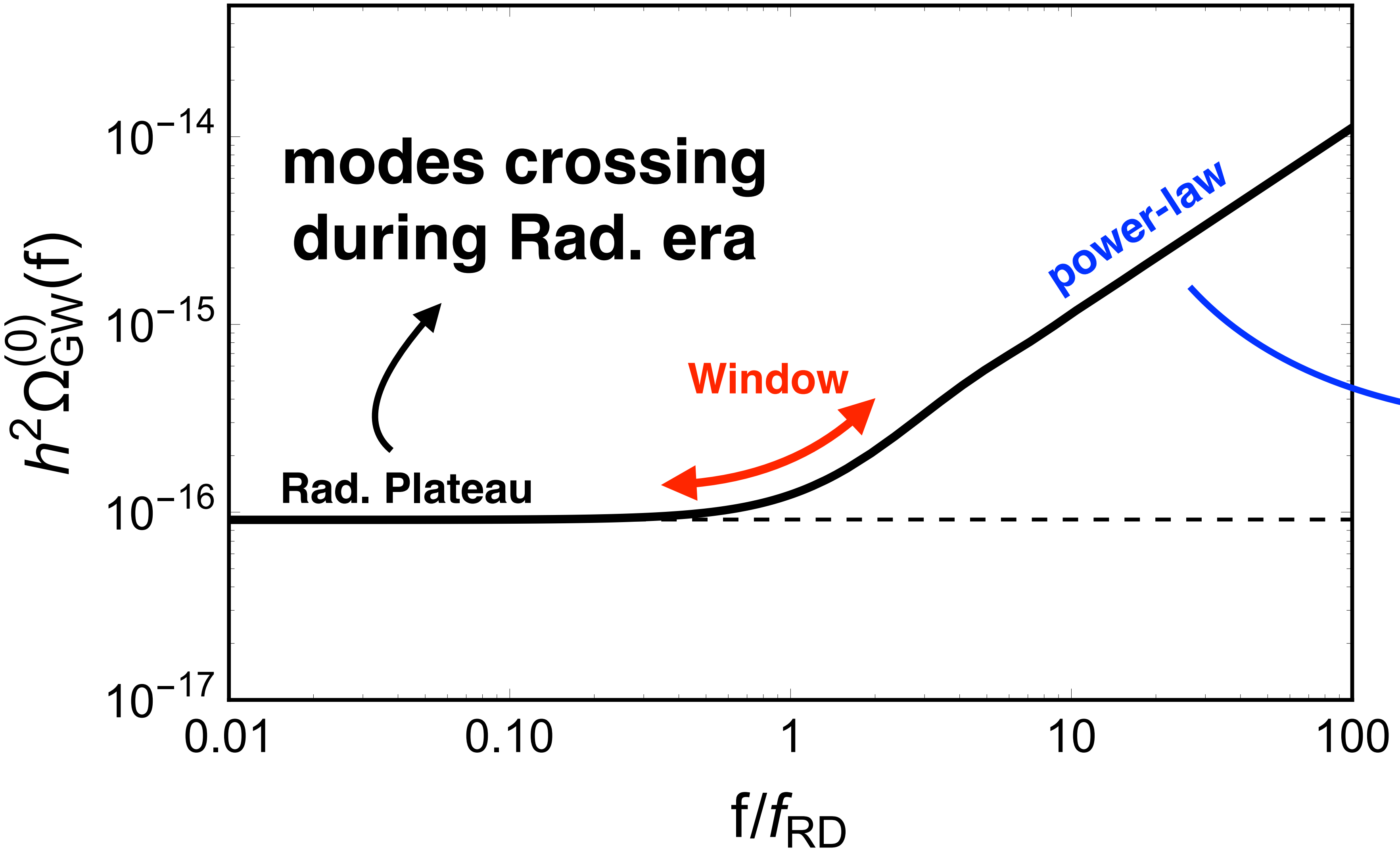
Rad. Plateau

Transfer Funct. Stiff Period

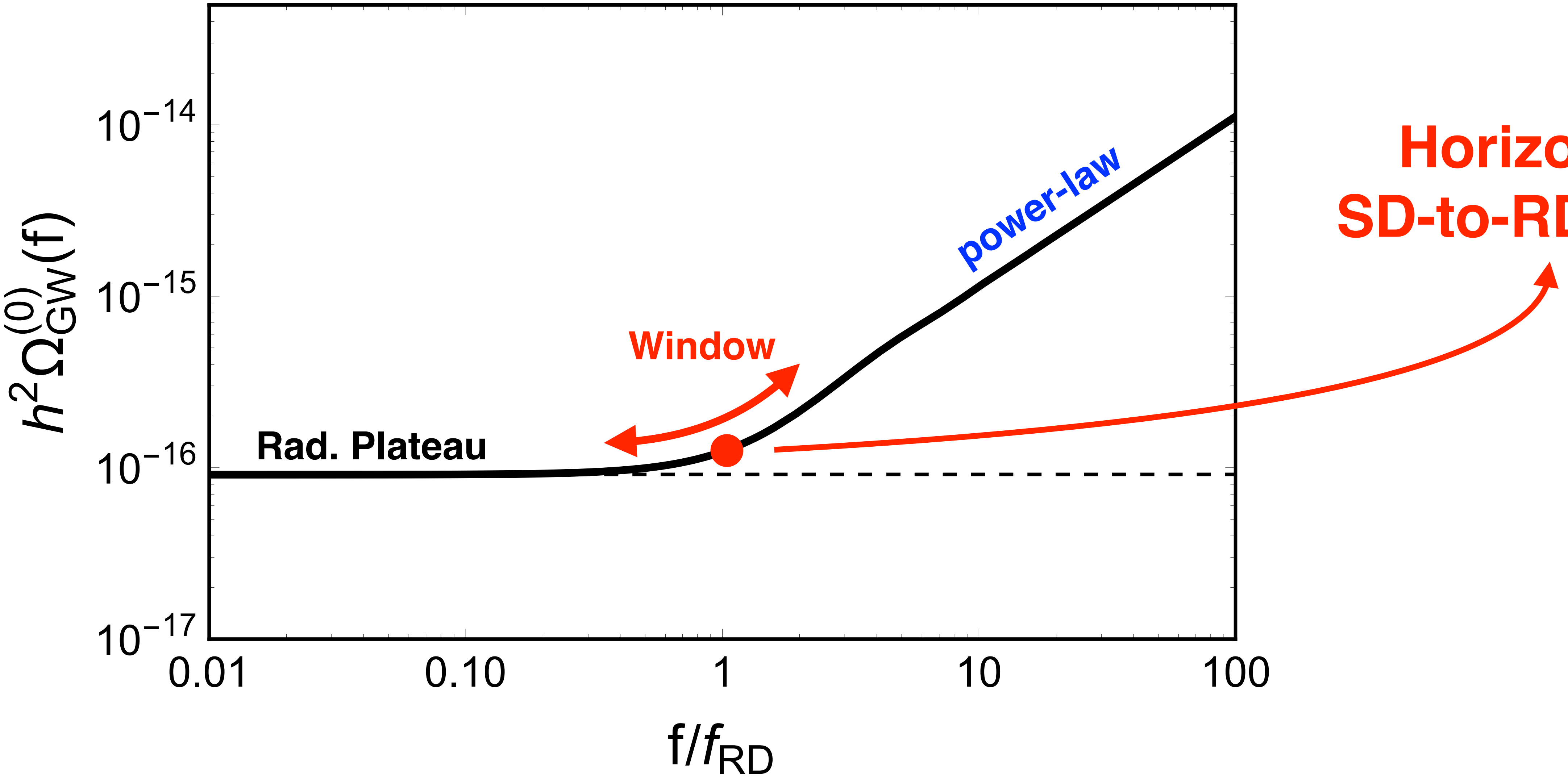
Window x *power-law*



modes crossing during Stiff era

Inflationary GW background

$$\boxed{\Omega^{(0)}_{\rm GW}(f) = \underbrace{\Omega^{(0)}_{\rm GW}\Big|_{\rm plateau}}_{\substack{\textbf{Rad.}\\ \textbf{Plateau}}} \times \underbrace{\mathcal{W}(f/f_{\rm RD}) \times \mathcal{A}_{\rm s} \left(\frac{f}{f_{\rm RD}}\right)^{n_t(w_s)}}_{\substack{\textbf{Transfer Funct. Stiff Period}\\ \textit{Window x power-law}}}}\,, \quad \boxed{\Omega^{(0)}_{\rm GW}\Big|_{\rm plateau} \simeq 2\cdot 10^{-16}\left(\frac{H_*}{H_{\rm max}}\right)^2}$$



Horizon size @ SD-to-RD transition

(

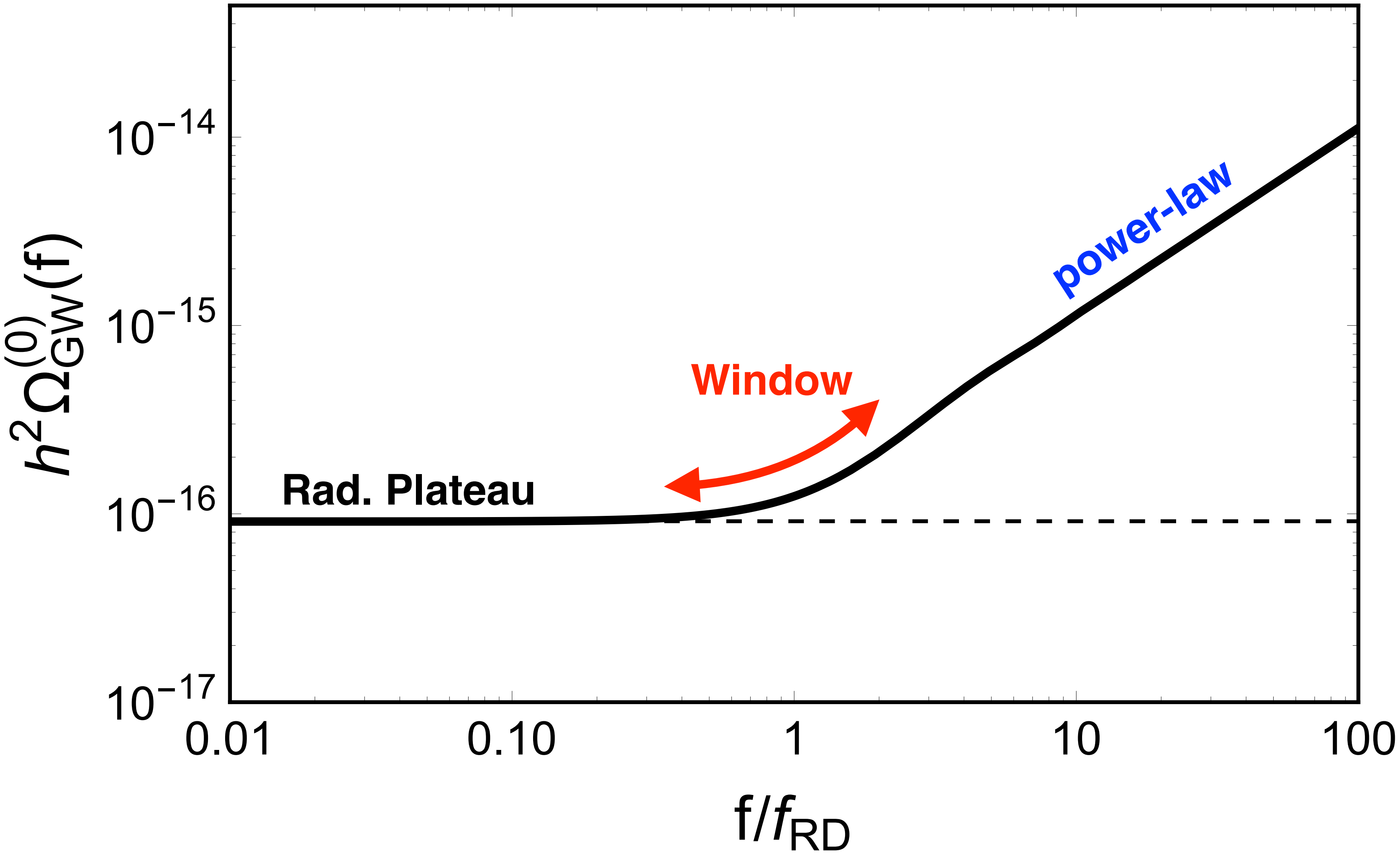
Inflationary GW background

$$\boxed{\Omega^{(0)}_{\rm GW}(f) = \underbrace{\Omega^{(0)}_{\rm GW}\Big|_{\rm plateau}} \times \underbrace{\mathcal{W}(f/f_{\rm RD}) \times \mathcal{A}_{\rm s}\left(\frac{f}{f_{\rm RD}}\right)^{n_t(w_s)}}}\,, \quad \boxed{\Omega^{(0)}_{\rm GW}\Big|_{\rm plateau} \simeq 2\cdot 10^{-16}\left(\frac{H_*}{H_{\rm max}}\right)^2}$$

Rad. Plateau

Transfer Funct. Stiff Period

Window x *power-law*



Inflationary GW background

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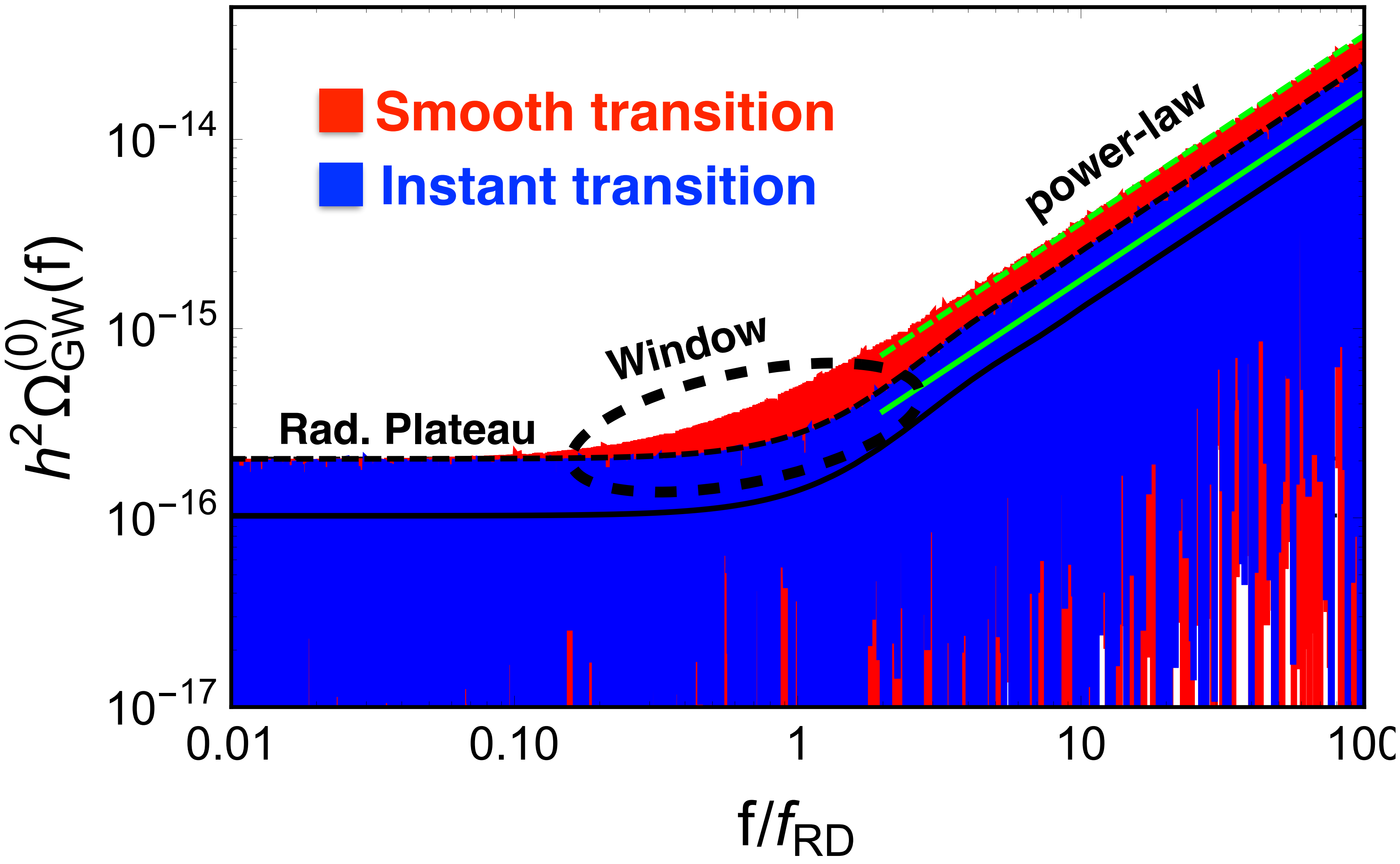
Window x *power-law*



Real signal:
highly oscillatory

Stochastic Signal:
average measurement

$$\langle \dot{h}_{ij}(f)\dot{h}_{ij}(f)\rangle = \mathcal{P}_h(f)$$

)

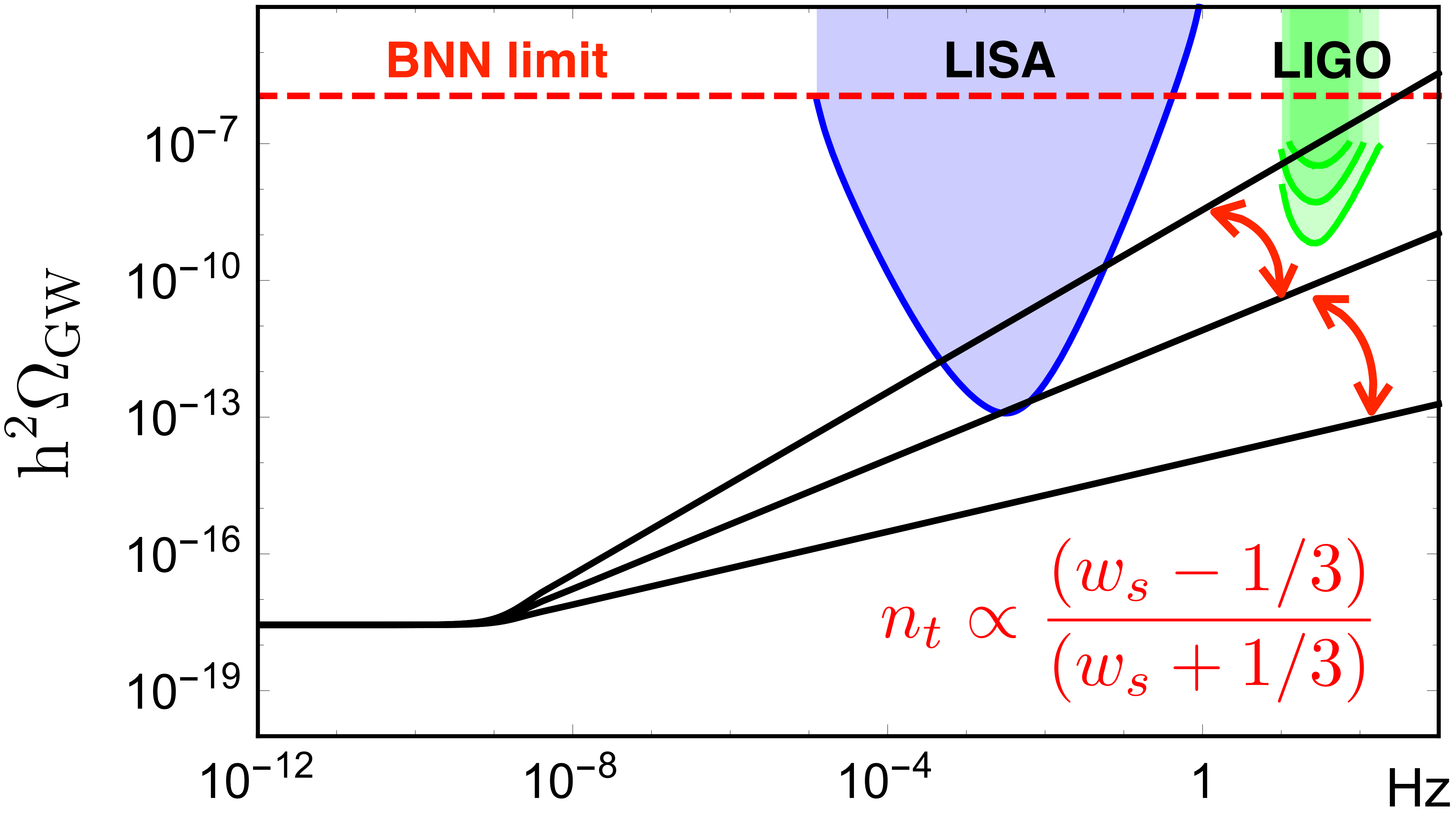
Inflationary GW background

$$\boxed{\Omega^{(0)}_{\rm GW}(f) = \underbrace{\Omega^{(0)}_{\rm GW}\Big|_{\rm plateau}} \times \underbrace{\mathcal{W}(f/f_{\rm RD}) \times \mathcal{A}_{\rm s}\left(\frac{f}{f_{\rm RD}}\right)^{n_t(w_s)}}}\,, \quad \boxed{\Omega^{(0)}_{\rm GW}\Big|_{\rm plateau} \simeq 2\cdot 10^{-16}\left(\frac{H_*}{H_{\rm max}}\right)^2}$$

Rad. Plateau

Transfer Funct. Stiff Period

Window x *power-law*



$$n_t \propto \frac{(w_s - 1/3)}{(w_s + 1/3)}$$

Slope/Tilt
(EoS
Stiff
Period)

Inflationary GW background

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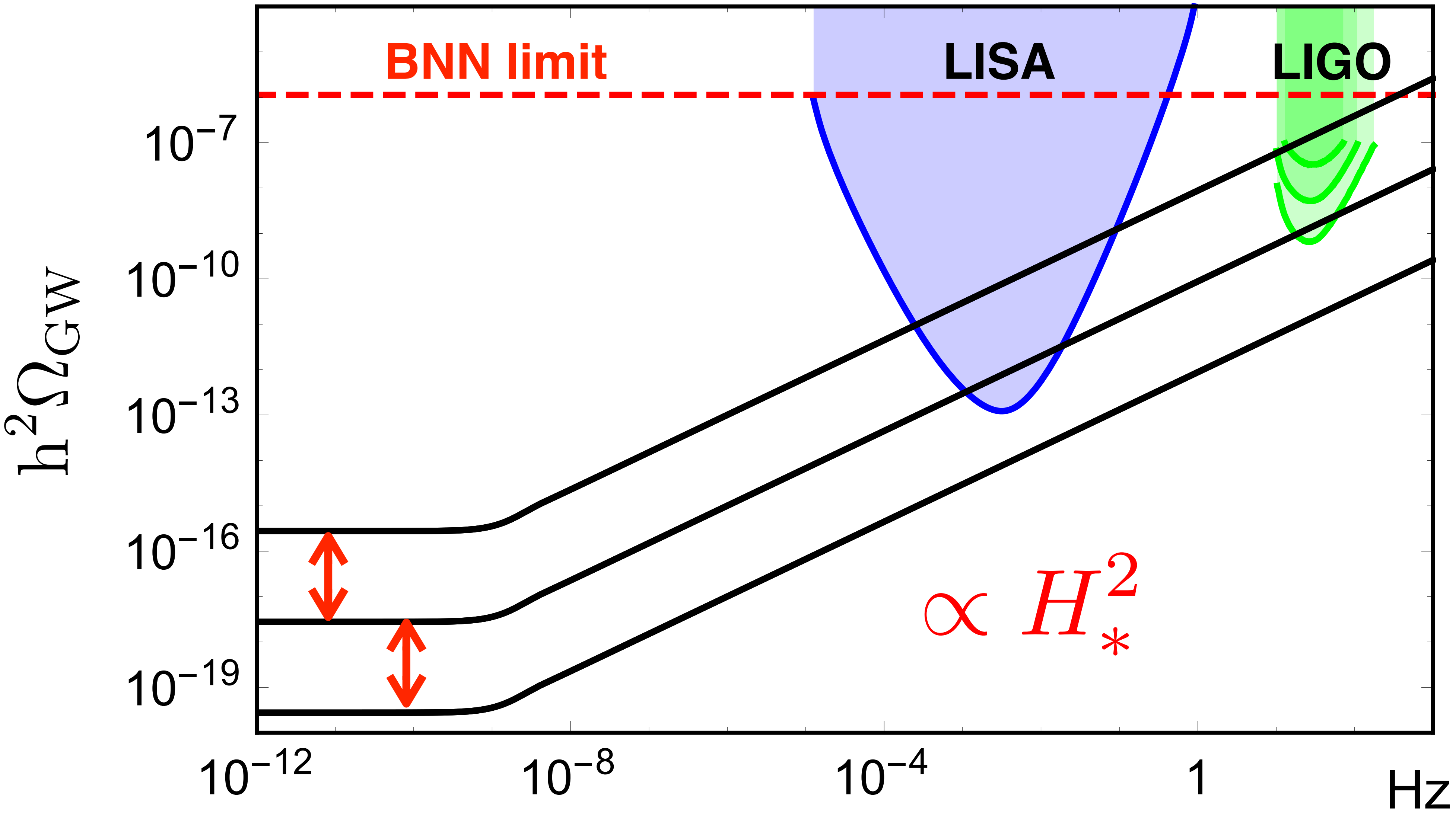
Rad. Plateau

Transfer Funct. Stiff Period

Window x power-law



Overall Amplitude
(Energy Scale Inflation)

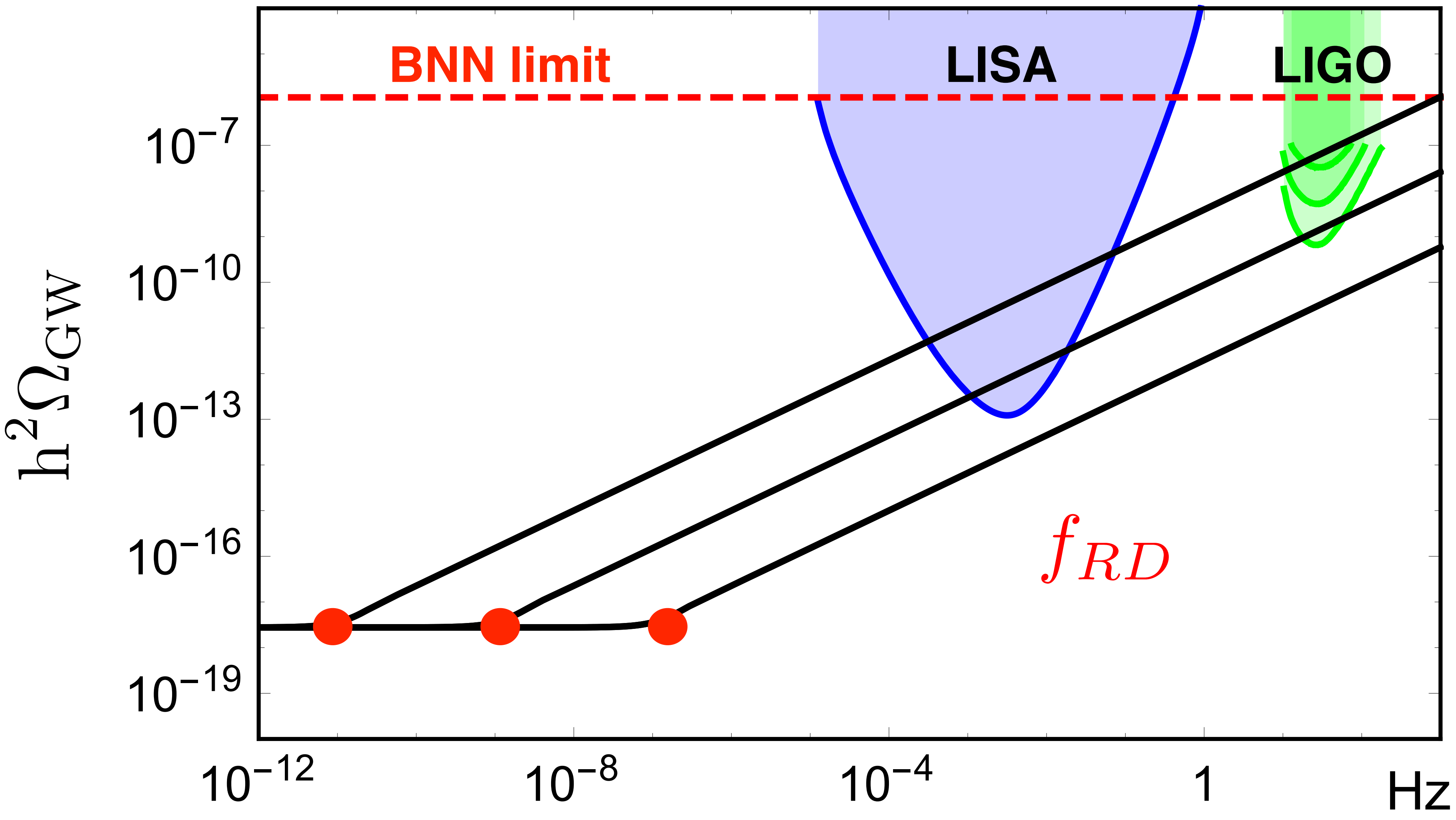
Inflationary GW background

$$\boxed{\Omega_{\rm GW}^{(0)}(f) = \underbrace{\Omega_{\rm GW}^{(0)}\Big|_{\rm plateau}}_{} \times \underbrace{\mathcal{W}(f/f_{\rm RD}) \times \mathcal{A}_{\rm s}\left(\frac{f}{f_{\rm RD}}\right)^{n_t(w_s)}}_{}}\,, \quad \boxed{\Omega_{\rm GW}^{(0)}\Big|_{\rm plateau} \simeq 2\cdot 10^{-16}\left(\frac{H_*}{H_{\rm max}}\right)^2}$$

Rad. Plateau

Transfer Funct. Stiff Period

Window x power-law



Freq. RD

$$k_{\rm RD} = a_{\rm RD} H_{\rm RD}$$

$$f_{\rm RD} \equiv k_{\rm RD}/(2\pi a_0)$$

SD-to-RD transition

Inflationary GW background

$$\boxed{\Omega^{(0)}_{\rm GW}(f) = \underbrace{\Omega^{(0)}_{\rm GW}\Big|_{\rm plateau}}_{\text{Rad. Plateau}} \times \underbrace{\mathcal{W}(f/f_{\rm RD}) \times \mathcal{A}_{\rm s} \left(\frac{f}{f_{\rm RD}}\right)^{n_t(w_s)}}_{\text{Transfer Funct. Stiff Period}}}\,, \quad \boxed{\Omega^{(0)}_{\rm GW}\Big|_{\rm plateau} \simeq 2 \cdot 10^{-16} \left(\frac{H_*}{H_{\rm max}}\right)^2}$$

Window x *power-law*

$$\Omega^{(0)}_{\rm GW}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}})$$

Energy Scale Inflation

EoS Stiff Period

Duration Stiff Period

GW background

$$\Omega_{\rm GW}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}})$$

Energy Scale

EoS Stiff

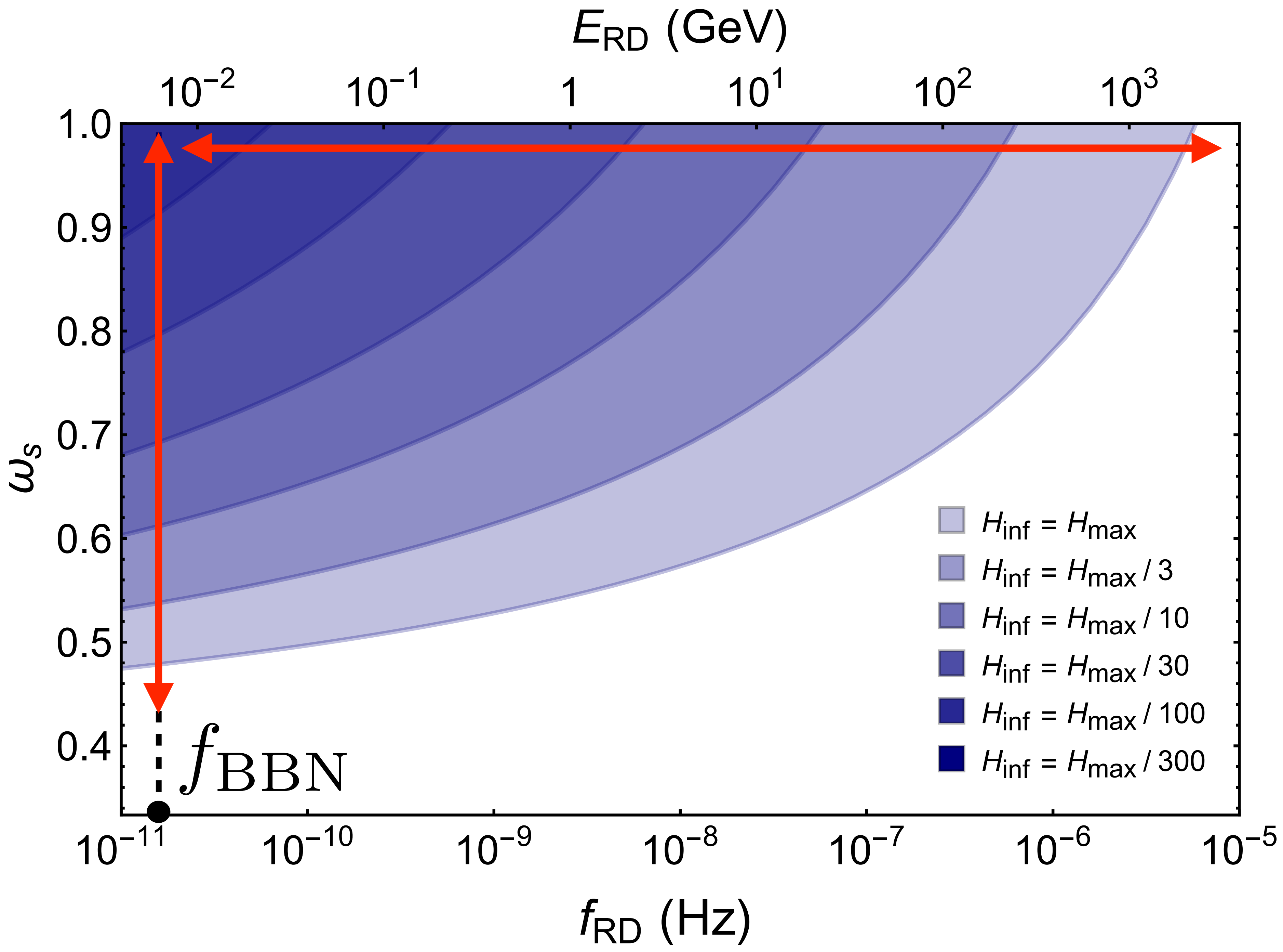
Duration Stiff

Observability @ LISA (~ 2034)

GW background

$$\Omega^{(0)}_{\rm GW}(f;\underset{\text{Energy Scale}}{\underline{H_*}},\underset{\text{EoS Stiff}}{\underline{w_s}},\underset{\text{Duration Stiff}}{\underline{f_{RD}}})$$

Observability @ LISA (~ 2034)



GW background $\Omega^{(0)}_{\rm GW}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}})$

Energy Scale

EoS Stiff

Duration Stiff

Observability @ LISA (~ 2034)

$$9.1 \times 10^{10}\ \text{GeV} < H_{\rm inf} < 6.6 \times 10^{13}\ \text{GeV}$$

$$0.47 < w_{\rm S} < 1$$

$$10^{-11}\ \text{Hz} \lesssim f_{\rm RD} < 4.6 \times 10^{-6}\ \text{Hz}$$

$$10^{-3}\ \text{GeV} \lesssim E_{\rm RD} < 5.91 \times 10^{3}\ \text{GeV}$$

Significant fraction of param. space observable !

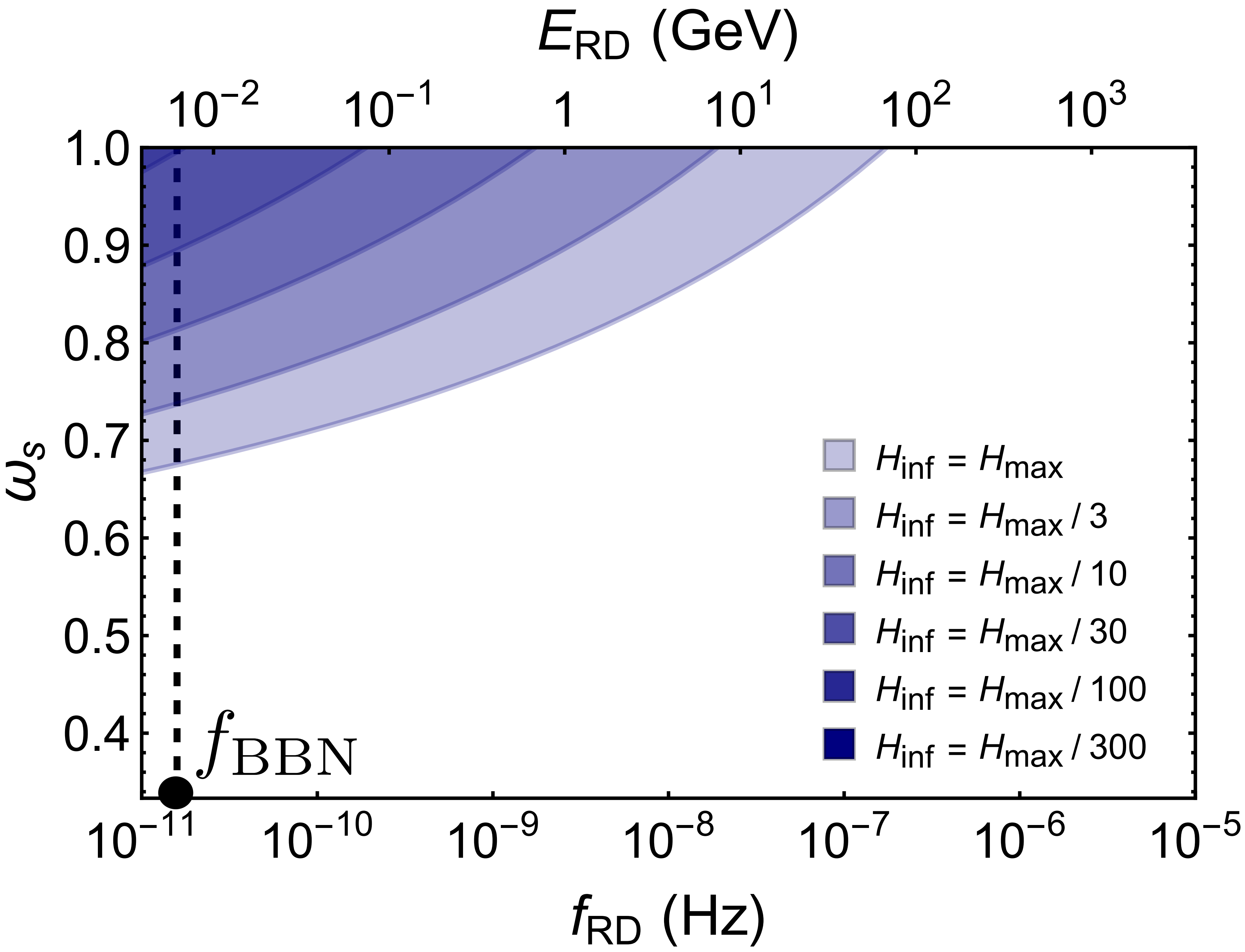
GW background $\Omega^{(0)}_{\rm GW}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}})$

Energy Scale

EoS Stiff

Duration Stiff

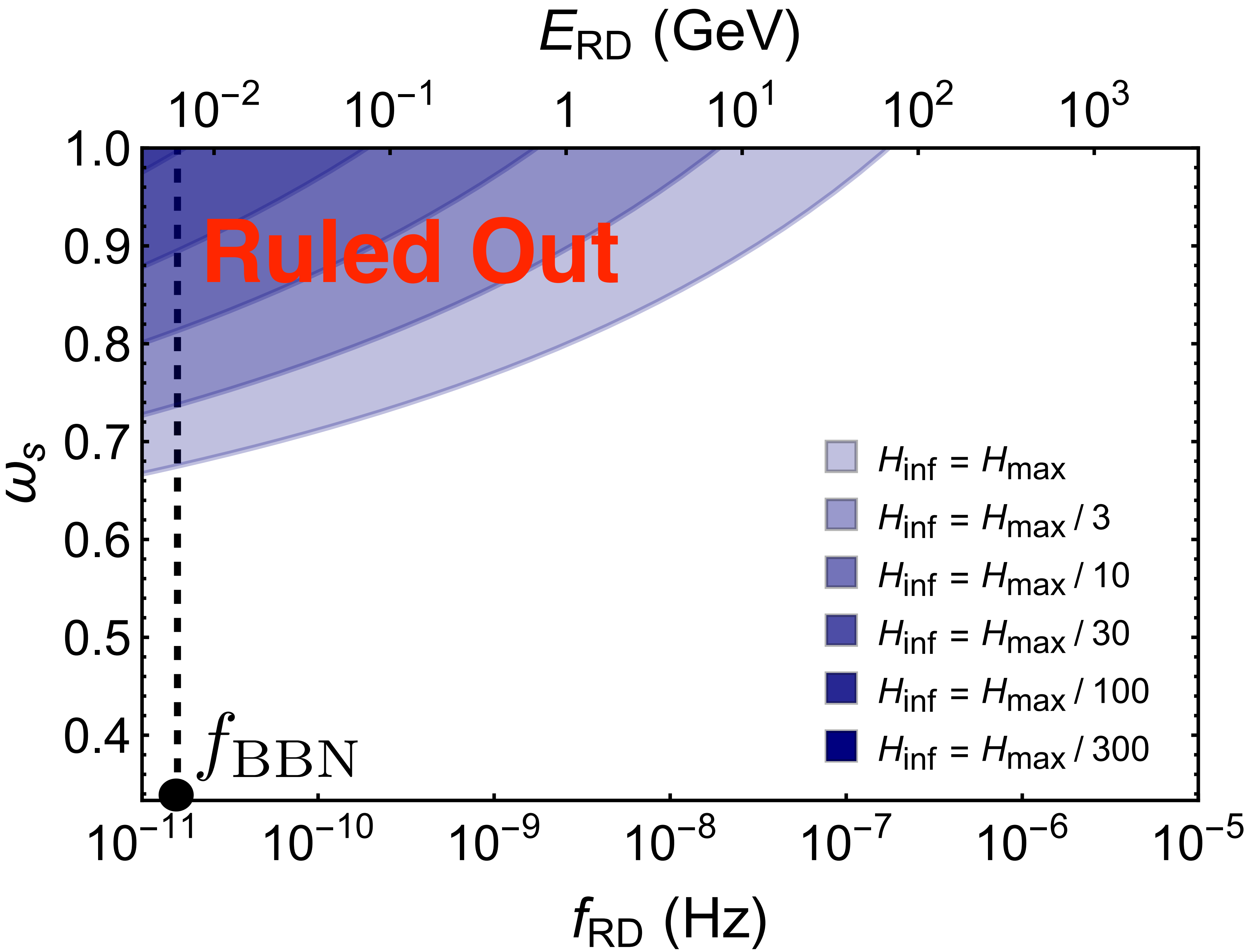
Observability @ LIGO (today)



GW background $\Omega_{\rm GW}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}})$

Energy Scale EoS Stiff Duration Stiff

Observability @ LIGO (today)



LIGO reduces parameter space probe-able by LISA !

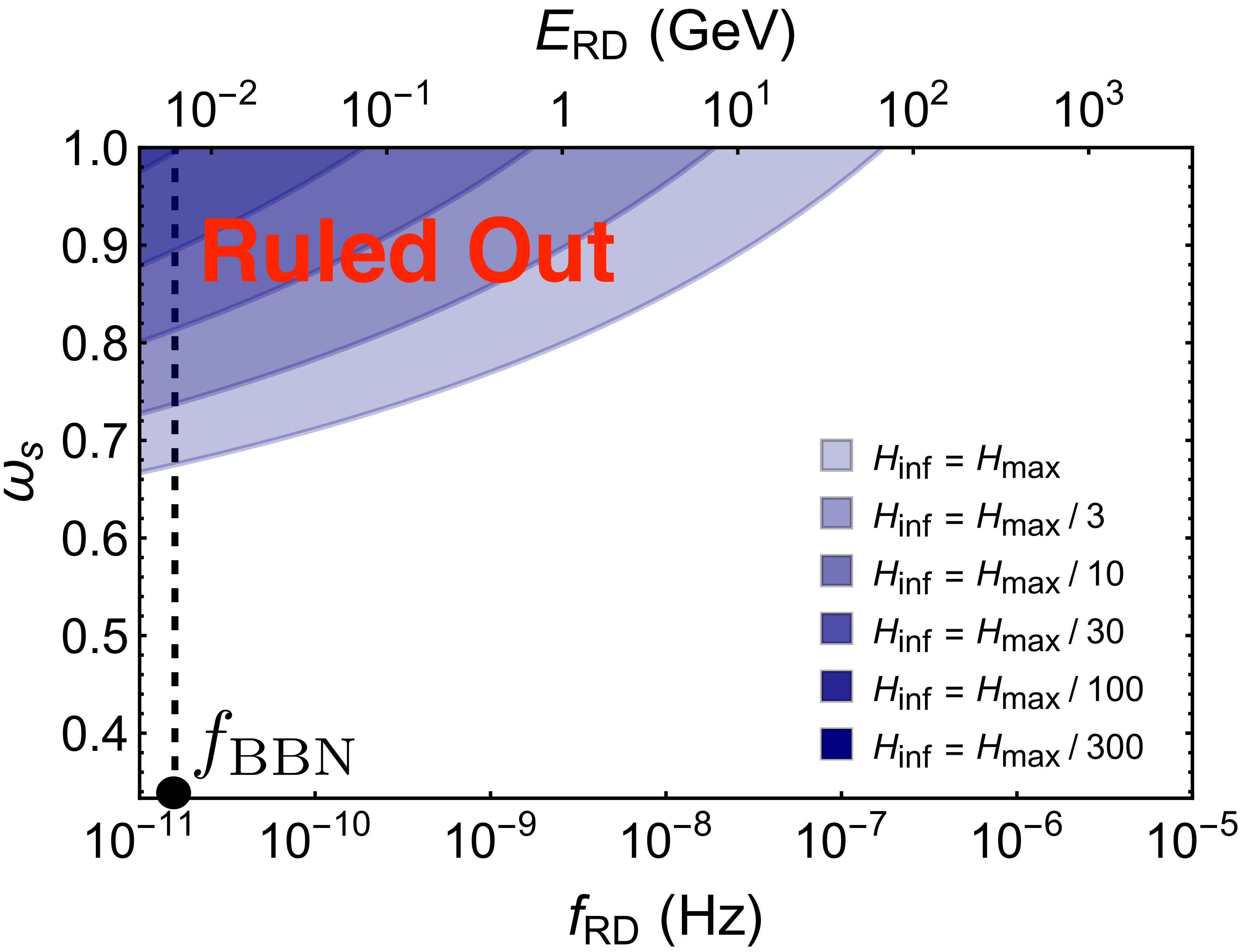
GW background $\Omega_{\rm GW}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}})$

Energy Scale

EoS Stiff

Duration Stiff

Observability @ LIGO (today)



Let's first look at consistency of scenarios

Part 2

REHEATING
(via GRAVITATION)

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\mathcal{L} = \frac{1}{\sqrt{g}} \left\{ \underbrace{(\partial\phi)^2 - V_{\rm inf}(\phi)}_{\text{inflaton}} + \underbrace{\frac{1}{2} m_{pl}^2 R}_{\substack{\text{gravity} \\ \text{(GR)}}} + \underbrace{(\partial\chi)^2 - V(\chi) - \xi\chi^2 R}_{\text{matter/rad}} - g^2\chi^2\phi^2 \right\}$$

Need to excite matter
(to reheat the Universe)

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\mathcal{L} = \frac{1}{\sqrt{g}}\left\{\underbrace{(\partial\phi)^2 - V_{\rm inf}(\phi)}_{\text{inflaton}} + \underbrace{\frac{1}{2}m_{pl}^2 R}_{\text{gravity (GR)}} + \underbrace{(\partial\chi)^2 - V(\chi) - \xi\chi^2 R}_{\text{matter/rad}} - \xcancel{g^2\chi^2\phi^2}\right\}$$

Weak coupling

Need to excite matter
(to reheat the Universe)

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\mathcal{L} = \frac{1}{\sqrt{g}}\left\{\underbrace{(\partial\phi)^2 - V_{\inf}(\phi)}_{\text{inflaton}} + \underbrace{\frac{1}{2}m_{pl}^2 R}_{\substack{\text{gravity}\\ \text{(GR)}}} + \underbrace{(\partial\chi)^2 - V(\chi) - \xi\chi^2 R}_{\text{matter/rad}} - \underset{\substack{\text{Weak}\\ \text{coupling}}}{\xcancel{g^2\chi^2\phi^2}}\right\}$$

Need to excite matter
(to reheat the Universe) → $\boxed{\rho_{\rm rad} = \delta \times 10^{-2} H_*^4}$ **Inflation does the job !**

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\mathcal{L} = \frac{1}{\sqrt{g}}\left\{\underbrace{(\partial\phi)^2 - V_{\rm inf}(\phi)}_{\text{inflaton}} + \underbrace{\frac{1}{2}m_{pl}^2 R}_{\text{gravity (GR)}} + \underbrace{(\partial\chi)^2 - V(\chi) - \xi\chi^2 R}_{\text{matter/rad}} - \cancel{g^2\chi^2\phi^2}\right\}$$

Weak coupling

Need to excite matter
(to reheat the Universe) →

$$\boxed{\rho_{\rm rad} = \delta \times 10^{-2} H_*^4} \qquad \boxed{\delta \lesssim 1\,,}$$

Inflation does the job !

$$\delta \sim \begin{cases} \mathcal{O}(m^2/H_*^2) & , \text{quantum} - \text{fluct.}\ (light\ dof) \\ \mathcal{O}(1) & , \text{quantum} - \text{fluct.}\ (self - interact.) \\ \mathcal{O}(1)/\xi & , \text{non} - \text{min grav},\ \xi \gtrsim 1 \\ \mathcal{O}(|1-6\xi|^2) & , \text{non} - \text{min grav},\ |1-6\xi| \ll 1 \end{cases}$$

Linde '83

Starobinsky & Yokoyama '94

Rajantie et al '15

Ford '87

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\mathcal{L} = \frac{1}{\sqrt{g}} \left\{ \underbrace{(\partial\phi)^2 - V_{\rm inf}(\phi)}_{\text{inflaton}} + \underbrace{\frac{1}{2} m_{pl}^2 R}_{\text{gravity (GR)}} + \underbrace{(\partial\chi)^2 - V(\chi) - \xi\chi^2 R}_{\text{matter/rad}} - \underset{\text{Weak coupling}}{\xcancel{g^2\chi^2\phi^2}} \right\}$$

Need to excite matter →

(to reheat the Universe)

$$\boxed{\rho_{\rm rad} = \delta \times 10^{-2} H_*^4}$$

$$\boxed{\delta \lesssim 1,}$$

Inflation does the job !

$$\boxed{\Delta_* \equiv \frac{\rho_{\rm rad}}{3m_p^2 H_*^2} = \frac{\delta}{300}\left(\frac{H_*}{m_p}\right)^2 \ll 1}$$

Fraction energies radiation-to-total

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\mathcal{L} = \frac{1}{\sqrt{g}}\left\{\underbrace{(\partial\phi)^2 - V_{\rm inf}(\phi)}_{\text{inflaton}} + \underbrace{\frac{1}{2}m_{pl}^2 R}_{\text{gravity (GR)}} + \underbrace{(\partial\chi)^2 - V(\chi) - \xi\chi^2 R}_{\text{matter/rad}} - \underset{\text{Weak coupling}}{\xcancel{g^2\chi^2\phi^2}}\right\}$$

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Inflation does the job !

$$\boxed{\Delta_* \equiv \frac{\rho_{\rm rad}}{3m_p^2 H_*^2} = \frac{\delta}{300}\left(\frac{H_*}{m_p}\right)^2 \lesssim \delta \cdot 10^{-12}} \qquad \boxed{\delta \lesssim 1\,,}$$

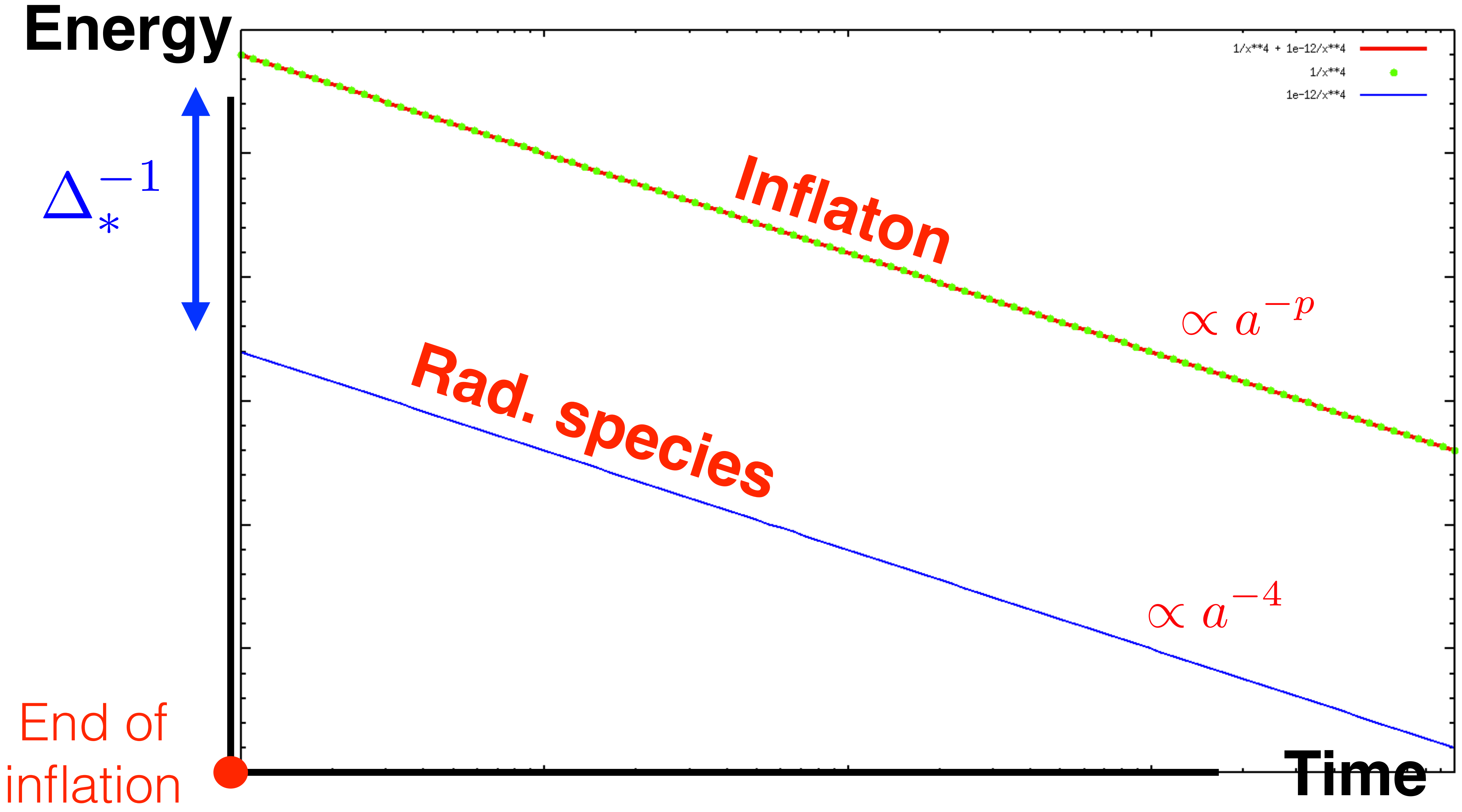
Fraction energies radiation-to-total

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\boxed{\rho^*_{\rm rad} \ll H_*^2 m_{pl}^2}$$

Fields excited

but subdominant



Rad. Excited

$$\rho^*_{\rm rad} \ll H_*^2 m_{pl}^2$$

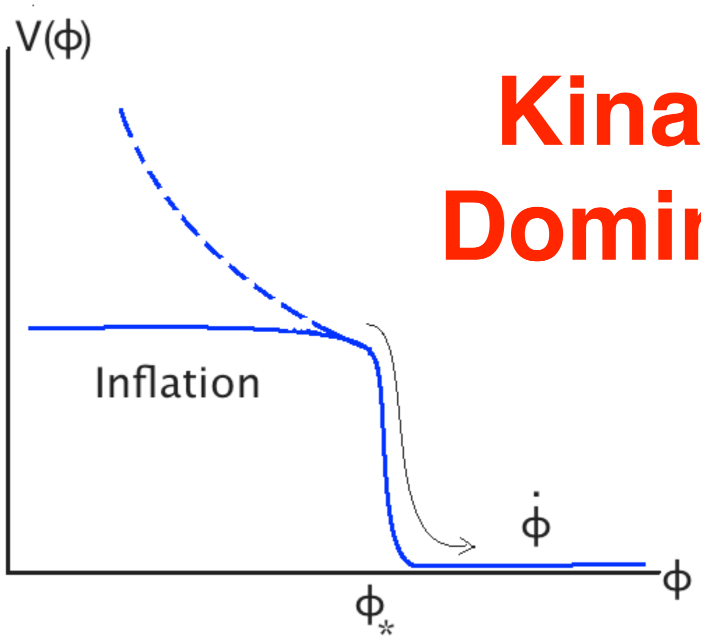
Rad. produced,
but subdominant ☹

?

Rad. Excited

$$\rho^*_{\rm rad} \ll H_*^2 m_{pl}^2$$

Rad. produced, and dominant !



Kination-Domination

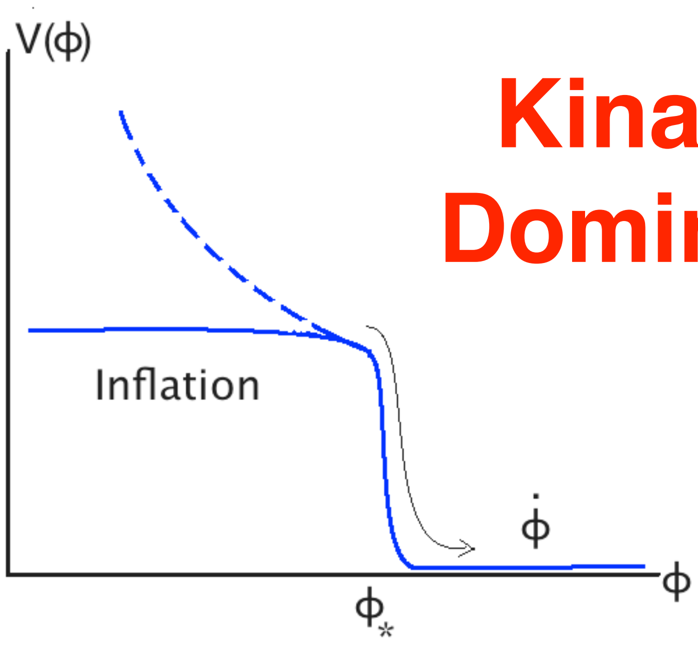
$$\boxed{\text{Energy} \propto \frac{1}{a^6}}$$

[Ford '86, Spokoiny '93]

Rad. Excited

$$\rho^{*}_{\rm rad} \ll H_*^2 m_{pl}^2$$

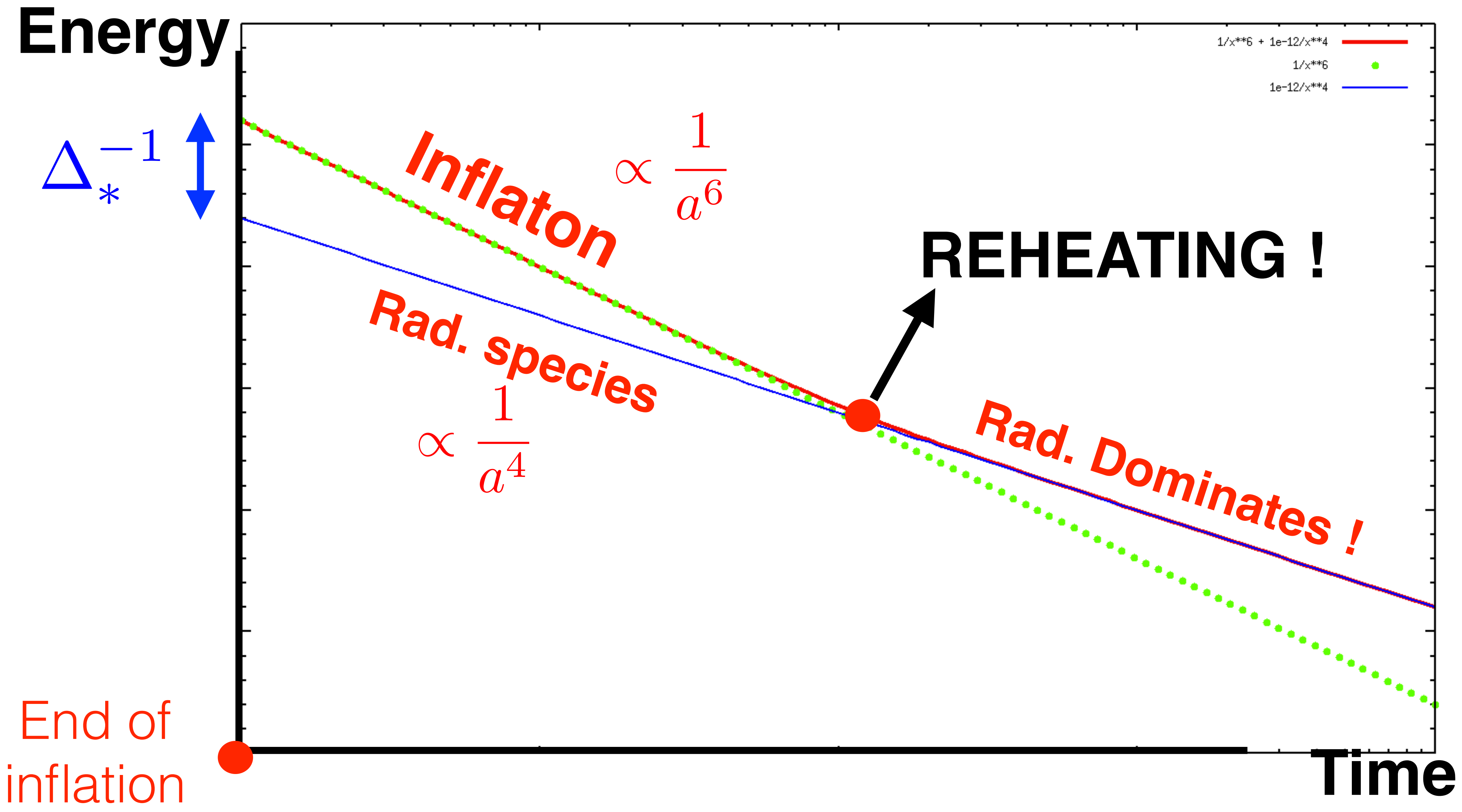
Rad. produced, and dominant !



Kination-Domination

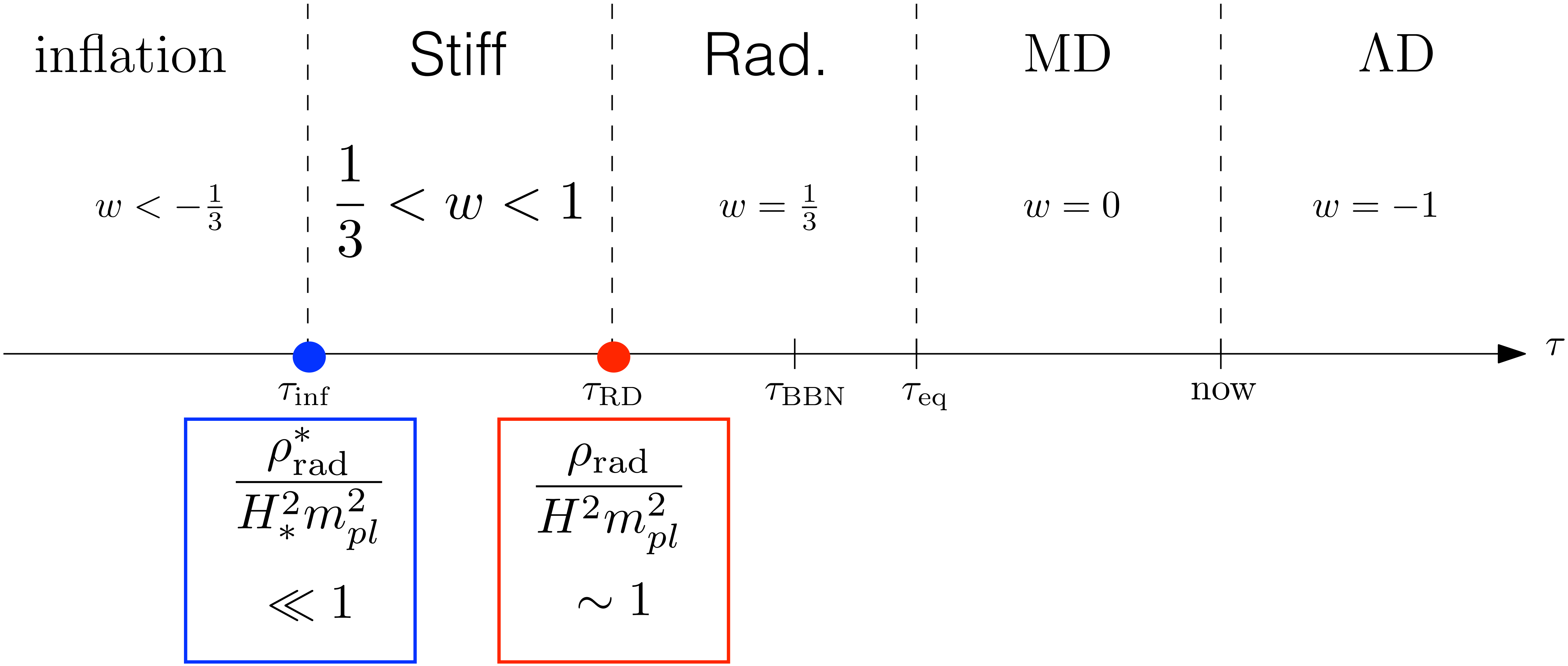
$$\text{Energy} \propto \frac{1}{a^6}$$

[Ford '86, Spokoiny '93]



GRAVITATIONAL REHEATING

Ford ’86, Spokoiny ’93, Joyce ’97,
Giovannini ’98/99, Vilenkin & Damour ’95,
Peebles & Vilenkin ’98, […], DGF & Tanin ’18/19



inflation | Stiff | Rad. | MD | ΛD

$w < -\frac{1}{3}$ | $\frac{1}{3} < w < 1$ | $w = \frac{1}{3}$ | $w = 0$ | $w = -1$

τ

$\tau_{\rm inf}$ | $\tau_{\rm RD}$ | $\tau_{\rm BBN}$ | $\tau_{\rm eq}$ | now

$$\frac{\rho^*_{\rm rad}}{H_*^2 m_{pl}^2} \ll 1$$

$$\frac{\rho_{\rm rad}}{H^2 m_{pl}^2} \sim 1$$

$$\rho_s \sim m_p^2 H^2 \propto a^{-p(w)}$$

$$(4 < p \lesssim 6)$$

GRAVITATIONAL REHEATING

Ford ’86, Spokoiny ’93, Joyce ’97,
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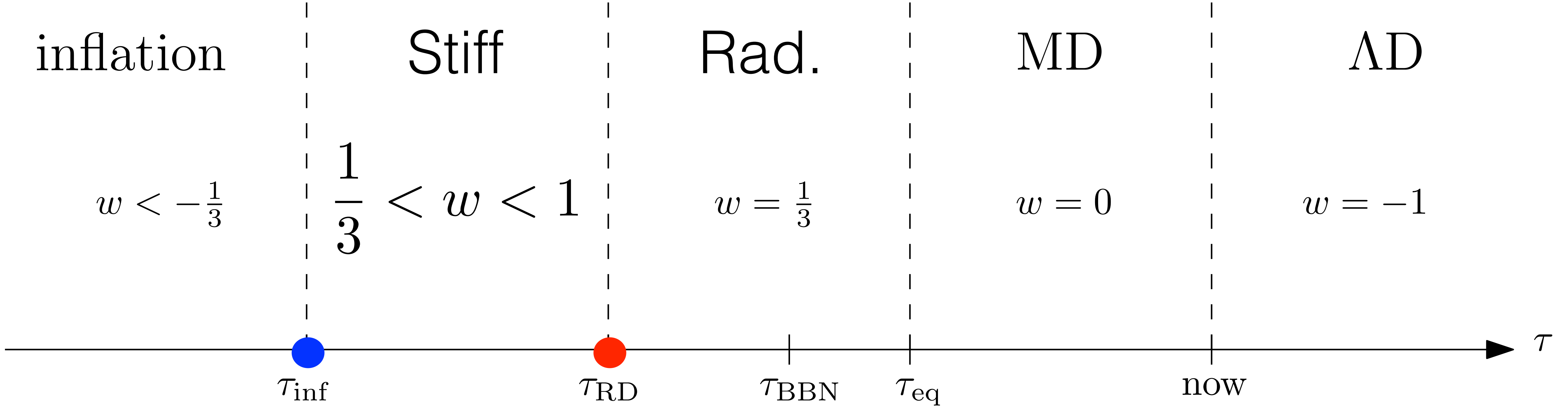

inflation | Stiff | Rad. | MD | ΛD

$w < -\frac{1}{3}$ | $\frac{1}{3} < w < 1$ | $w = \frac{1}{3}$ | $w = 0$ | $w = -1$

$\tau_{\rm inf}$ | $\tau_{\rm RD}$ | $\tau_{\rm BBN}$ | $\tau_{\rm eq}$ | now | τ

$$\frac{\rho^*_{\rm rad}}{H_*^2 m_{pl}^2} \ll 1$$

$$\frac{\rho_{\rm rad}}{H^2 m_{pl}^2} \sim 1$$

Requisites

- Control perturbations
- End before BBN

$$\rho_s \sim m_p^2 H^2 \propto a^{-p(w)}$$

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GRAVITATIONAL REHEATING

Ford ’86, Spokoiny ’93, Joyce ’97,
Giovannini ’98/99, Vilenkin & Damour ’95,
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$$\boxed{1/3 < w_s \lesssim 1}$$

Stiff Eq. of State

Requisites

* Control perturbations
* End before BBN

✓

But as we learnt before …

GRAVITATIONAL REHEATING

Ford ’86, Spokoiny ’93, Joyce ’97,
Giovannini ’98/99, Vilenkin & Damour ’95,
Peebles & Vilenkin ’98, […], DGF & Tanin ’18/19

$$\boxed{1/3 < w_s \lesssim 1}$$

Stiff Eq. of State

Requisites

- Control perturbations
- End before BBN

✓

↓

But as we learnt before …

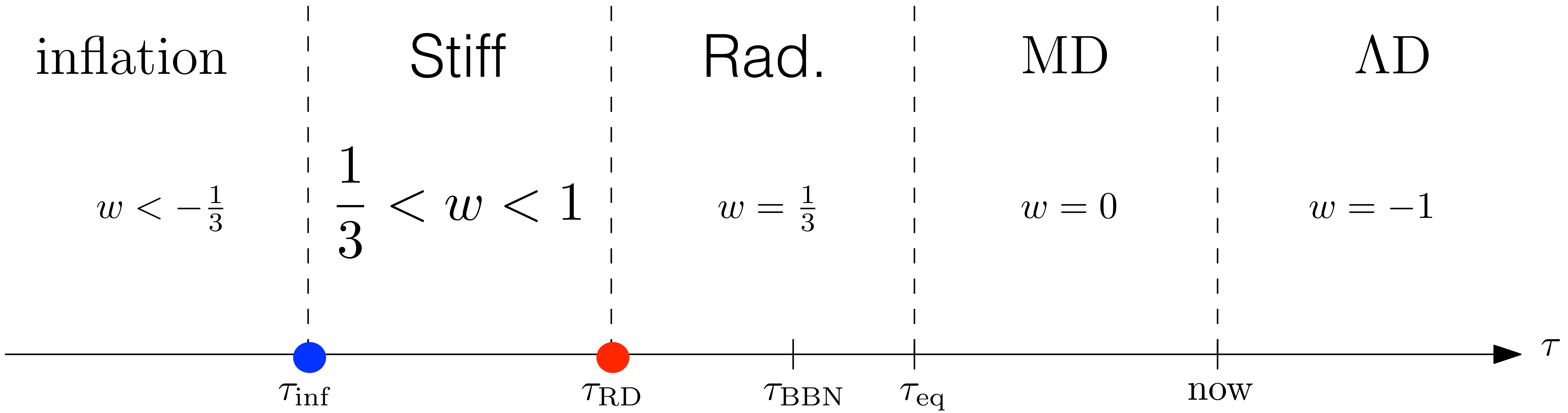
Enhancement of inflationary Gravitational Waves (GW) !

[Giovannini ’98/99, …, Boyle & Buonnano ’07, …, DGF & Tanin ’19]

Part 3

Gravitational waves from gravitational reheating

BACK to ... GRAVITATIONAL REHEATING



$$\boxed{\frac{\rho^*_{\rm rad}}{H_*^2 m_{pl}^2} \ll 1}$$

$$\boxed{\frac{\rho_{\rm rad}}{H^2 m_{pl}^2} \sim 1}$$

$$\boxed{\Omega_{\rm GW}^{\rm (TOT)} = \int \left(\frac{d\rho_{\rm GW}}{d\log f}\right) \frac{d\log f}{\rho_c}}$$

Requisites

- * Control perturbations
- * End before BBN
- * **Not too many GWs**

BIG BANG NUCLEOSYNTHESIS

Expansion rate (Rad. Dom): ~ Extra relativistic species

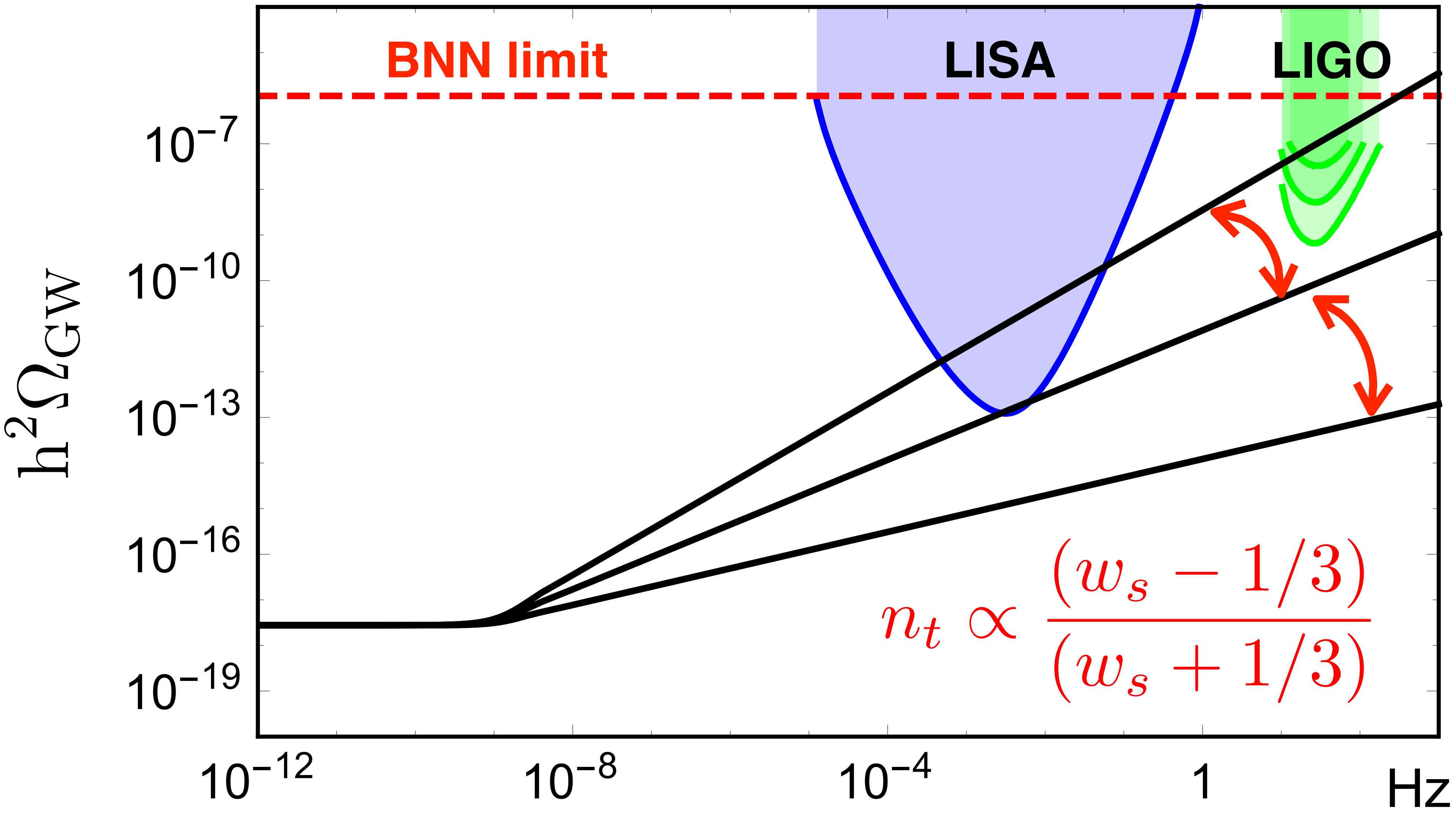
$$\int \frac{df}{f}\, h^2\, \Omega_{\rm GW}(f) \leq \; 1.12 \times 10^{-6}$$

$$\Delta N_\nu = 0.2 \;(95\% C.L.)$$ [latest CMB]

BIG BANG NUCLEOSYNTHESIS

Expansion rate (Rad. Dom): ~ Extra relativistic species

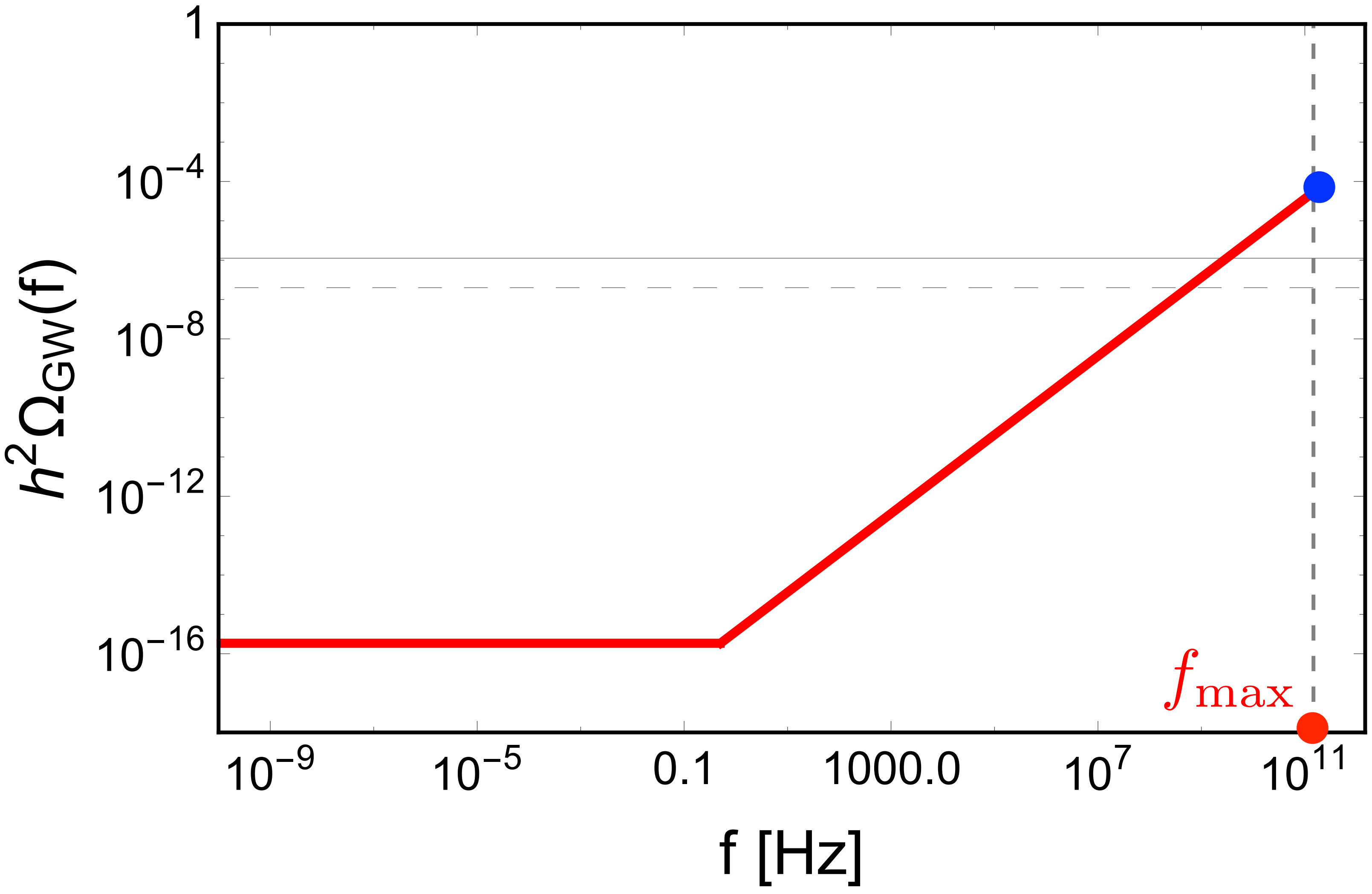
$$\int \frac{df}{f}\, h^2\, \Omega_{\rm GW}(f) \leq\ 1.12 \times 10^{-6}$$



BBN: $$\int \frac{df}{f}\, h^2\,\Omega_{\rm GW}(f) \leq \; 1.12 \times 10^{-6}$$

Grav. Reheating: $$\Omega_{\rm GW}(f) \propto (f/f_{\rm RD})^{2\left(\frac{w_s - 1/3}{w_s + 1/3}\right)}$$

Monotonically growing signal !



$h^2\Omega_{\rm GW}\Big|_{\rm max}$

BBN bound

@ $f_{\rm max}$

[Freq. horizon

crossing End

of Inflation]

BBN: $$\int \frac{df}{f}\, h^2\, \Omega_{\rm GW}(f) \leq \; 1.12 \times 10^{-6}$$

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Monotonically growing signal !

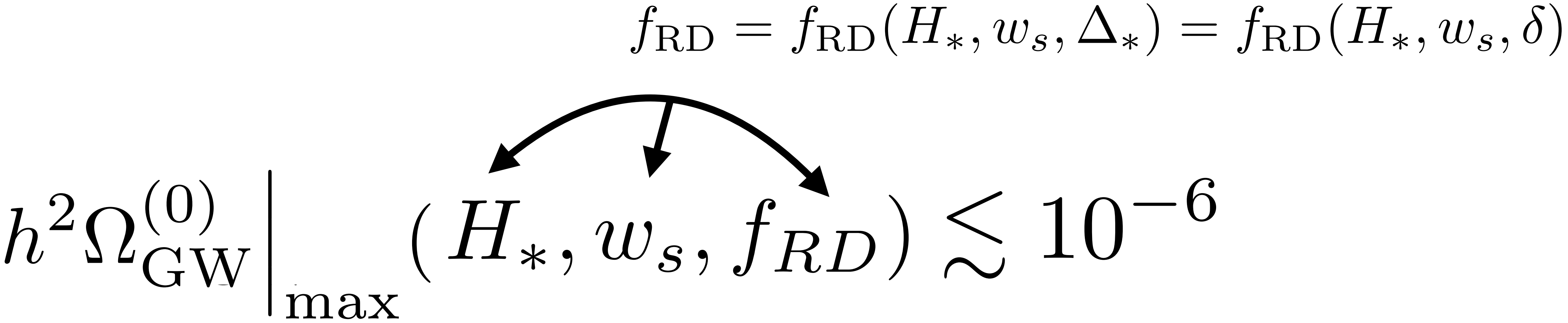
BBN bound @ $f_{\rm max}$ [Freq. horizon crossing End of Inflation]

$$h^2\Omega^{(0)}_{\rm GW}\Big|_{\rm max}(H_*, w_s, f_{RD}) \lesssim 10^{-6}$$

BBN: $h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max}(H_*, w_s, f_{RD}) \lesssim 10^{-6}$

BBN: $h^2\Omega^{(0)}_{\rm GW}\Big|_{\rm max}(\,H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $\Delta_* \equiv \frac{\rho_{\rm rad}}{3m_p^2H_*^2} = \frac{\delta}{300}\left(\frac{H_*}{m_p}\right)^2$, $\delta \lesssim 1\,,$



$$f_{\rm RD} = f_{\rm RD}(H_*, w_s, \Delta_*) = f_{\rm RD}(H_*, w_s, \delta)$$

$$h^2\Omega^{(0)}_{\rm GW}\Big|_{\rm max}(\,H_*, w_s, f_{RD}) \lesssim 10^{-6}$$

BBN: $h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max}(H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max}(H_*, w_s; \delta) \lesssim 10^{-6}$**,** $\delta \lesssim 1\,,$

BBN:

$$h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max}(H_*, w_s, f_{RD}) \lesssim 10^{-6}$$

Grav. Reheating:

$$h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max}(H_*, w_s; \delta) \lesssim 10^{-6}, \quad \delta \lesssim 1\,,$$

However …

$$h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max}(H_*, w_s; \delta) \simeq \underbrace{2.1\cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$$

BBN: $h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max}(H_*, w_s, f_{RD}) \lesssim 10^{-6}$

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$$\underset{(w_s = 1/3)}{1} \leq f(w_s) \leq \underset{(w_s = 1)}{2.54}$$

$$f(w_s) \equiv \frac{2^{\frac{3(1-w_s)}{(1+3w_s)}}\,\Gamma^2\left(\frac{5+3w_s}{2+6w_s}\right)}{\left(\frac{2}{1+3w_s}\right)^{\frac{4}{1+3w_s}}\Gamma^2\left(\frac{3}{2}\right)}$$

BBN: $$h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max}(H_*, w_s, f_{RD}) \lesssim 10^{-6}$$

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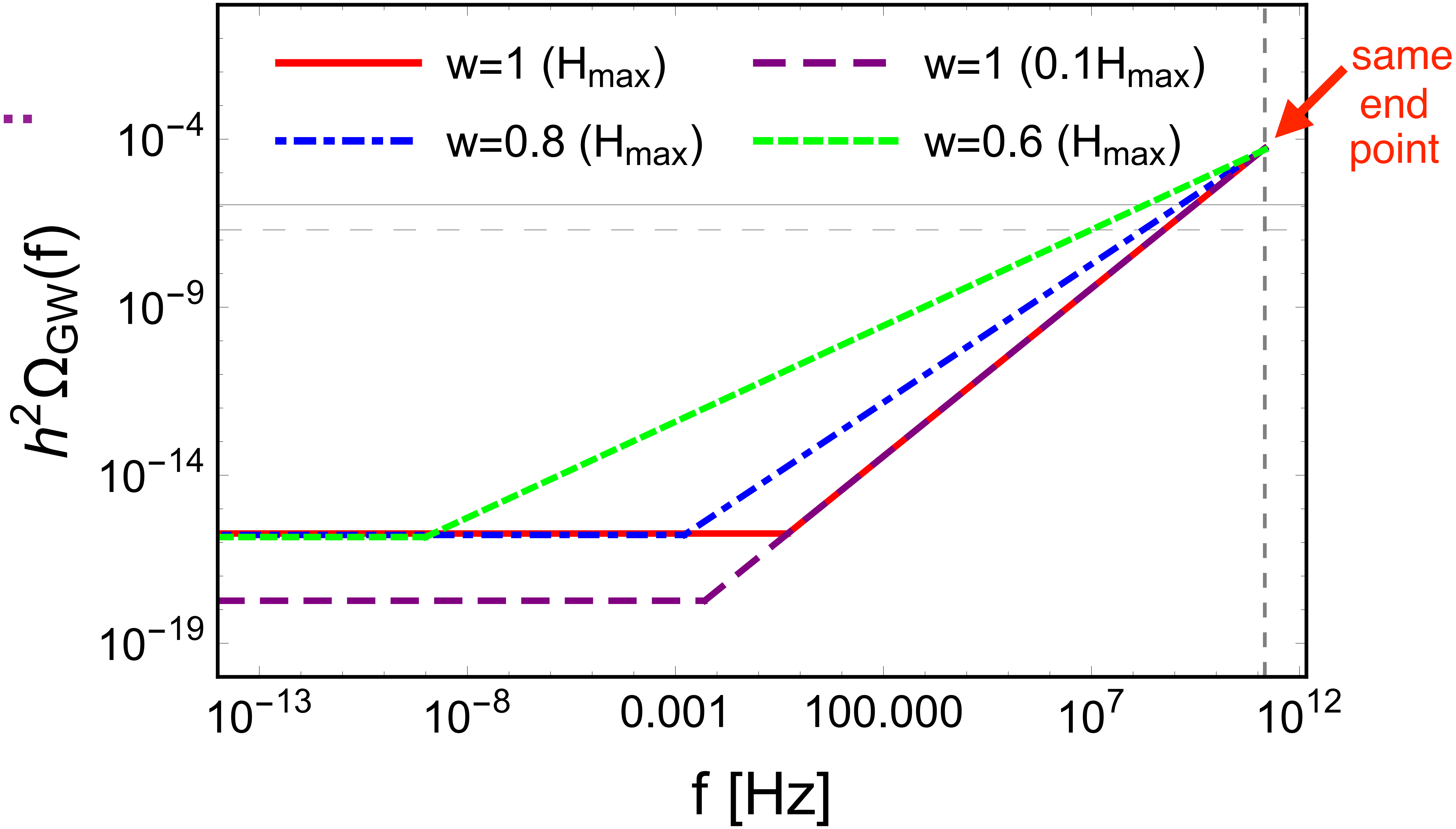

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However … $$h^2\Omega^{(0)}_{\rm GW}\Big|_{\rm max}(\underbrace{\cancel{H_*}, \cancel{w_s}}; \delta) \simeq \underbrace{2.1\cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$$

Why ?

$$\left.\begin{aligned}\rho_{\rm rad} &\propto H_*^4 a^{-4}\\ \rho_{\rm GW} &\propto H_*^4 a^{-4}\end{aligned}\right\} \Longrightarrow \frac{\rho_{\rm GW}}{\rho_{\rm rad}} \sim const.$$

BBN: $h^2\Omega^{(0)}_{\mathrm{GW}}\Big|_{\max}(H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2\Omega^{(0)}_{\mathrm{GW}}\Big|_{\max}(H_*, w_s; \delta) \lesssim 10^{-6}$, $\boxed{\delta \lesssim 1\,,}$

However … $h^2\Omega^{(0)}_{\mathrm{GW}}\Big|_{\max}(\cancel{H_*}, \cancel{w_s}; \boxed{\delta}) \simeq \underbrace{2.1\cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\boxed{\frac{1}{\delta}}}_{\text{initial fraction}}$

$$\Delta_* \equiv \frac{\rho_{\mathrm{rad}}}{3m_p^2H_*^2} = \frac{\delta}{300}\left(\frac{H_*}{m_p}\right)^2$$

BBN: $$h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max}(H_*, w_s, f_{RD}) \lesssim 10^{-6}$$

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So ... $$h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max} \simeq \frac{const.}{\delta} \lesssim 10^{-6} \quad \Leftrightarrow \quad \delta \gtrsim 50$$

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(!)

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However … $h^2\Omega^{(0)}_{\rm GW}\Big|_{\rm max}(\cancel{H_*}, \cancel{w_s}; \delta) \simeq \underbrace{2.1 \cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$

So … $h^2\Omega^{(0)}_{\rm GW}\Big|_{\rm max} \simeq \frac{const.}{\delta} \lesssim 10^{-6} \quad \Leftrightarrow \quad \boxed{\delta \gtrsim 50}$ **(!)**

$\frac{\rho_{\rm GW}}{\rho_{\rm rad}} \sim \underline{const. \gg 1}$ **Universe dominated by GWs!**

BBN: $$h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max}(H_*, w_s, f_{RD}) \lesssim 10^{-6}$$

Grav. Reheating: $$h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max}(H_*, w_s; \delta) \lesssim 10^{-6}, \quad \boxed{\delta \lesssim 1\,,}$$

However … $$h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max}(\cancel{H_*}, \cancel{w_s}; \delta) \simeq \underbrace{2.1\cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$$

So … $$h^2\Omega_{\rm GW}^{(0)}\Big|_{\rm max} \simeq \frac{const.}{\delta} \lesssim 10^{-6} \quad \Leftrightarrow \quad \boxed{\delta \gtrsim 50}$$

(!)

CMB: $$\boxed{\delta \gtrsim 200}$$ **(!!)**

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore…

1) Either we modify Grav. Reheating

2) We use modified gravity in Inflationary Sector

3) We couple the inflaton and reheat via such couplings

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Standard (P)reheating

(standard) Grav. Reheating incompatible with BBN/CMB !

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it’s up to you, me I’m happy with General Relativity …

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore…

1) Either we modify Grav. Reheating

2) We use modified gravity in Inflationary Sector

But if you are not …

Y. Watanabe and E. Komatsu, Phys. Rev. **D75**, 061301 (2007), gr-qc/0612120.
Y. Watanabe, Phys. Rev. **D83**, 043511 (2011), 1011.3348.
A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980), [,771(1980)].
A. De Felice and S. Tsujikawa, Living Rev. Rel. **13**, 3 (2010), 1002.4928.

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$$\Delta_* \equiv \frac{\rho_{\rm rad}}{3m_p^2 H_*^2} = \frac{\delta}{300}\left(\frac{H_*}{m_p}\right)^2 \longrightarrow \mathcal{N}_f \Delta_*$$

All $\mathcal{N}_f$ fields same properties !

$$\delta = \delta_1 \times \mathcal{N}_f\ ,$$

$$\mathcal{N}_f \gtrsim \mathcal{O}(10^3)$$

Ad hoc tuning !

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore…

1) Either we modify Grav. Reheating

Radiation field is the SM Higgs ? We need non-min coupling

$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \xi\chi^2 R$$

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore…

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Standard Grav. RH ?

(standard) Grav. Reheating incompatible with BBN/CMB !

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$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \xi\chi^2 R$$

Standard Grav. RH wrong !

$m_\chi^2 < 0$ @ Stiff Period,

(standard) Grav. Reheating incompatible with BBN/CMB !

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Radiation field is the SM Higgs ? We need non-min coupling

$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \underbrace{\xi\chi^2 R}_{}$$

Standard Grav. RH wrong !
$m_\chi^2 < 0$ @ Stiff Period, and
self-interactions regularize

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore…

1) Either we modify Grav. Reheating

Radiation field is the SM Higgs ? We need non-min coupling

$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \xi\chi^2 R$$

Standard Grav. RH wrong !

Corrected in DGF & Byrnes '16
Phys.Lett. B767 (2017) 272-277
Arxiv: 1604.03905

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore…

1) Either we modify Grav. Reheating

Radiation field is the SM Higgs ? We need non-min coupling

$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \xi\chi^2 R$$

Standard Grav. RH wrong !

$$\boxed{\delta \sim \mathcal{O}(10^3)\frac{\xi^2}{\lambda} \gg 1}$$

Grav. Reheating OK !

$$\lambda > 0 \ (\text{stability})\,, \quad \xi \gtrsim 1$$

Corrected in DGF & Byrnes '16
Phys.Lett. B767 (2017) 272-277
Arxiv: 1604.03905

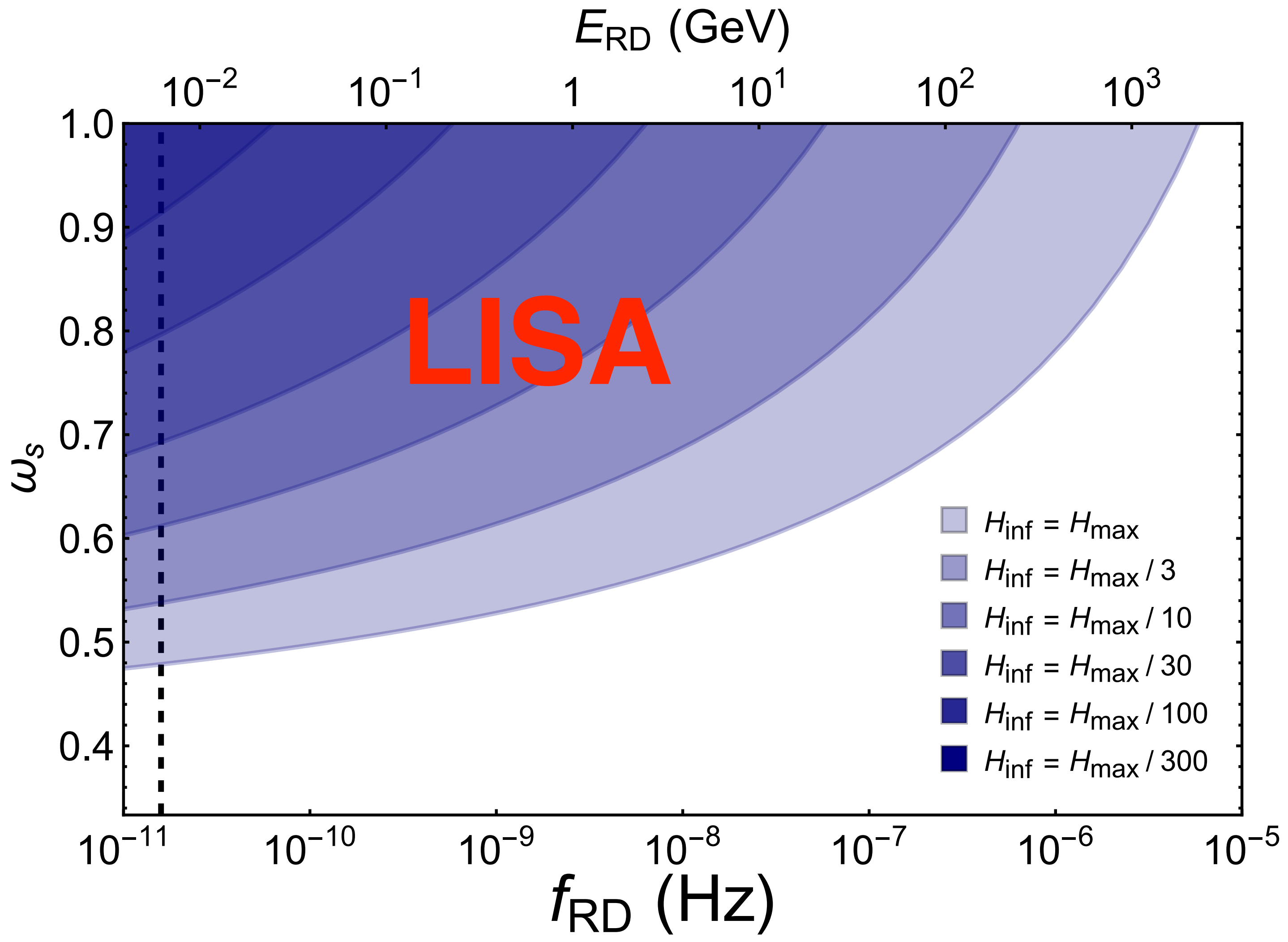
See also **1803.07399** **1905.06823** for generic $\lambda\chi^4$

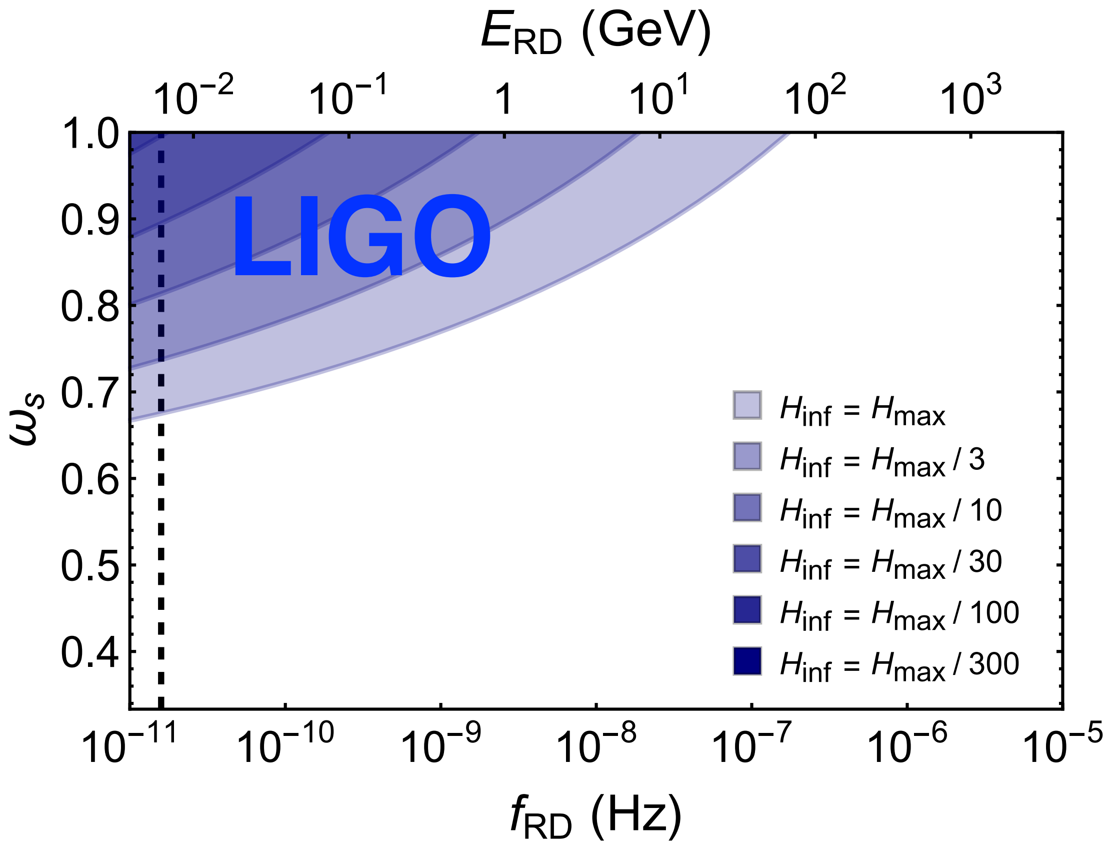
Part 4

BBN/CMB constraints: further implications

BBN Bound

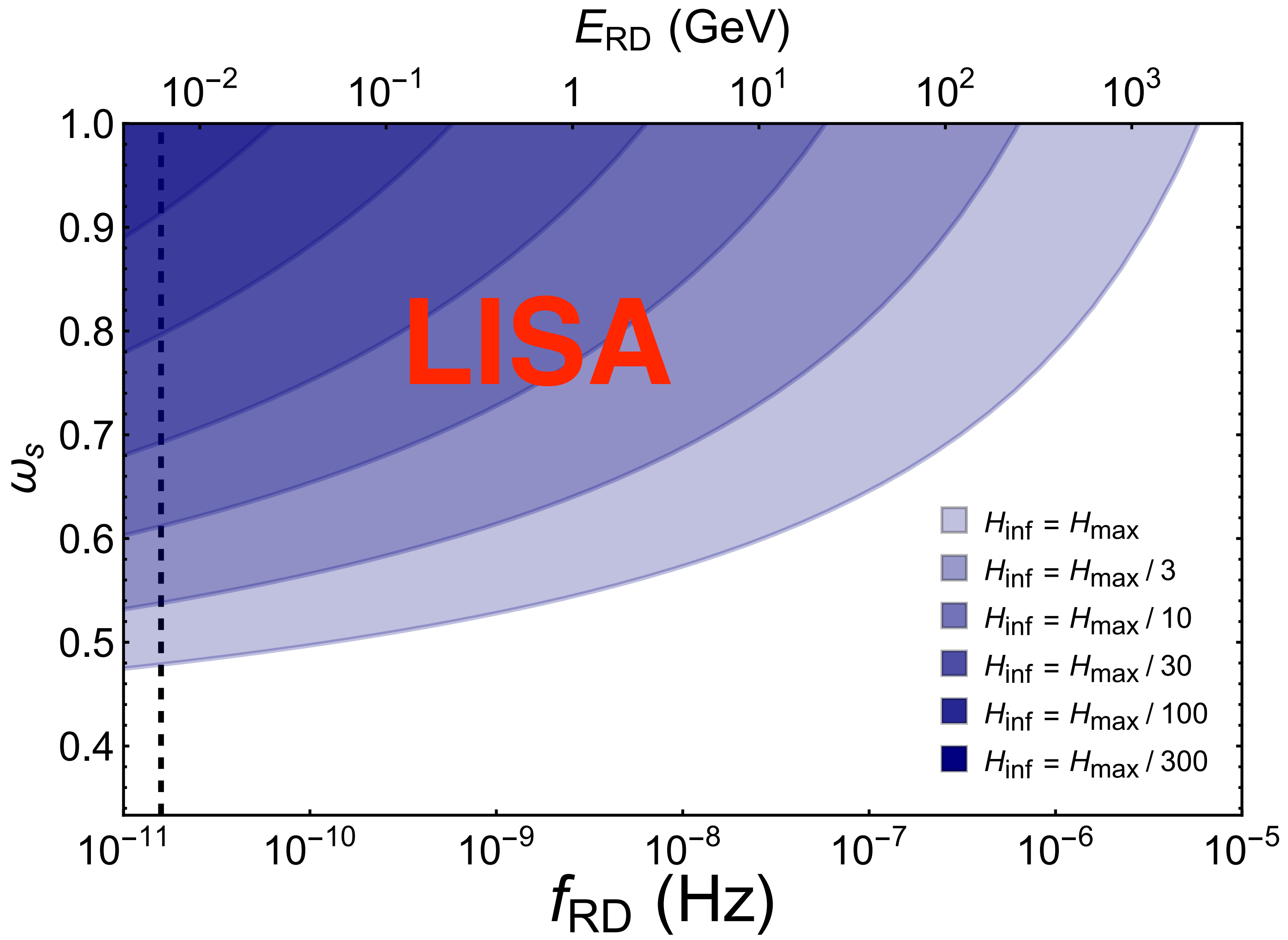
$$\Omega_{\rm GW}^{(0)}(f;\underbrace{H_*}_{\text{Energy Scale}},\underbrace{w_s}_{\text{EoS Stiff}},\underbrace{f_{RD}}_{\text{Duration Stiff}})\lesssim 10^{-6}$$

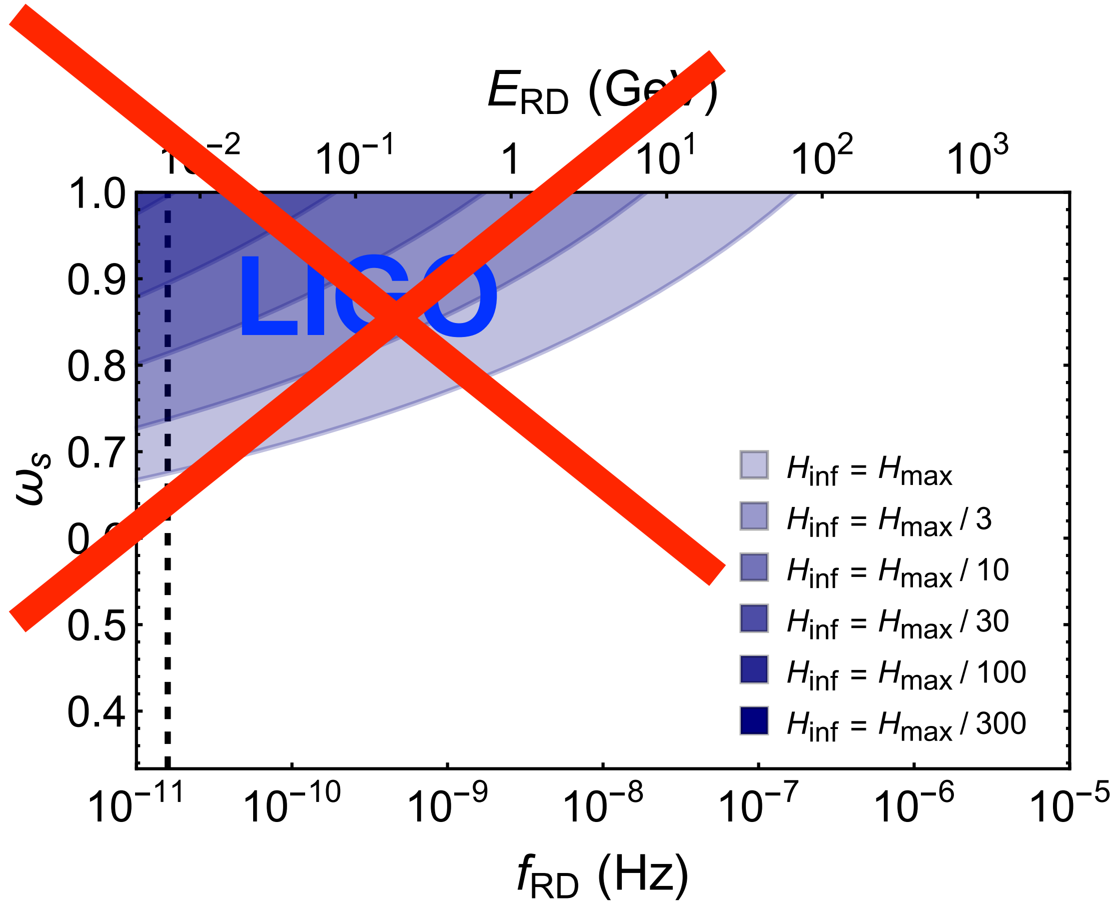




BBN Bound $\Omega^{(0)}_{\rm GW}(f; \underbrace{H_*}_{\text{Energy Scale}}, \underbrace{w_s}_{\text{EoS Stiff}}, \underbrace{f_{RD}}_{\text{Duration Stiff}}) \lesssim 10^{-6}$

LIGO cannot probe it !





LIGO cannot probe parameter space compatible with BBN !

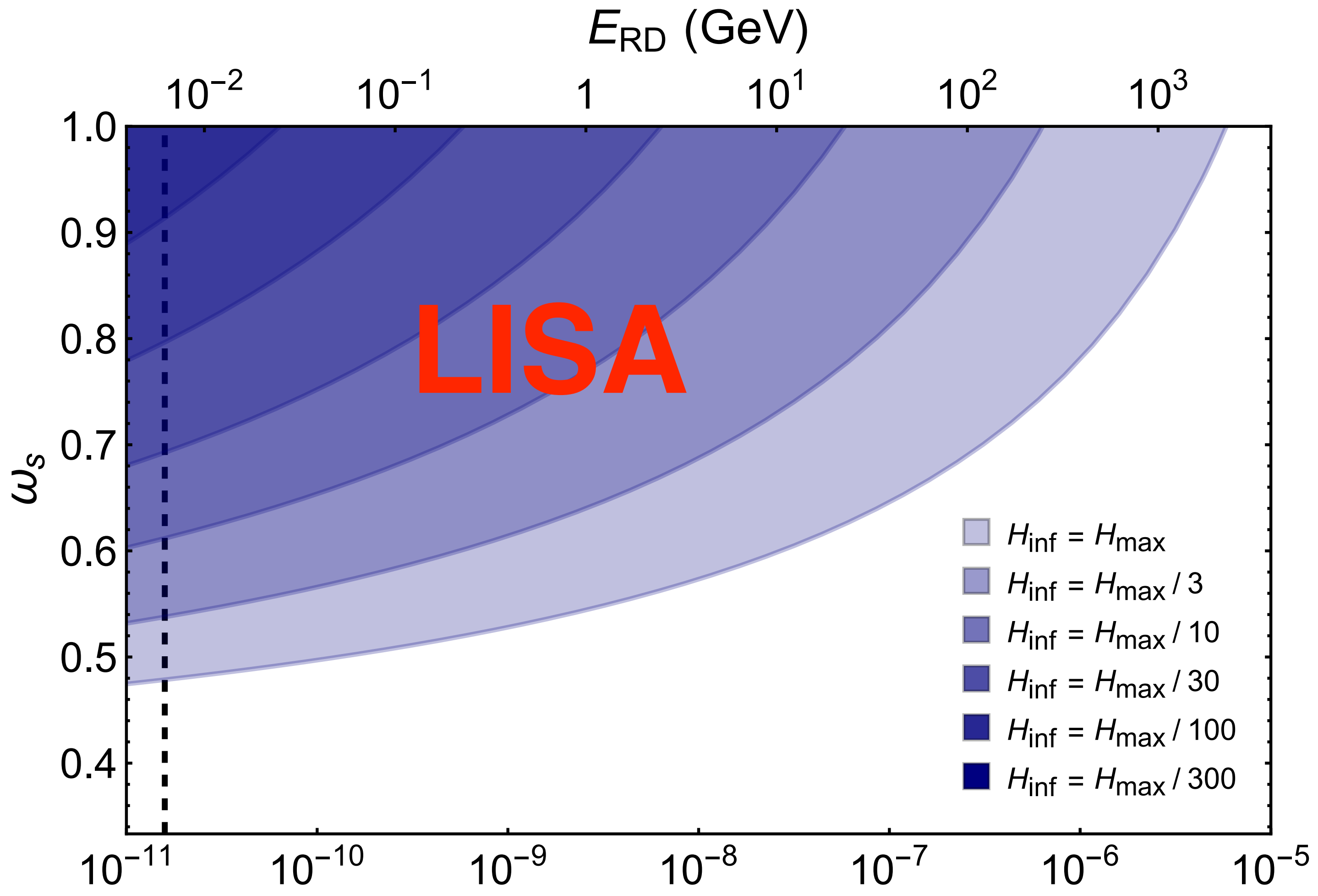
BBN Bound $\Omega^{(0)}_{\rm GW}(f;\underline{H_*},\underline{w_s},\underline{f_{RD}}) \lesssim 10^{-6}$

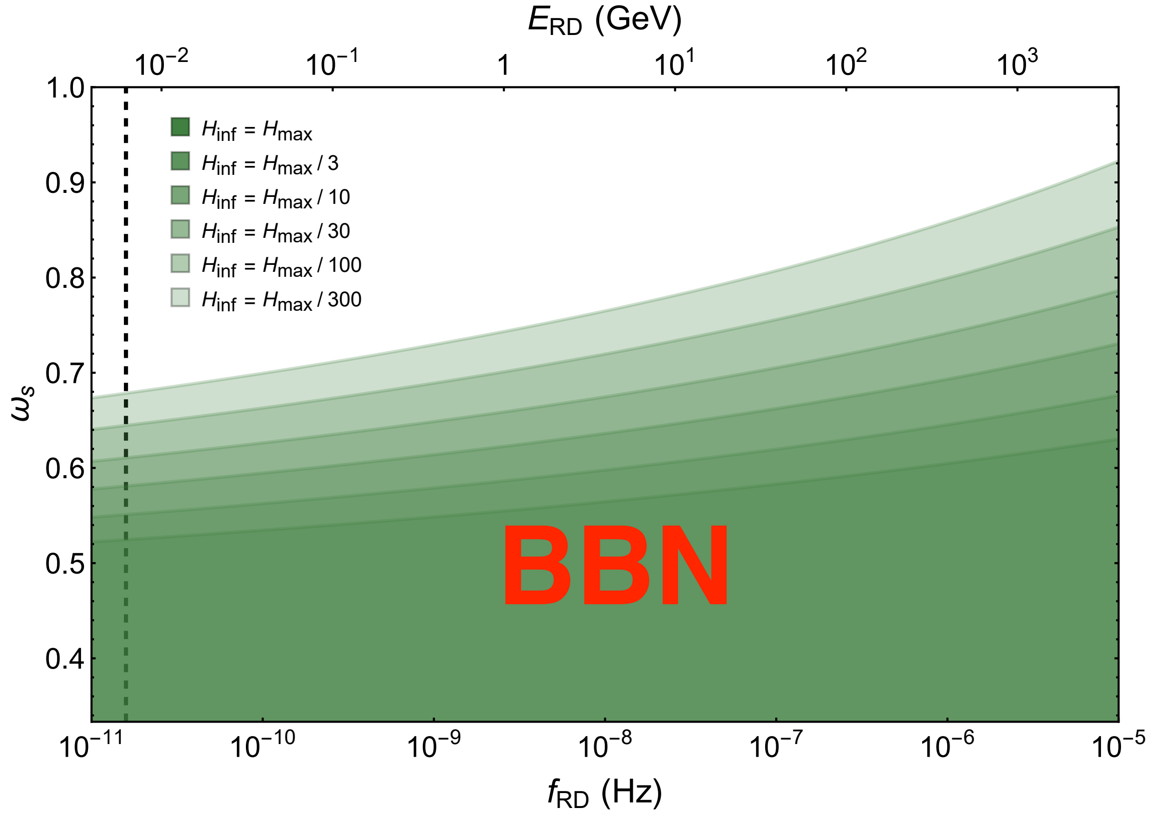
Why ?

Energy Scale

EoS Stiff

Duration Stiff





BBN Bound

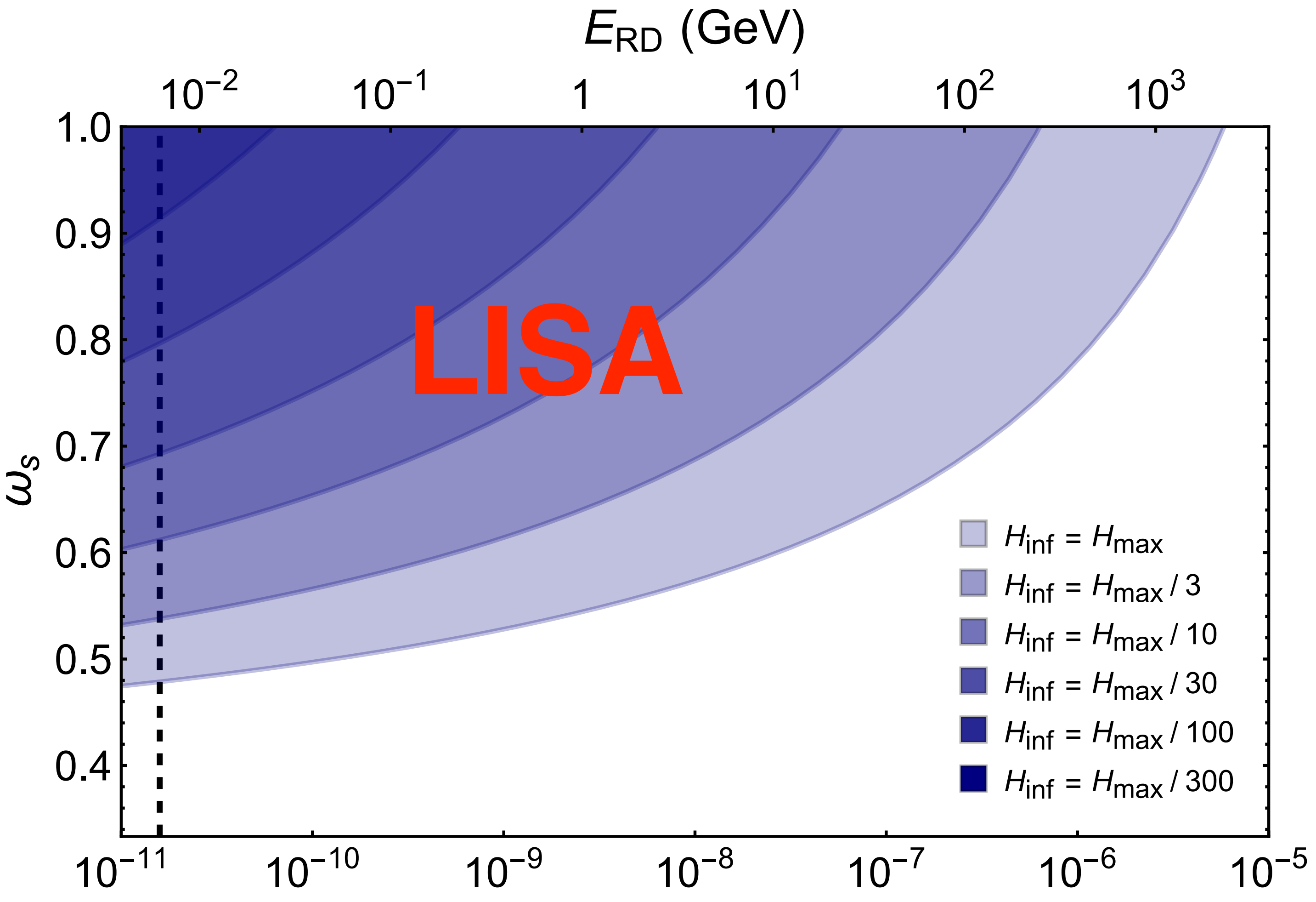
$$\Omega^{(0)}_{\rm GW}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}}) \lesssim 10^{-6}$$

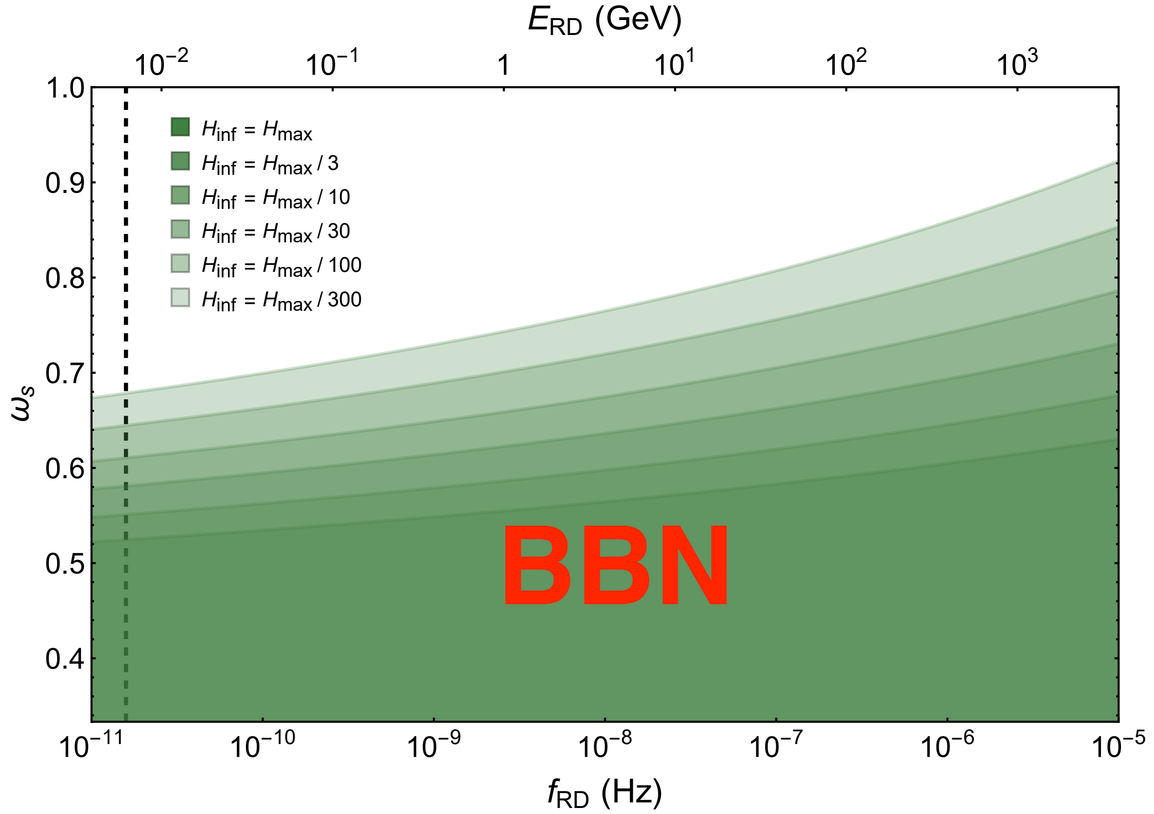
Energy Scale

EoS Stiff

Duration Stiff

LISA ?

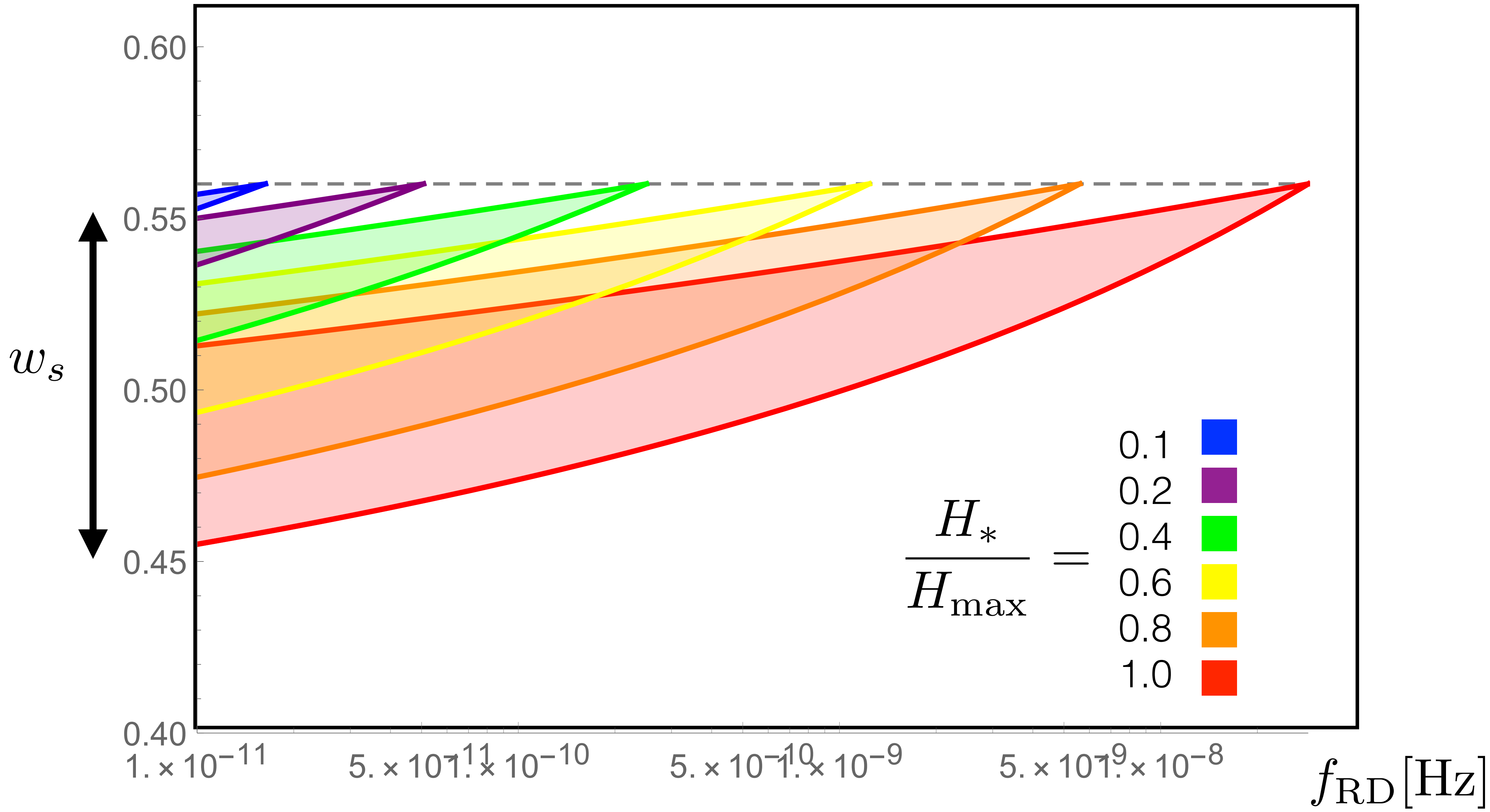




BBN Bound $\Omega_{\rm GW}^{(0)}(f; \underbrace{H_*}_{\text{Energy Scale}}, \underbrace{w_s}_{\text{EoS Stiff}}, \underbrace{f_{RD}}_{\text{Duration Stiff}}) \lesssim 10^{-6}$

LISA ✓

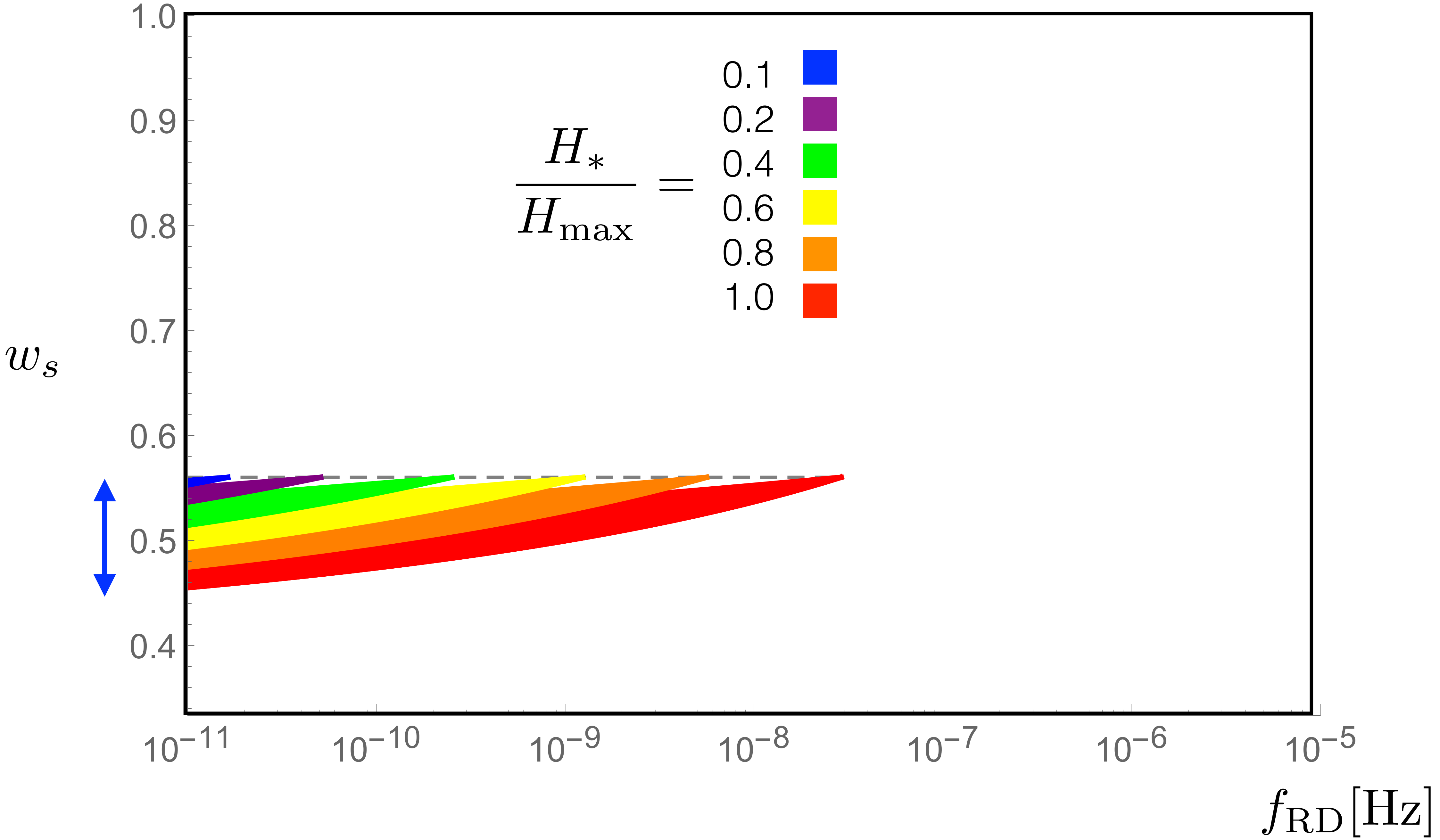
ZOOM



BBN Bound

$$\Omega_{\rm GW}^{(0)}(f; \underbrace{H_*}_{\text{Energy Scale}}, \underbrace{w_s}_{\text{EoS Stiff}}, \underbrace{f_{RD}}_{\text{Duration Stiff}}) \lesssim 10^{-6}$$

LISA ✓



Part 5

Outlook

OUTLOOK

0) Reheating w/o couplings requires imagination:

Grav. Reheating or Modified Gravity

1) (Standard) Grav. Reheating is inconsistent

Too many GWs (violates BBN/CMB bounds)

2) Inf. sectors only (minimally) coupled

to gravity inconsistent unless:

i) Inflation ~ Modify gravity: Up to you…

ii) O(1000) spectator fields identical: ad hoc tuning

iii) SM Higgs + Non-Min coupling: works (not observable)

3) Stiff Era (in general): not observable @ LIGO, barely @ LISA

PROPAGANDA

If you want to go ‘numerical' in your early universe computations…

CosmoLattice

Figueroa, Florio, Torrenti, Valkenburg

arXiv: 2102.01031

('GW computation' module about to be available)

Backslides

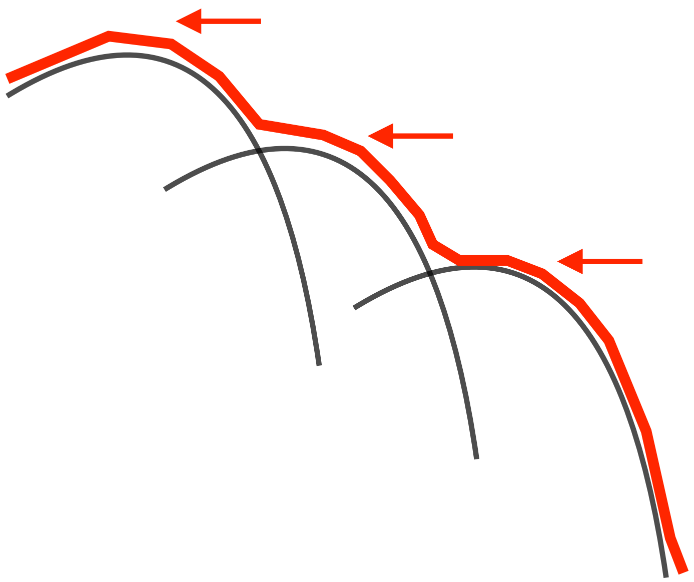
INFLATIONARY PREHEATING

Parameter Dependence (Peak amplitude)

Chaotic Models: $\Omega_{\rm GW}^{(o)} \sim 10^{-11}\,, \quad @ \quad f_o \sim 10^8 - 10^9~{\rm Hz}$

Large amplitude ! **... at high Frequency !**

$\Omega_{\rm GW} \propto q^{-1/2} \longrightarrow$ **Spectroscopy of particle couplings ?**



different couplings
... different peaks ?

DGF & Loayza, work in progress

INFLATIONARY PREHEATING

Parameter Dependence (Peak amplitude**)**

Hybrid Models: $\Omega_{\rm GW}^{(o)} \propto \left(\frac{v}{m_p}\right)^2 \times f(\lambda, g^2)$, $f_o \sim \lambda^{1/4} \times 10^9$ Hz

$$\Omega_{\rm GW}^{(o)} \sim 10^{-11}\,, \quad @ \quad \begin{cases} f_o \sim 10^8 - 10^9 \text{ Hz} \\ f_o \sim 10^2 \text{ Hz} \end{cases}$$

$\boxed{\lambda \sim 0.1}$ (natural)

$\boxed{\lambda \sim 10^{-28}}$ (fine-tuning)

Large amplitude !

(for $v \simeq 10^{16}$ GeV)

realistically speaking ...