New twists in compact binary waveform modelling: a fast time domain model for precession

Héctor Estellés, Marta Colleoni, Cecilio García-Quirós, Sascha Husa, David Keitel, Maite Mateu, Maria de Lluc Planas, Antoni Ramos-Buades

11th Iberian Gravitational Wave Meeting

Image credit: Rafel Jaume

Universitat de les Illes Balears
IAC3 Institute of Applied Computing & Community Code.
GOVERN ILLES BALEARS
EUROPEAN REGIONAL DEVELOPMENT FUND "A way to make Europe"
MINISTERIO DE ECONOMÍA, INDUSTRIA Y COMPETITIVIDAD
PRACE Partnership for Advanced Computing in Europe
RES Red Española de Supercomputación
GOIB Conselleria Educació, Universitat i Recerca
Direcció General Política Universitària i Recerca
Illes Sostenibles

Compact Binary GW Signals

Compact binary coalescing signals:

- High dimensional parameter space: individual masses, spins, orientation parameters, sky location, matter parameters, eccentricity …
- Knowledge about signals from different approaches during different stages of the evolution:
  - Early inspiral: Post-Newtonian theory, Self-force
  - Numerical relativity: late inspiral, plunge, merger and ringdown
  - Ringdown: BH perturbation theory

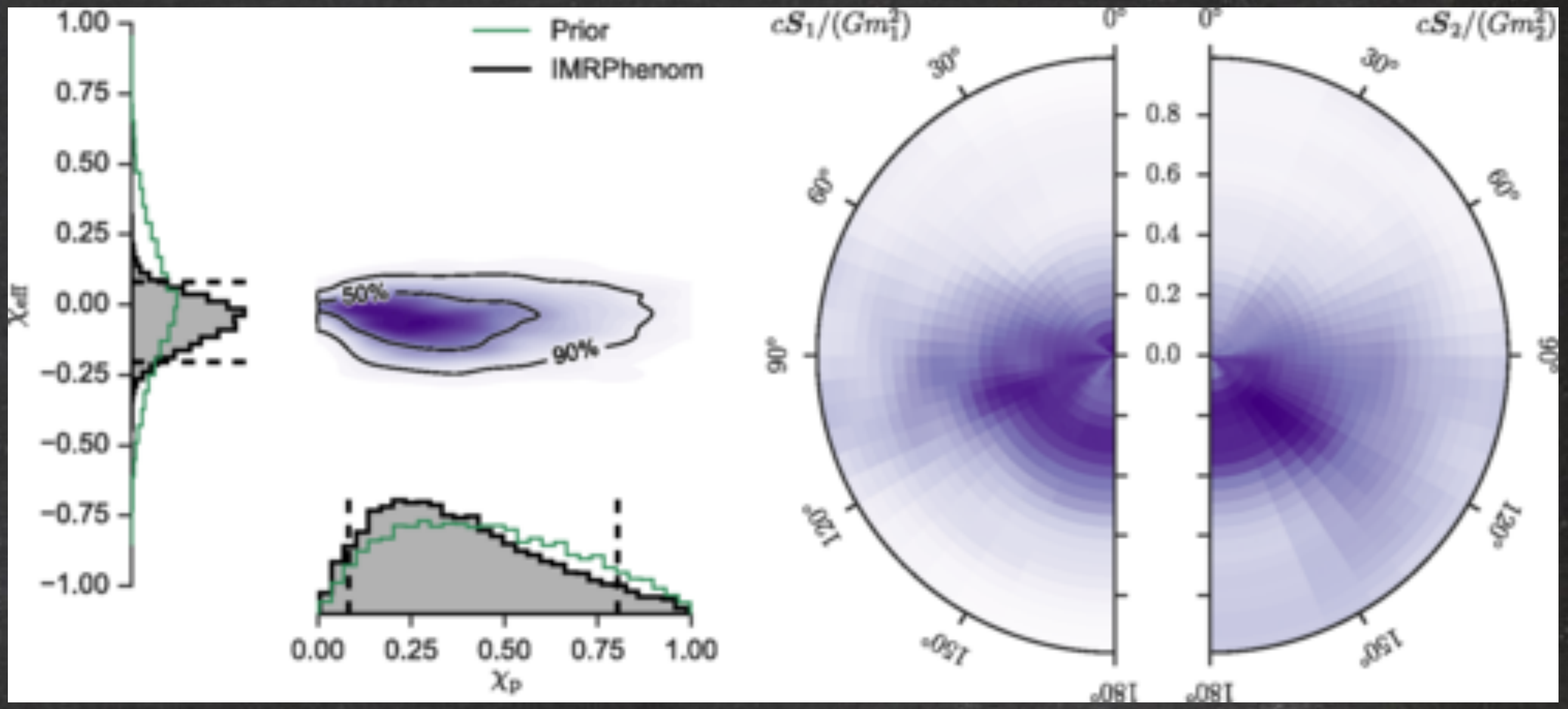


Image credit: B. P. Abbott *et al.* Phys. Rev. Lett. **116**, 241102

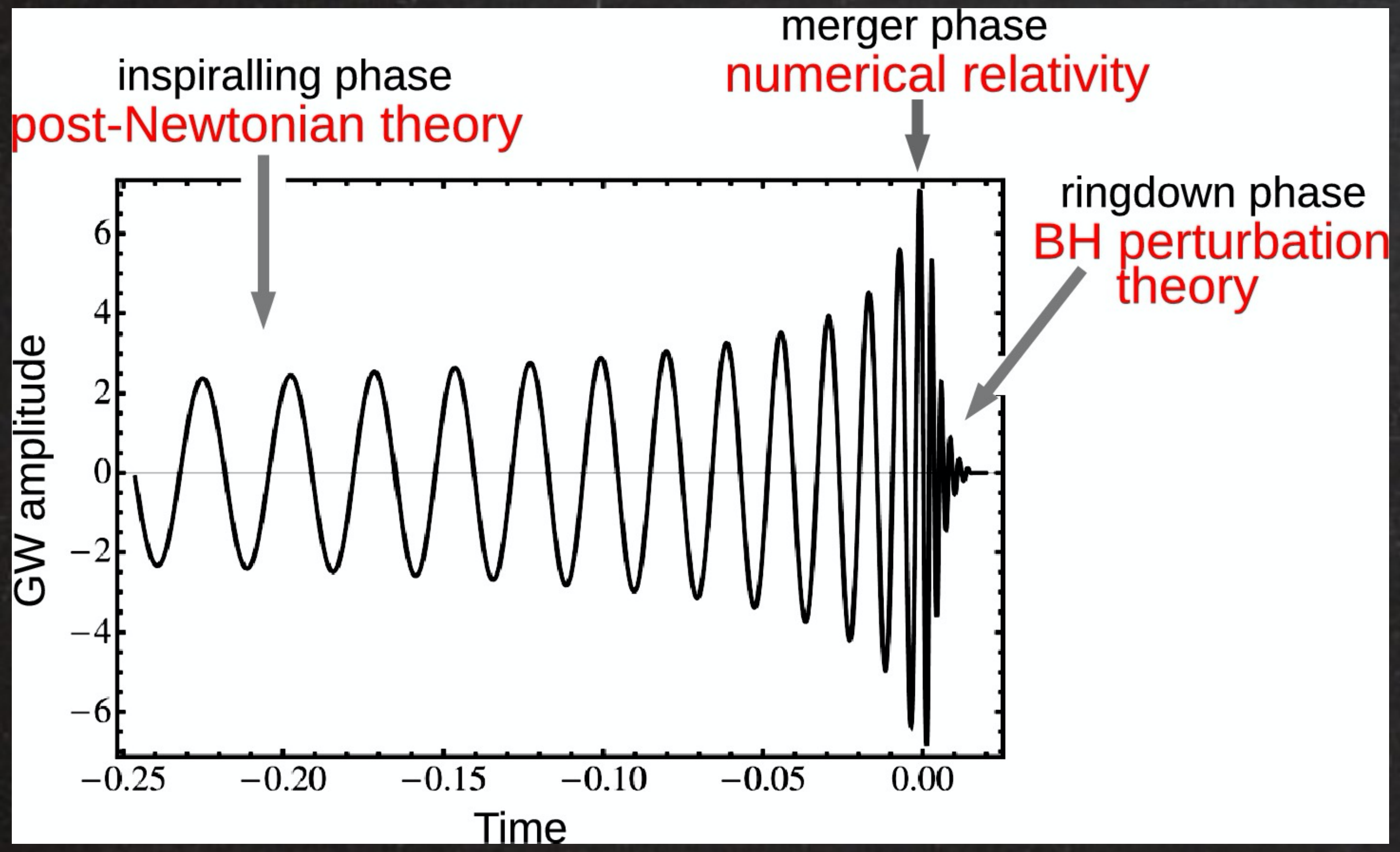


Image credit: Luc Blanchet, doi.org/10.1016/j.crhy.2019.02.004

Waveform models are crucial for extracting the best info from the detectors data:

- Parameter estimation of source properties
- Tests of general relativity
- Searches/event rates, …

Accurate, general and efficient waveform models needed for the challenges of next observing runs and future observatories (LISA, ET)

Phenomenological waveform modelling program

Phenom(enological) waveform modelling: accurate and fast representations of GW signals.

- Extreme compression of available information (PN theory, BH perturbation theory, Numerical Relativity) in terms of fast closed-form expressions for the waveforms.

In Fourier domain (best suited for most data analysis applications).

Continuous development towards modelling generic CBC signals:

- Non-spinning (PhenomA/B)
- Spin aligned (dominant mode): PhenomC/D/X
- Precessing: PhenomP/Pv2/Pv3/XP
- Higher harmonics: PhenomHM/XHM/Pv3HM/XPHM
- Eccentricity: PhenomXE
- Matter: PhenomNRTidal/NSBH

Motivation for a time domain Phenom family:

- Dispense with the Stationary Phase Approximation (SPA) for modelling the precession transfer functions.
- Closer relation to system dynamics (aims to help in the modelling of generic systems).
- Easier to approximate precessing ringdown.
- Cleaner inspiral-merger-ringdown separation for testing GR.
- While maintaining Phenom philosophy:
    - Efficient and accurate representation of the waveforms.

Phenom modelling in the time domain: non-precessing

GW polarisations decomposed in (spin-weighted) spherical harmonic basis:

$$h_+(t) - ih_\times(t) = \sum_l \sum_{-l \le m \le l} h_{lm}(t)\ {}_{-2}Y_{lm}(\iota, \phi)$$

IMRPhenomT/P: Estellés et al 2020

IMRPhenomTHM: Estellés et al 2020

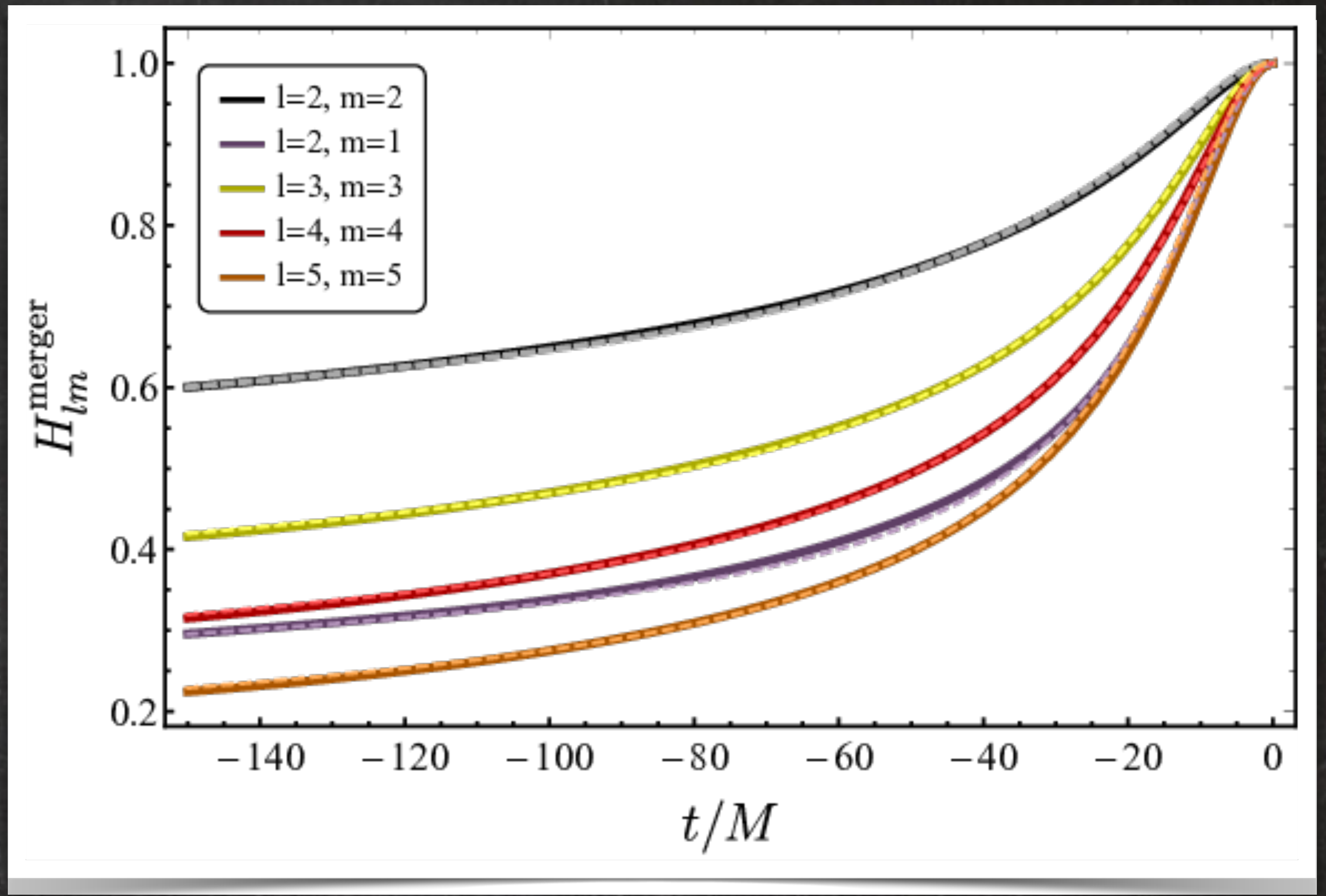
IMRPhenomTPHM: Estellés et al 2021

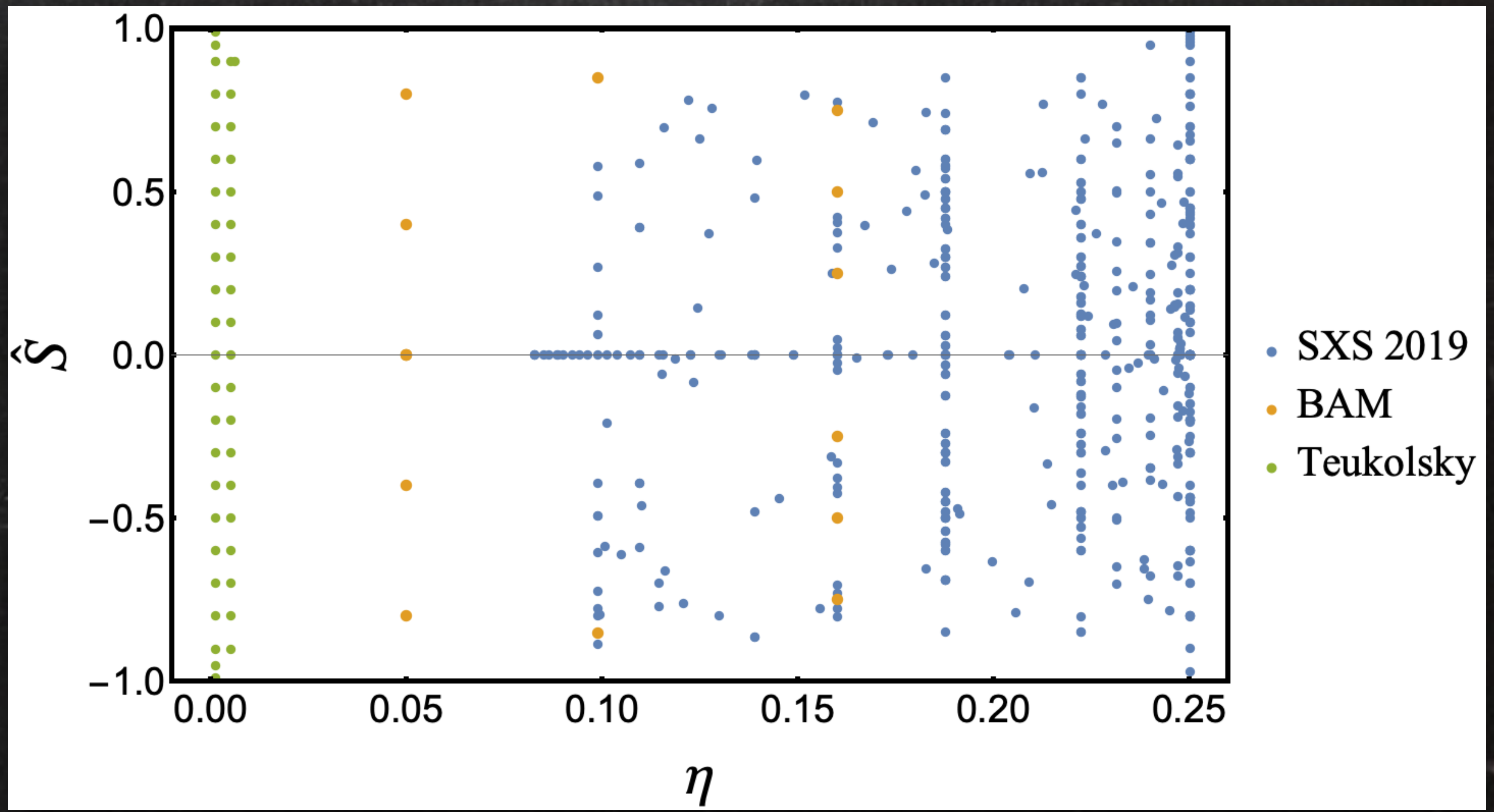
Model separately each mode (2,±2), (2,±1), (3,±3), (4,±4), (5,±5):

- Piecewise C^1 expressions for amplitude and phase (derivative) of each mode.

$$H_{lm} = |h_{lm}|,\ \phi_{lm} = \arg(h_{lm}),\ \omega_{lm} = \dot{\phi}_{lm}$$

- Inspiral: PN analytical expressions (3.5PN spinning TaylorT3 for orbital frequency, 3PN amplitudes from Blanchet+, 2PN corrections from Buonanno+, 1.5PN corrections from Arun+ + 3.5PN for (2,2) amplitude from Faye+)
- Intermediate/plunge: phenomenological expressions based on hyperbolic functions.
- Ringdown: adaptation of analytical expressions based on QNM decomposition from Damour+
- Calibrated with 531 BBH non-precessing NR simulations from SXS Collaboration Catalog (2019) Boyle+, 15 BAM simulations and 61 numerical waveforms from TeukCode.





Phenom modelling in the time domain: precessing

Precessing extension based on "twisting-up" technique:

$$h_+(t) - ih_\times(t) = \sum_l \sum_{-l\leq m\leq l} h^I_{lm}(t)\ {}_2Y_{lm}(\iota, \phi)$$

- Inertial frames modes obtained from rotation of non-inertial (co-precessing) modes with simpler morphology:

$$h^I_{lm}(t) = \mathscr{D}^l_{mm'}[\alpha(t), \beta(t), \gamma(t)] h^{\text{coprec}}_{lm'}(t)$$

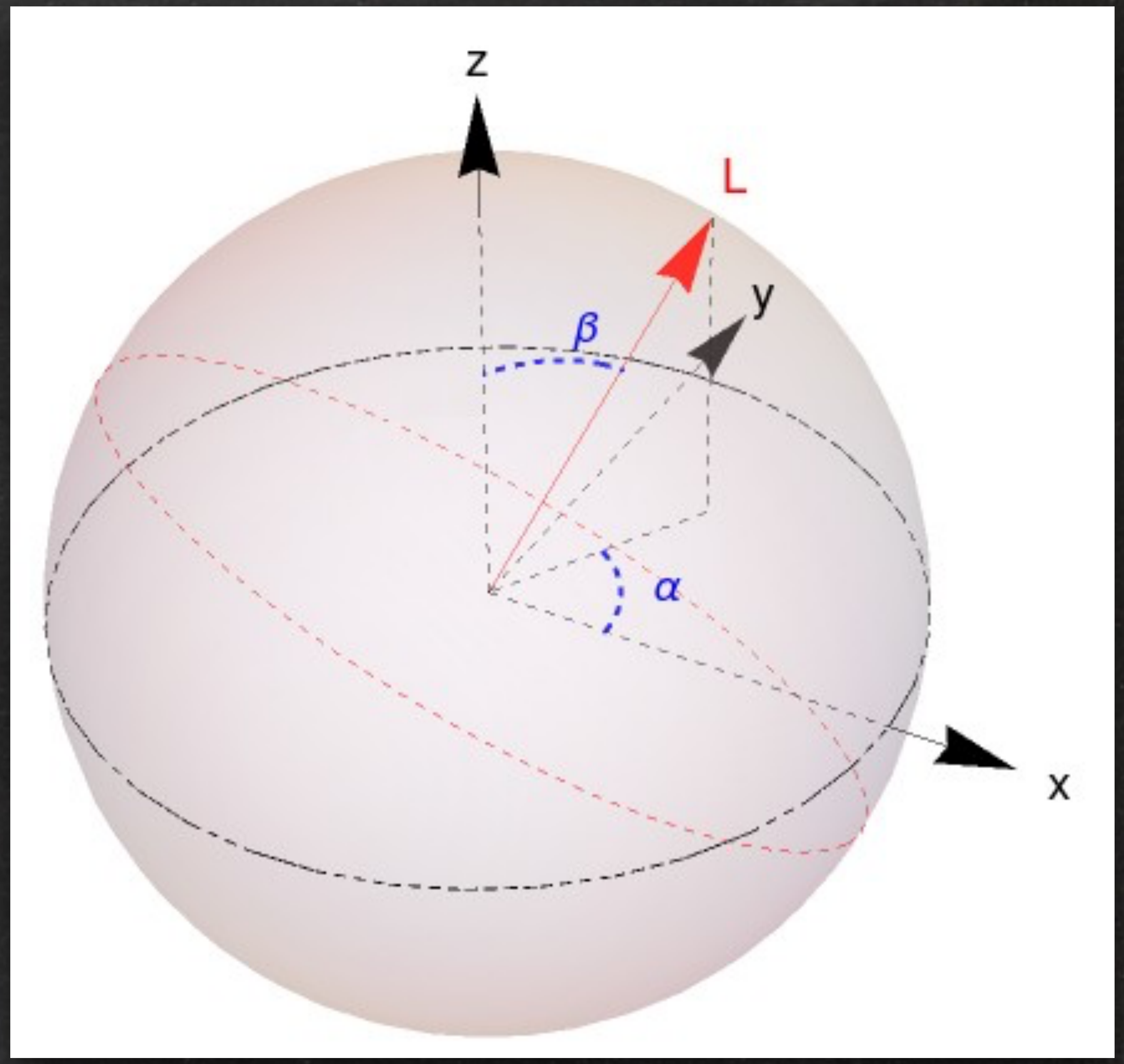
- Euler angles encode precessing motion of the orbital plane:

$$\alpha = \arctan(\hat{\ell}_y, \hat{\ell}_x) \qquad \dot{\gamma} = -\dot{\alpha}\cos\beta$$

$$\cos\beta = \hat{\boldsymbol{J}} \cdot \hat{\boldsymbol{\ell}} = \hat{\ell}_z$$

- Co-precessing modes approximated from non-precessing model (with modified precessing final state):

$$h^{\text{coprec}}_{lm}(t; m_1/m_2, \chi_1, \chi_2) \approx h^{\text{nonprec}}_{lm}(t; m_1/m_2, \chi_{1l}, \chi_{2l})$$



Credit: Maria de Lluc Planas

$\hat{\boldsymbol{\ell}}$: Unit vector perpendicular to the orbital plane (Newtonian orbital angular momentum).

$\boldsymbol{J}$: Total angular momentum of the system.

Euler angles: analytical approaches

Main analytical approaches to precessing Euler angles:

- Next-to-next-to-leading order (**NNLO**) (Bohe+) effective single spin.
- Multiscale analysis (**MSA**) (Chatziioannou+) doble spin.

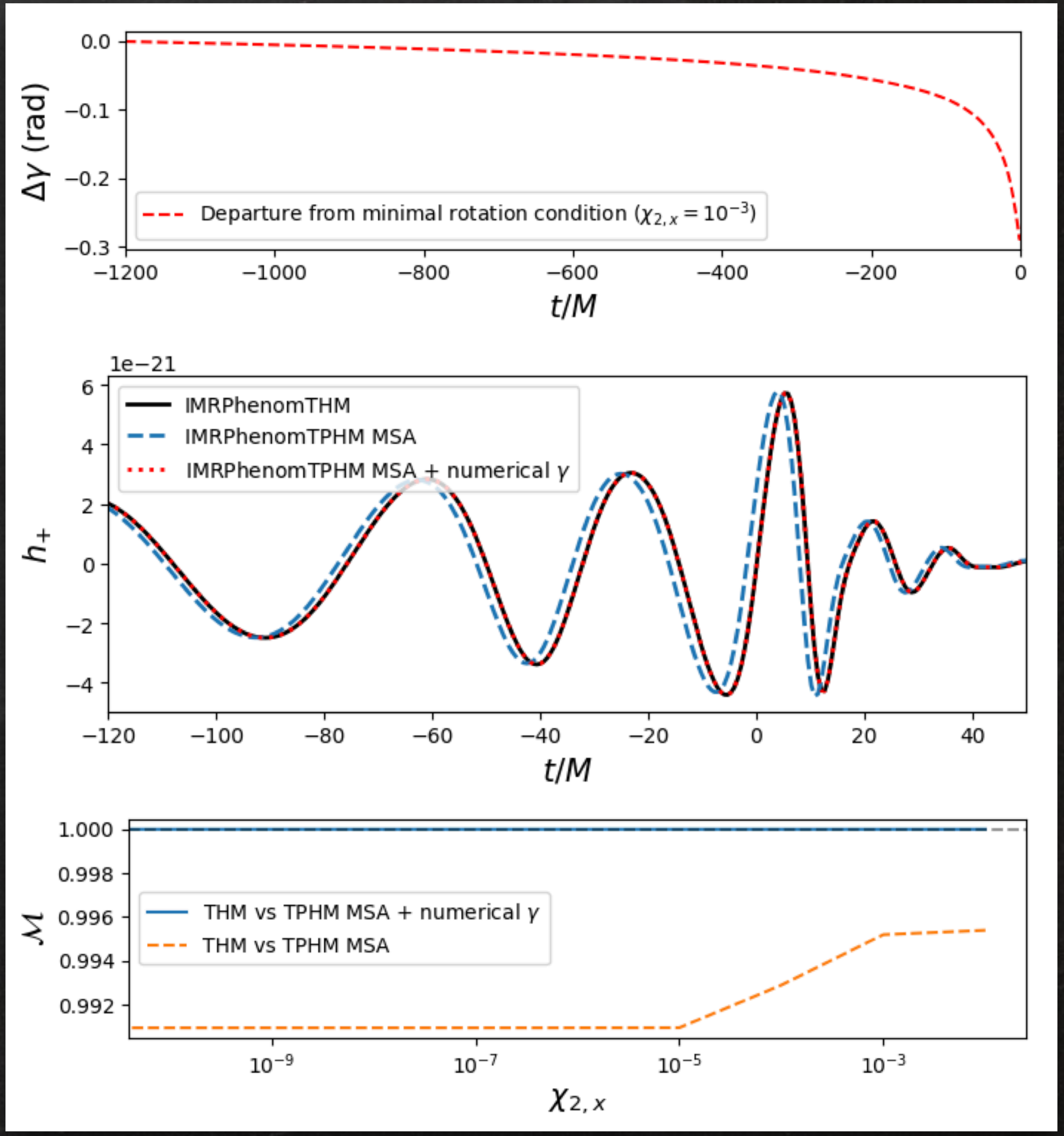
Aimed for more direct comparisons with other Phenom models.

Evaluated with non-precessing analytical orbital frequency:

$$v(t) = \Omega_{orb}^{1/3},\ \Omega_{orb} = \frac{1}{2}\omega_{22}^{T}$$

Improvements over previous implementations:

- Numerical evaluation of minimal rotation condition (recovering of nonprecessing limit in MSA).
- Smooth plunge behaviour with linear continuation.



Euler angles: numerical evolution

Numerical evolution approach:

- Solve evolution equations for $\hat{\ell}$ (implies evolving individual spin vectors):

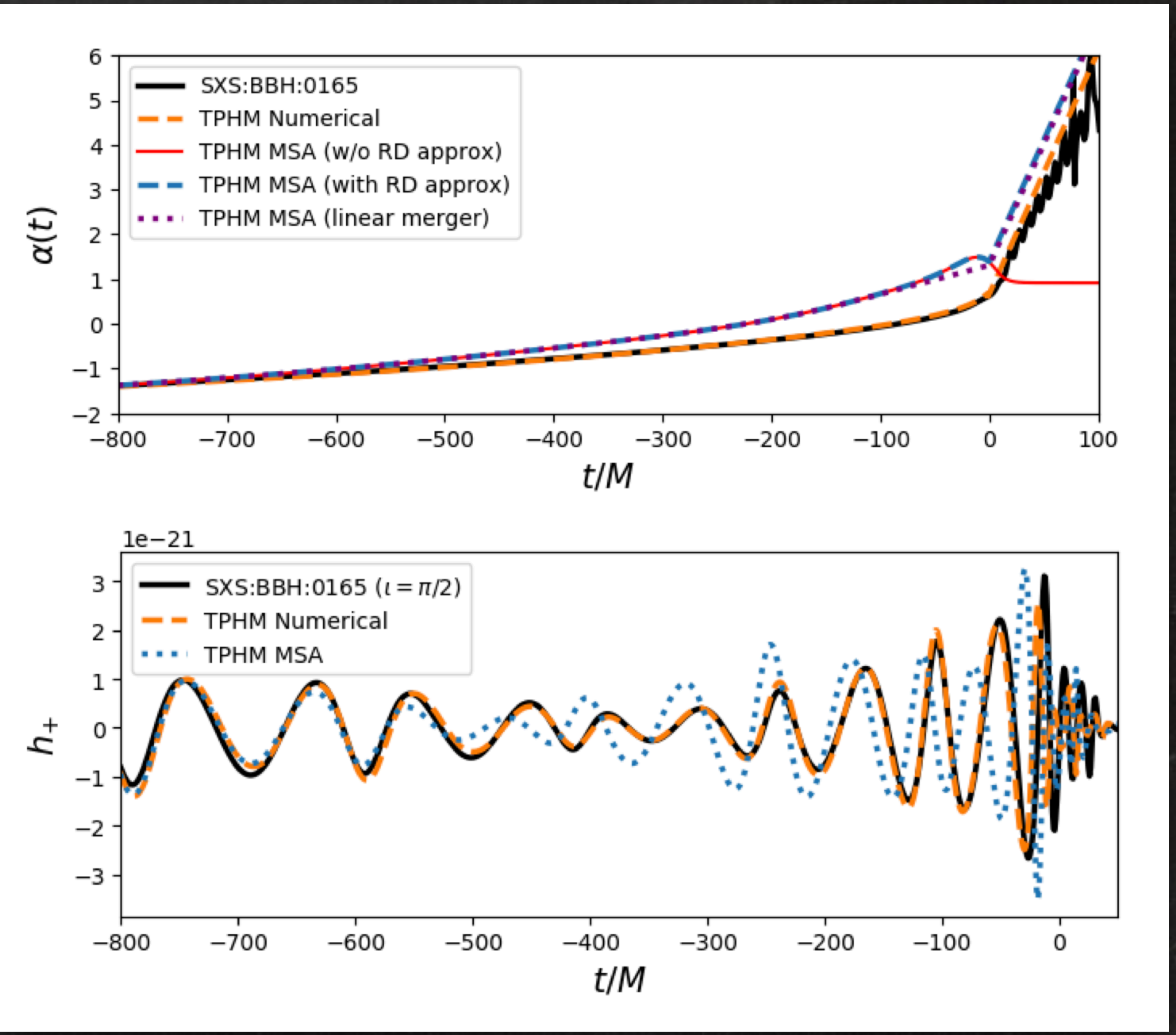
$$\frac{d\hat{\ell}}{dt} = \mathbf{\Omega}(v(t), q, \mathbf{S}_1, \mathbf{S}_2) \times \hat{\ell}$$

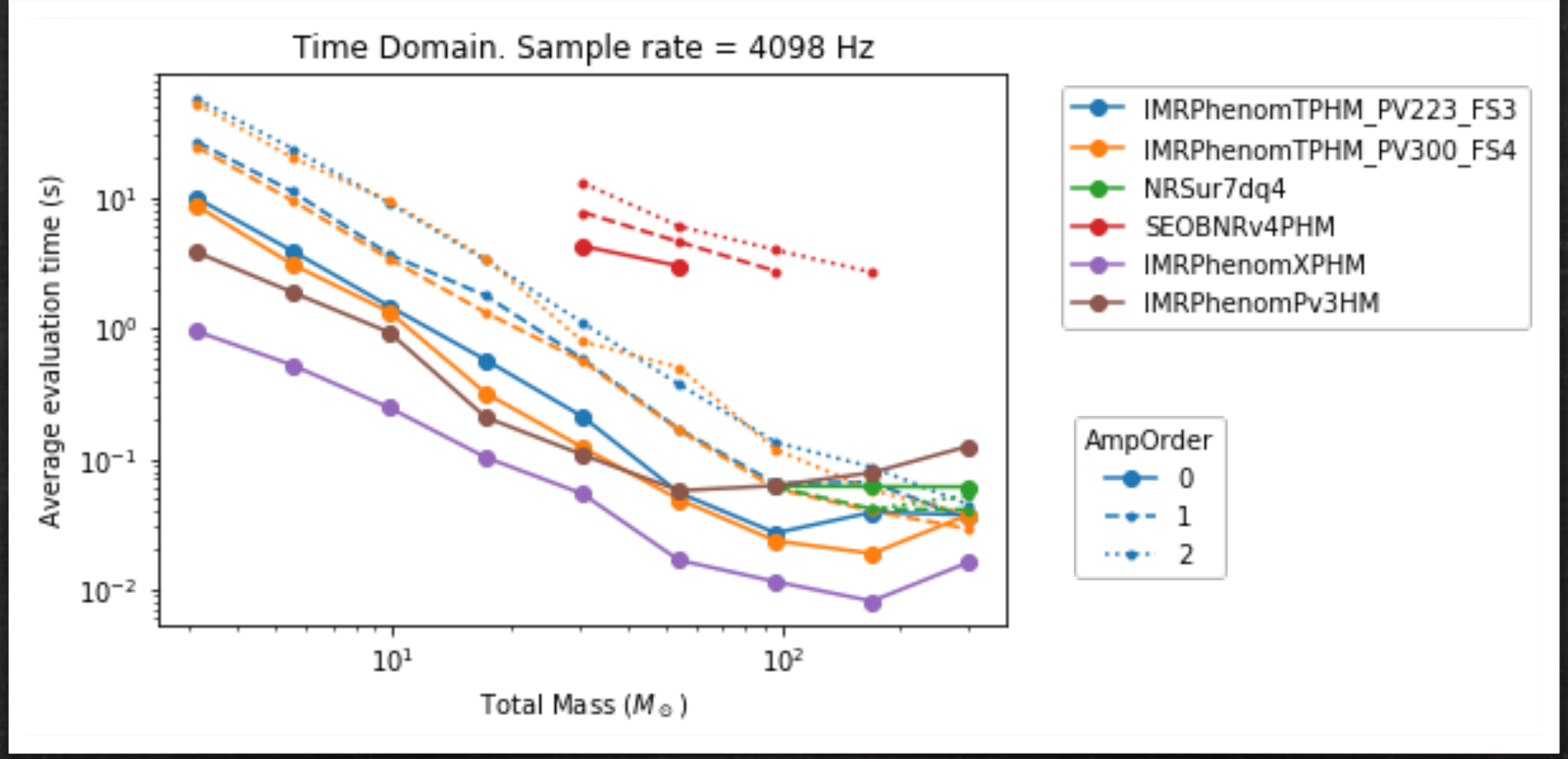
$$\frac{d\mathbf{S}_{1,2}}{dt} = \mathbf{\Omega}(v(t), q, \mathbf{S}_1, \mathbf{S}_2) \times \mathbf{S}_{1,2}$$

$$\dot{\ell} = -\dot{S}_1 - \dot{S}_2$$

- Orbit averaged N^4LO PN expressions included.
- Tracking all degrees of freedom: improvement over previous analytical expressions.
- Efficient evaluation in terms of analytical non-precessing orbital frequency: fast implementation.
- Simple analytical approximation attached at ringdown:

$$\alpha^{RD}(t) \simeq (\omega^{RD}_{122} - \omega^{RD}_{121})t + \alpha^{RD}_0$$





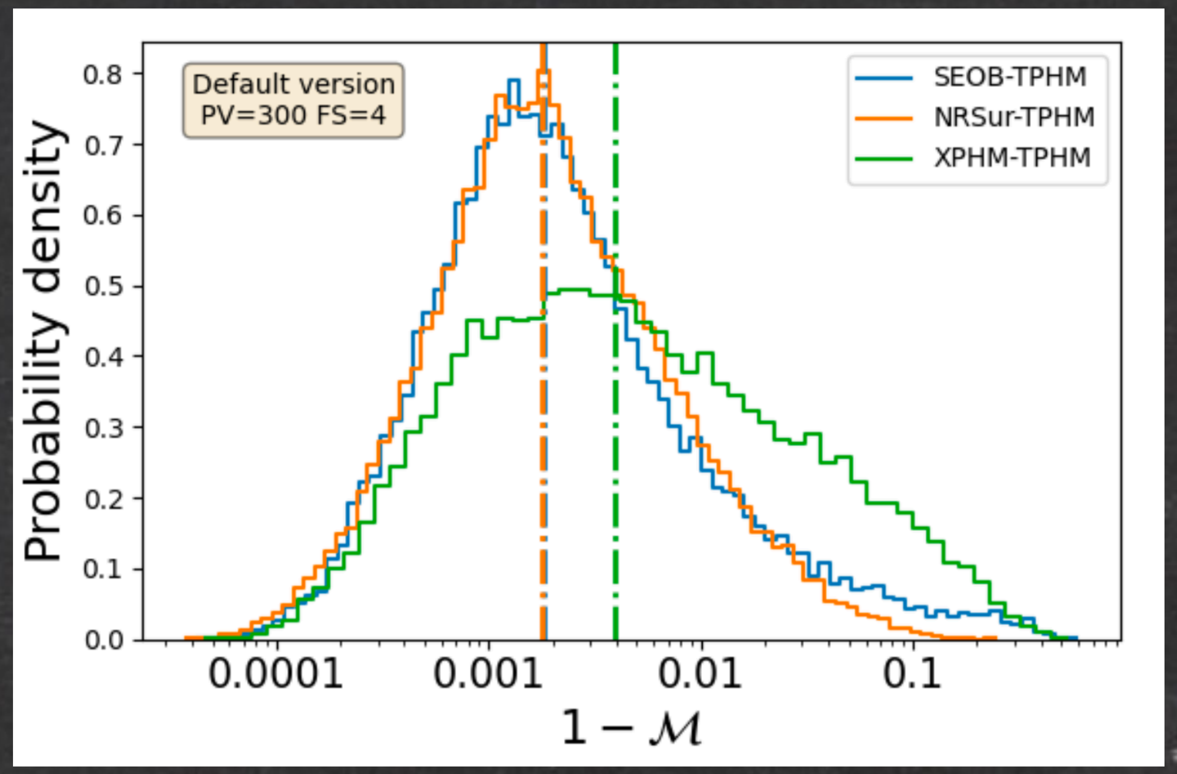
Comparison with other state-of-the-art waveform models

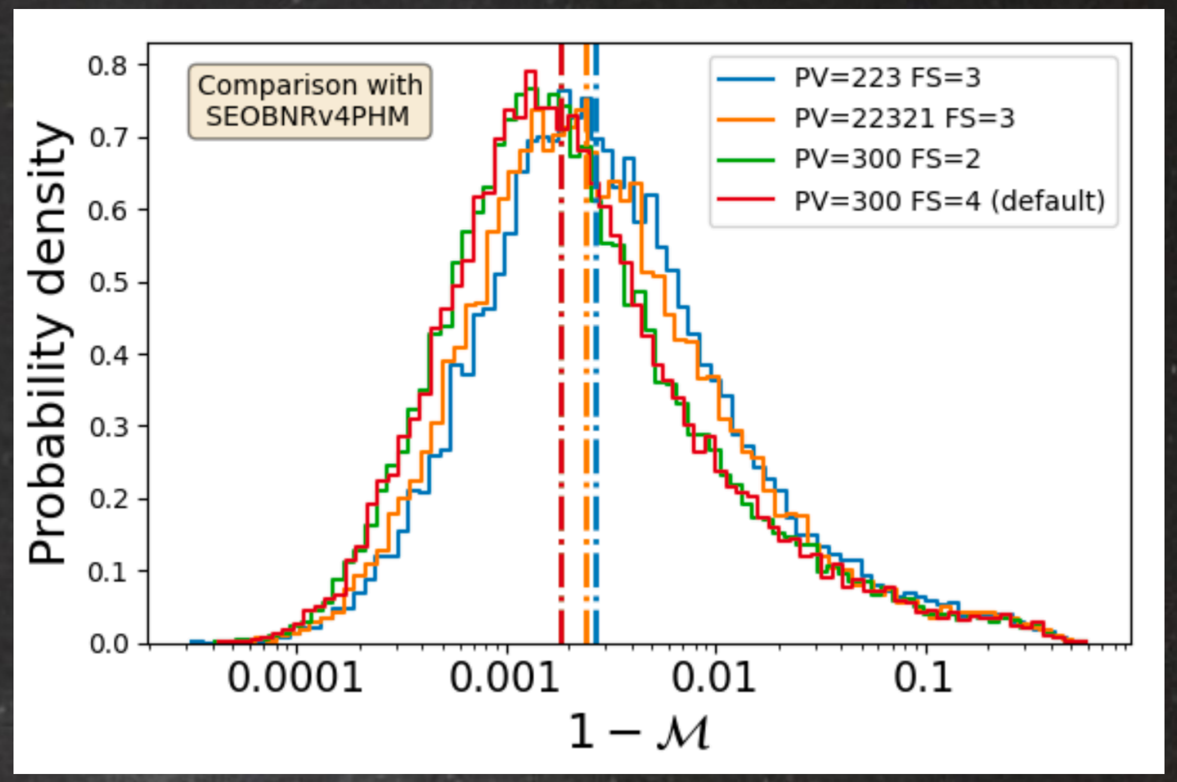
Unfaithfulness comparison with other state-of-the-art precessing multimode models: IMRPhenomXPHM, SEOBNRv4PHM and NRSur7dq4.

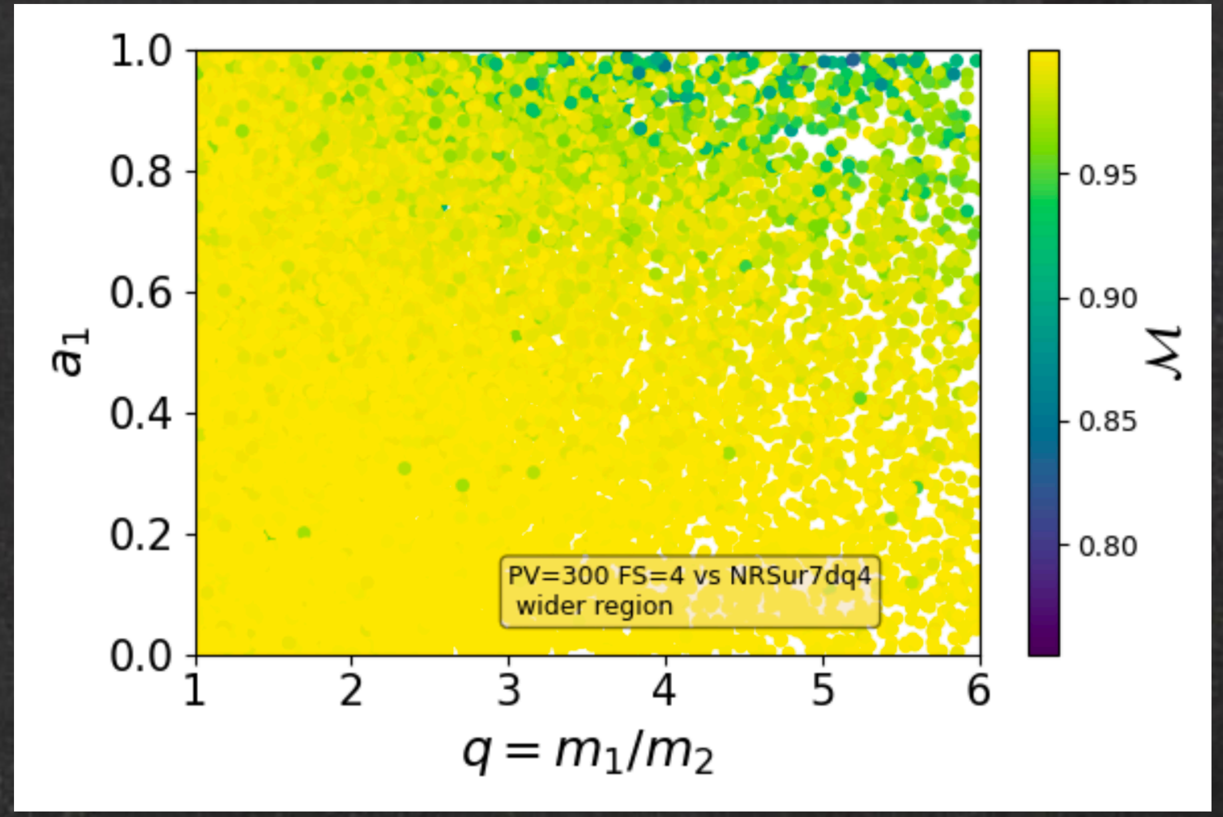
Great agreement with TD models (median ~ 0.2%): more consistent treatment of merger-RD.

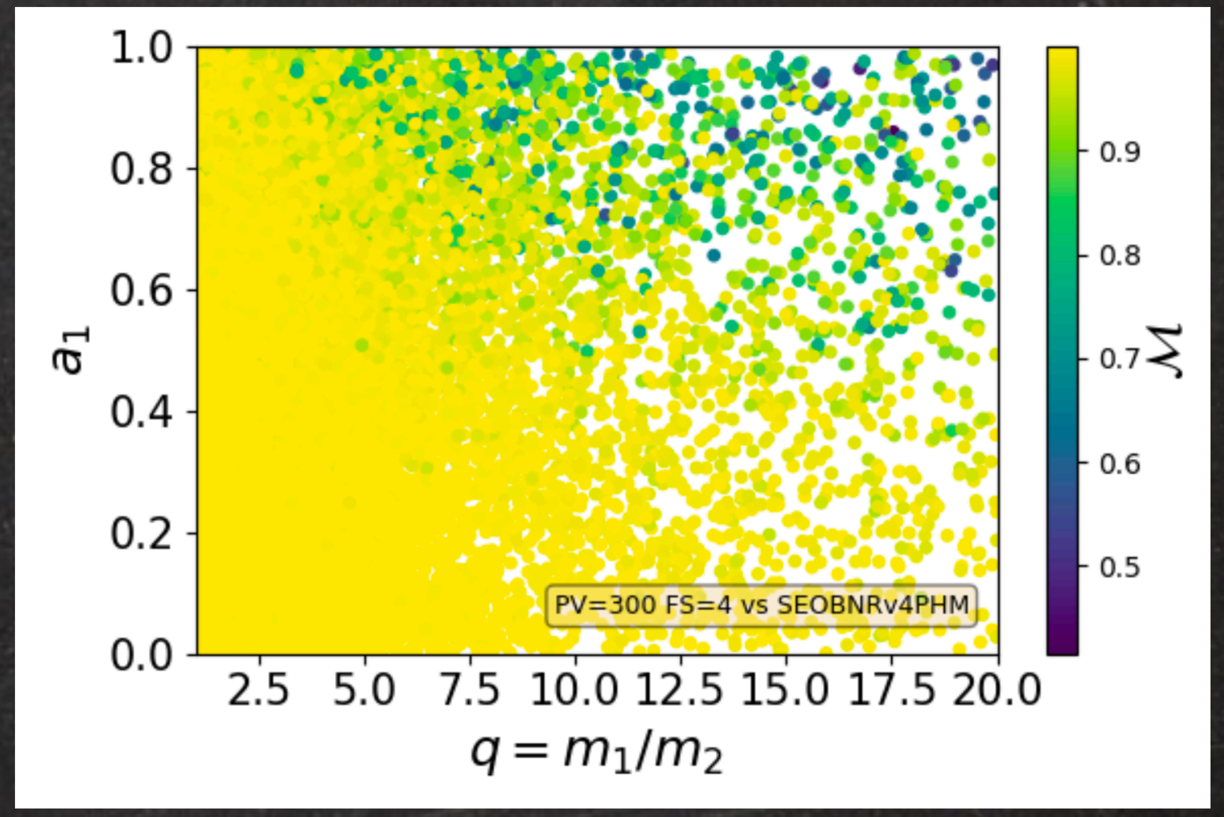
Better agreement of numerical approach with SEOB: more accurate inspiral than analytical approaches.

Disagreement for large mass asymmetry and high spins norm: possibly caveat of non-precessing orbital frequency.









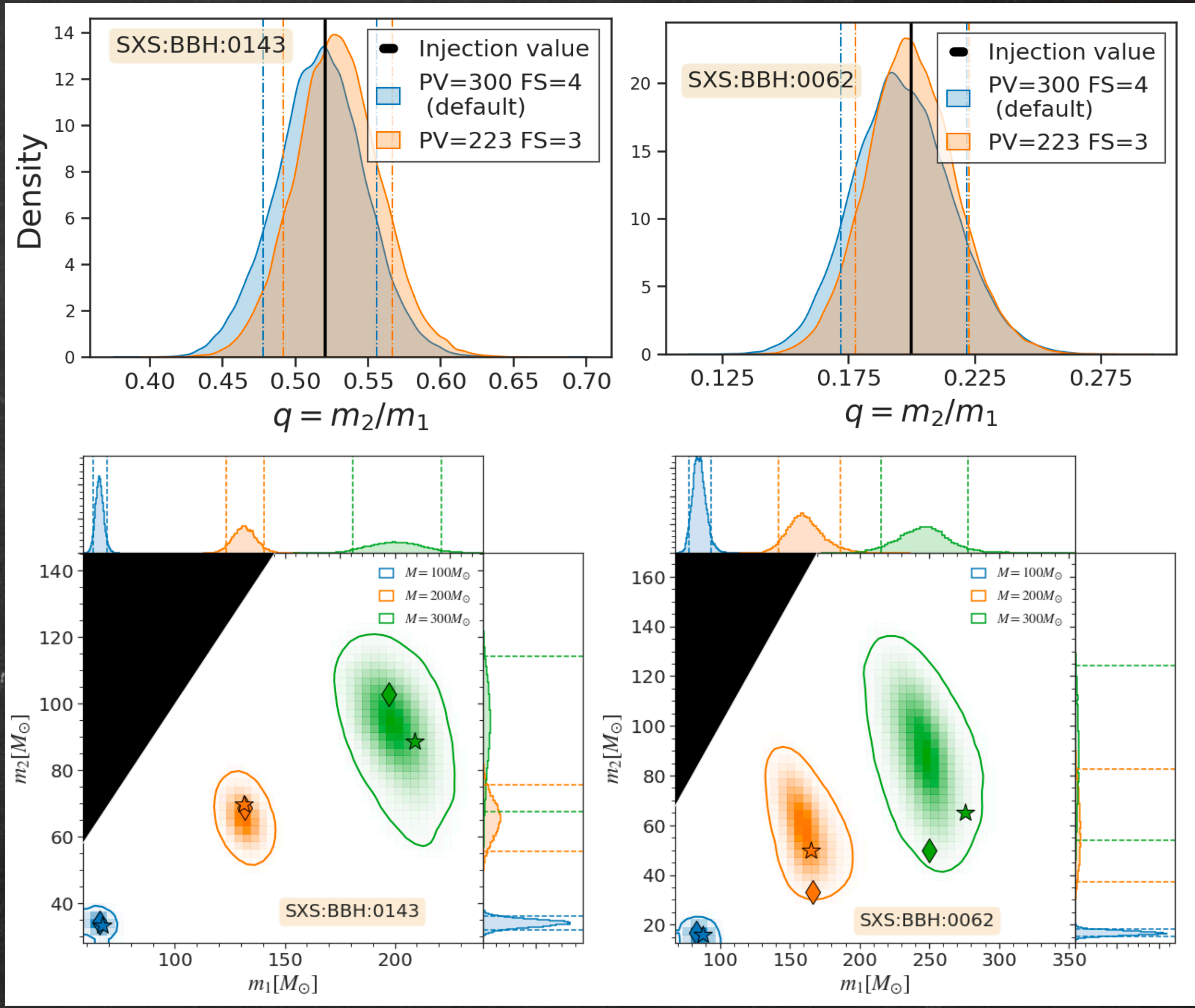
Comparison with Numerical Relativity

Unfaithfulness comparison with Numerical Relativity precessing simulations:

- Bulk of cases below 1% mismatch.
- 1 outlier (SXS:0165) with challenging parameters. Need to include further physics (mode asymmetry).

Parameter estimation of NR injected signals:

- Correct recovery at $M_T = 100 M_\odot$.
- Individual masses in 90% contour levels for higher masses.
- Need of more detailed systematic studies towards identifying recovery bias.



Parameter estimation: GW190412 re-analysis

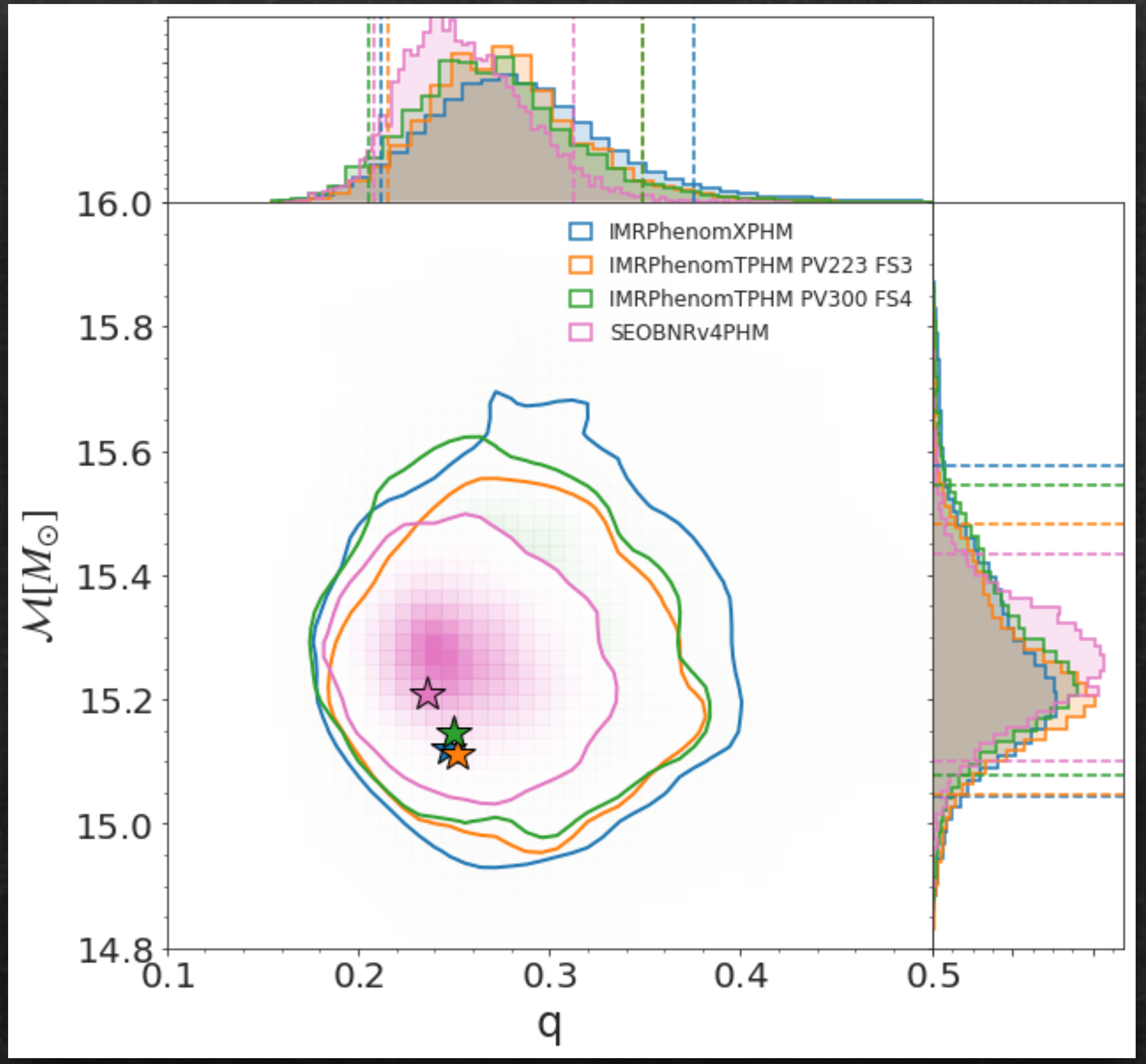
GW190412: first reported GW event with confident mass asymmetry: interesting to compare different multimode models.

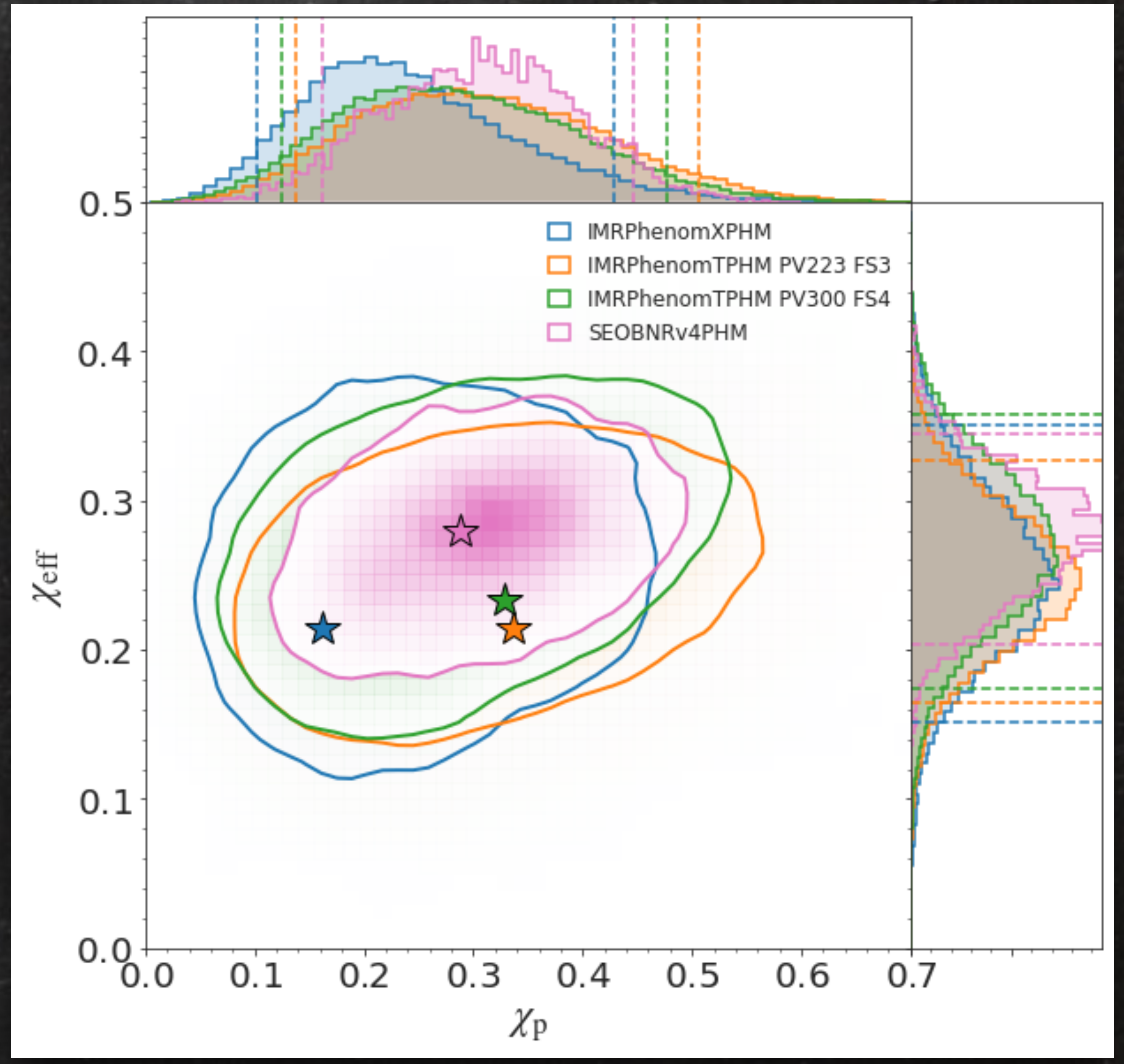
Non-precessing IMRPhenomTHM employed in published re-analysis (Colleoni+): great consistency with other NR-calibrated models.

Precessing re-analysis:

- Recovered medians and CI consistent with previous results.
- Slightly better agreement with SEOB results (in terms of medians and CIs). (SEOB results obtained with RIFT PE code)
- Higher SNR, likelihood and BF that for Fourier domain models.

parameter	SEOBNRv4PHM	IMRPhenomPv3HM	LVC Combined	IMRPhenomXPHM	IMRPhenomTPHM
$m_1^s/M_\odot$	$31.7^{+3.6}_{-3.5}$	$28.1^{+4.8}_{-4.3}$	$29.7^{+5.0}_{-5.3}$	$30.0^{+5.2}_{-4.3}$	$30.9^{+3.5}_{-3.2}$
$m_2^s/M_\odot$	$8.0^{+0.9}_{-0.7}$	$8.8^{+1.5}_{-1.1}$	$8.4^{+1.8}_{-1.0}$	$8.4^{+1.3}_{-1.1}$	$8.2^{+0.8}_{-0.7}$
$M^s/M_\odot$	$39.7^{+3.0}_{-2.7}$	$36.9^{+3.7}_{-2.9}$	$38.1^{+4.0}_{-3.7}$	$38.4^{+4.2}_{-3.2}$	$39.1^{+2.8}_{-2.5}$
$\mathcal{M}^s/M_\odot$	$13.3^{+0.3}_{-0.3}$	$13.2^{+0.5}_{-0.3}$	$13.3^{+0.4}_{-0.3}$	$13.3^{+0.5}_{-0.4}$	$13.3^{+0.3}_{-0.3}$
q	$0.25^{+0.06}_{-0.04}$	$0.31^{+0.12}_{-0.07}$	$0.28^{+0.13}_{-0.06}$	$0.28^{+0.09}_{-0.07}$	$0.27^{+0.06}_{-0.05}$
$\chi_{\rm eff}$	$0.28^{+0.06}_{-0.08}$	$0.22^{+0.08}_{-0.11}$	$0.25^{+0.08}_{-0.11}$	$0.25^{+0.1}_{-0.1}$	$0.27^{+0.07}_{-0.07}$
$\chi_{\rm p}$	$0.31^{+0.14}_{-0.15}$	$0.31^{+0.24}_{-0.17}$	$0.30^{+0.19}_{-0.15}$	$0.23^{+0.20}_{-0.13}$	$0.28^{+0.15}_{-0.13}$

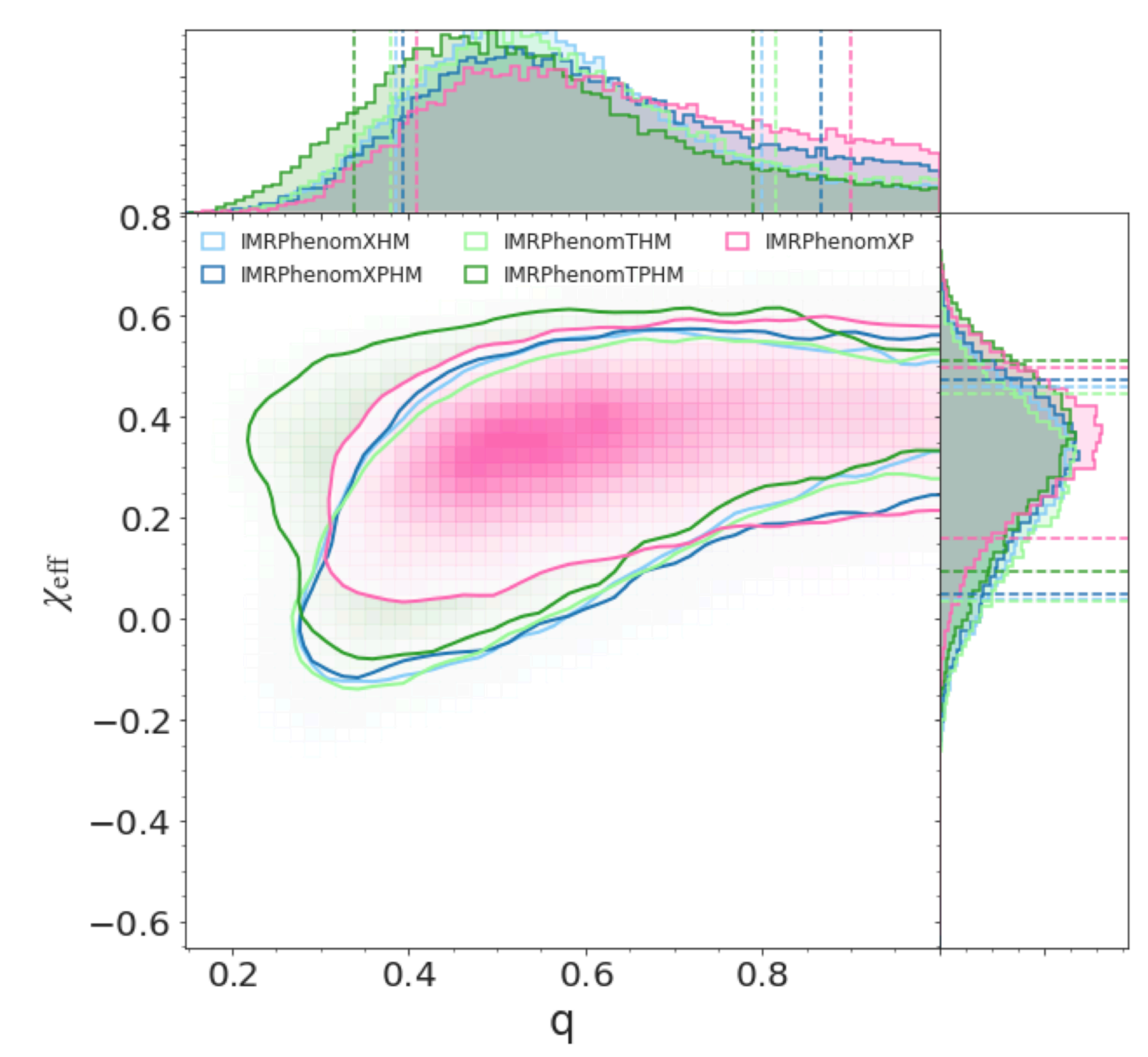


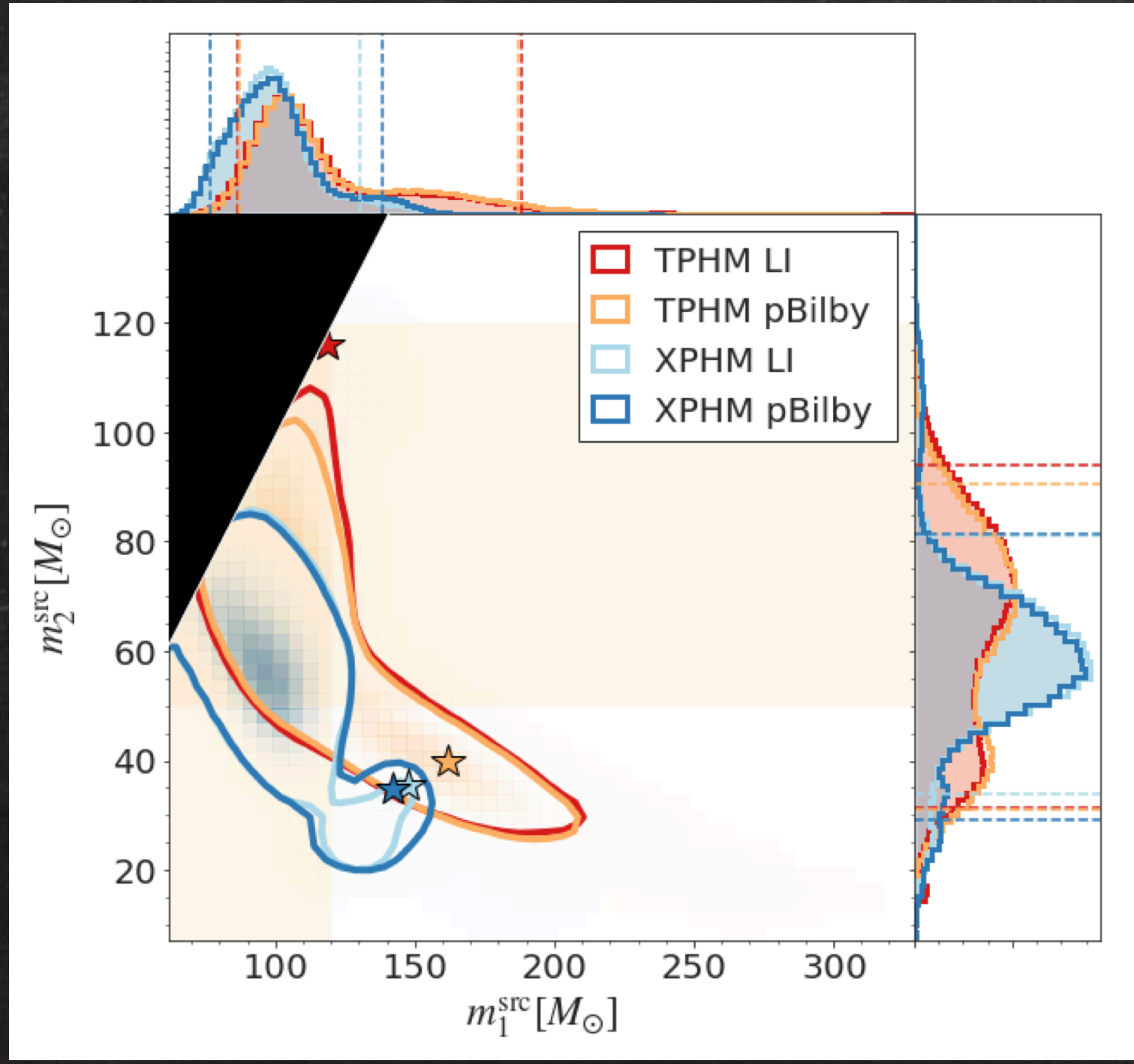


More on parameter estimation

Re-analysis of GWTC-1 with a new generation of phenomenological waveform models (Maite Mateu-Lucena, presented today at 12:40):

- Re-analysis with nonprecessing model IMRPhenomTHM for all events and precessing IMRPhenomTPHM for some of them.
- Consistent results with IMRPhenomXPHM, better inference for GW170729.





A detailed analysis of GW190521 with phenomenological waveform models (Marta Colleoni, Friday at 15:10):

- Discussion of recovery of high mass ratio support, higher mode content effects, probability of PISN mass gap, association with AGN flare ZTF19abanrhr …

Conclusions & future work

New precessing multimode model in the time domain for BBH signals:

- Phenom philosophy: fast and accurate representation of the waveforms.
- Improved inspiral Euler angle description: numerical evolution of spin evolution equations.
- Simple analytical approximation for the ringdown.
- Fast implementation.
- Candidate model for BBH coalescing signals.
- Reviewed by the LVC, publicly available with LALSuite.

Caveats and future improvements:

- Improve efficiency:
  - Inefficient evaluation time for low mass signals.
  - Bottleneck in ringdown evaluation for highly redshifted massive systems.
- Improve physics:
  - Consistent evolution of orbital frequency in terms of the evolving spin magnitudes.
  - Include mode asymmetry effects.
  - Better understanding of precessing ringdown.