Hybrid Waveforms for Precessing Binary Black holes for LIGO Data Analysis

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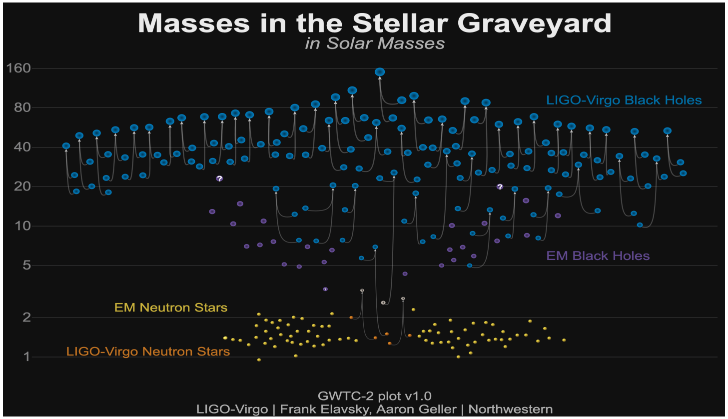
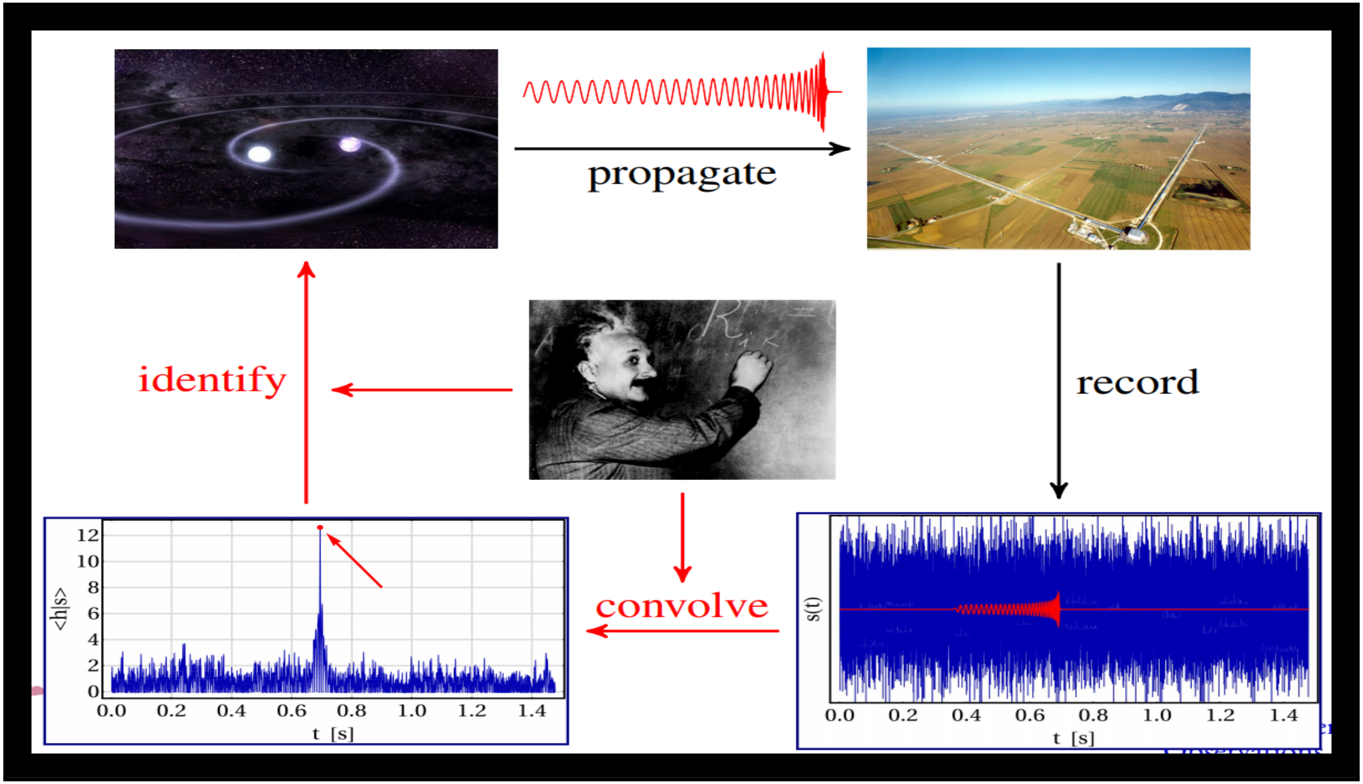
Detection of Gravitational Waves



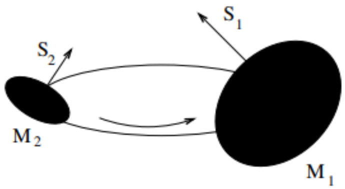
ICRR, Univ. of Tokyo/LIGO Lab/Caltech/MIT/Virgo Collaboration, Nature, 2019 [1]

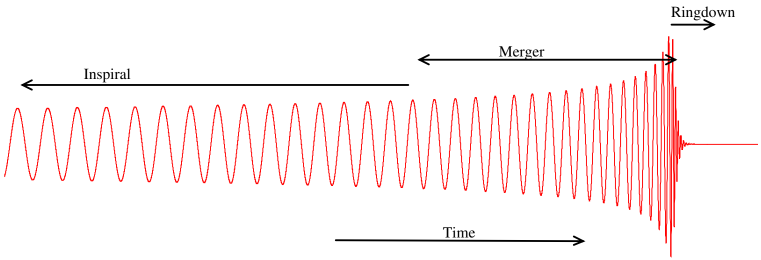
Binary Black-hole Observations and Analysis



Ohme, 2019 [2]

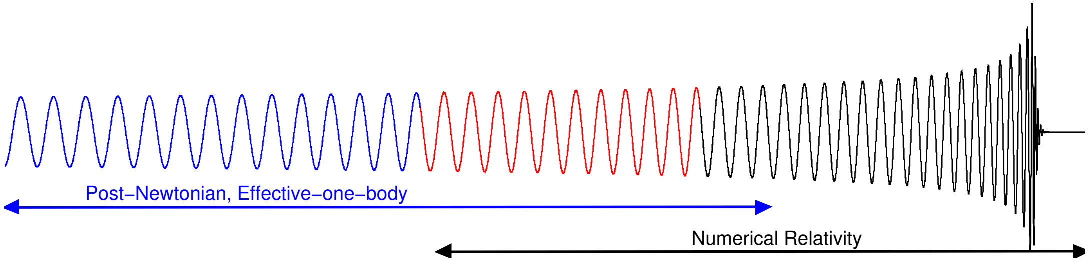
Binary Black-hole Waveforms





- Waveform divided into three parts
  - Inspiral: BHs far apart, described by post-Newtonian (PN) theory.
  - Merger: Highly relativistic, needs Numerical Relativity (NR).
  - Ringdown: Single BH, described by perturbation theory or NR.
- **Idea**: Match NR simulation to PN, before PN becomes inaccurate

Hybrid Waveforms



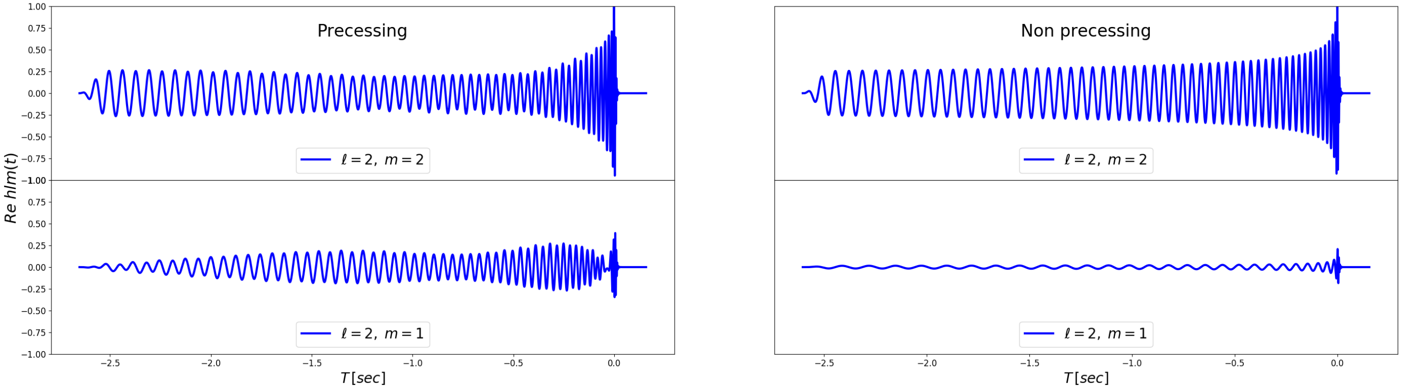
- With the improvements in low frequency detector sensitivity, longer waveforms will be required
- Numerical relativity waveforms are accurate but mostly available for late inspiral merger and ring down phase
- Analytical approximate model waveforms provide good accuracy waveforms in the early inspiral regime
- Numerical relativity waveforms have been **hybridized** to analytical model waveforms

Objective of the Work

- Non-precessing binaries have been hybridized and used for parameter estimation as done in Lange et. al (2017) [3]
- The precessing-binary parameter space has been sampled by only a relatively small number of numerical simulations.
- We want to **hybridize precessing binaries** for LIGO data analysis for such events
- Hybridizing precessing waveforms is a complicated process, as shown below

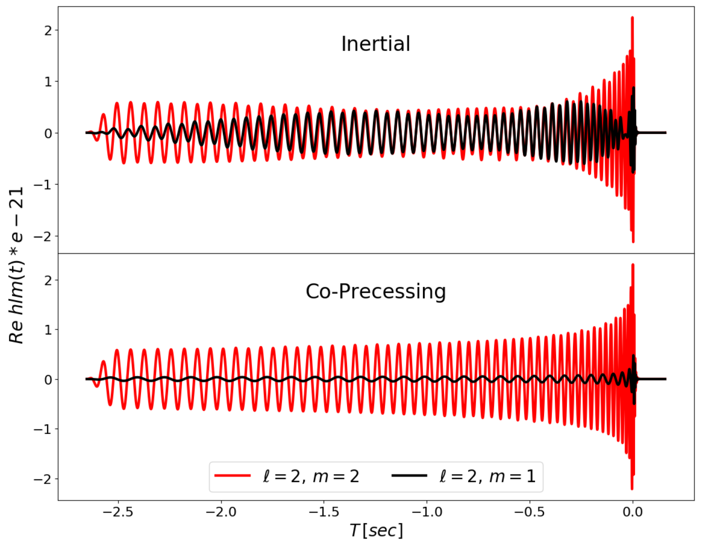
Precessing versus Non-Precessing Dynamics

- In precessing binaries the orbital precession strongly affects the gravitational waveforms by modulating both amplitude and phase
- $\ell = 2,\ m = \pm 2$ are not necessarily the dominant mode for the waveforms



Co-precessing Frame

- Complications of precession can be reduced by transforming the waveform into co-precessing frame
- Time dependent 3D rotation of the waveform to align the orbital angular momentum along z-direction (non-inertial)
- In this co-precessing frame the waveform behaves like a non-precessing waveform



SXS:BBH:0058 [?]

Co-precessing Frame

- Developed in O'Shaughnessy et al. (2011), Schmidt et al. (2011), Boyle et al. (2011) [4, 5, 6]
- The idea is to continually rotate (3D) the waveform. Two of the Euler angles obtained from principal eigenvector of the orientation-averaged tensor

$$\langle L_{(ab)} \rangle = \frac{1}{\sum_{lm} |h_{lm}|^2} \begin{bmatrix} I_0 + \mathrm{Re}(I_2) & \mathrm{Im} I_2 & \mathrm{Re} I_1 \\ \mathrm{Im} I_2 & I_0 - \mathrm{Re}(I_2) & \mathrm{Im} I_1 \\ \mathrm{Re} I_1 & \mathrm{Im} I_1 & I_{zz} \end{bmatrix}$$

where I_0, I_1, I_2, I_{zz} are related to quantum mechanical angular momentum notations.

Co-precessing Frame Rotations

- Two of the Euler angles are related to the principal eigenvector $\hat{V}$ of the orientation-averaged tensor $\langle L_{(ab)} \rangle$, O'Shaughnessy et al. (2011)[4]

$$\alpha = \cos^{-1}[\hat{v}_z]$$

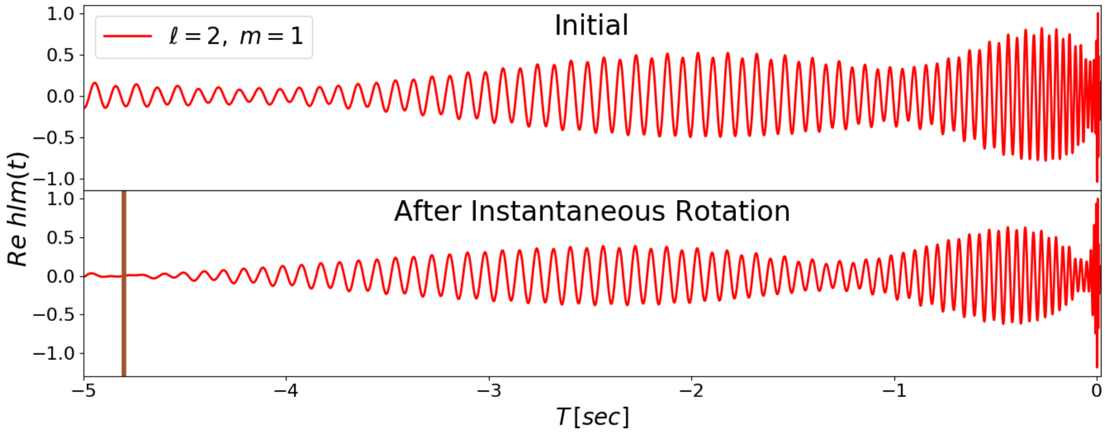
$$\beta = \mathrm{Arg}[\hat{v}_x + i\hat{v}_y] - \frac{\pi}{2}$$

- The third Euler angle chosen to account for the phase evolution of the waveform. Boyle et al. (2011)[6]

$$\gamma = -\int \dot{\alpha} cos\beta \quad dt$$

Hybridization Procedure: Instantaneous Rotation

- We choose a time and the corresponding instantaneous rotations (α, β, γ) of two waveforms that place the waveforms in the co-precessing frame at that time. This minimize the non-quadrupole modes for a short period $H^{\mathrm{rot}}_{lm}(t) = \sum_{m'=-l}^{l} e^{im'\gamma + im\alpha} d^{l}_{mm'}(\beta) h_{lm}(t)$



Hybridization Procedure: Instantaneous Rotation

- We rotate the two waveforms at these fixed angles

$$H_{lm}^{\mathrm{rot}}(t) = \sum_{m'=-l}^{l} e^{im'\gamma + im\alpha} d^{l}_{mm'}(\beta) h_{lm}(t)$$

Hybridization Procedure: Time and Phase shifts

- We rotate the two waveforms at these fixed angles

$$H^{\mathrm{rot}}_{lm}(t) = \sum_{m'=-l}^{l} e^{im'\gamma+im\alpha} d^{l}_{mm'}(\beta) h_{lm}(t)$$

- we compute phase and time shifts to align waveforms in a hybridizing interval

$$\Delta = min_{t_0,\phi_0,\Psi} \int_{t_1}^{t_2} \sum_{l,m} |H^{\mathrm{NR}}_{lm}(t) - H^{\mathrm{PN}}_{lm}(t-t_0) e^{i(m\phi_0+2\Psi)}|\mathrm{dt}$$

Hybridization Procedure: Hybridization

- We rotate the two waveforms at these fixed angles
$$H_{lm}^{\mathrm{rot}}(t) = \sum_{m'=-l}^{l} e^{im'\gamma + im\alpha} d_{mm'}^{l}(\beta) h_{lm}(t)$$
- we compute phase and time shifts to align waveforms in a hybridizing interval

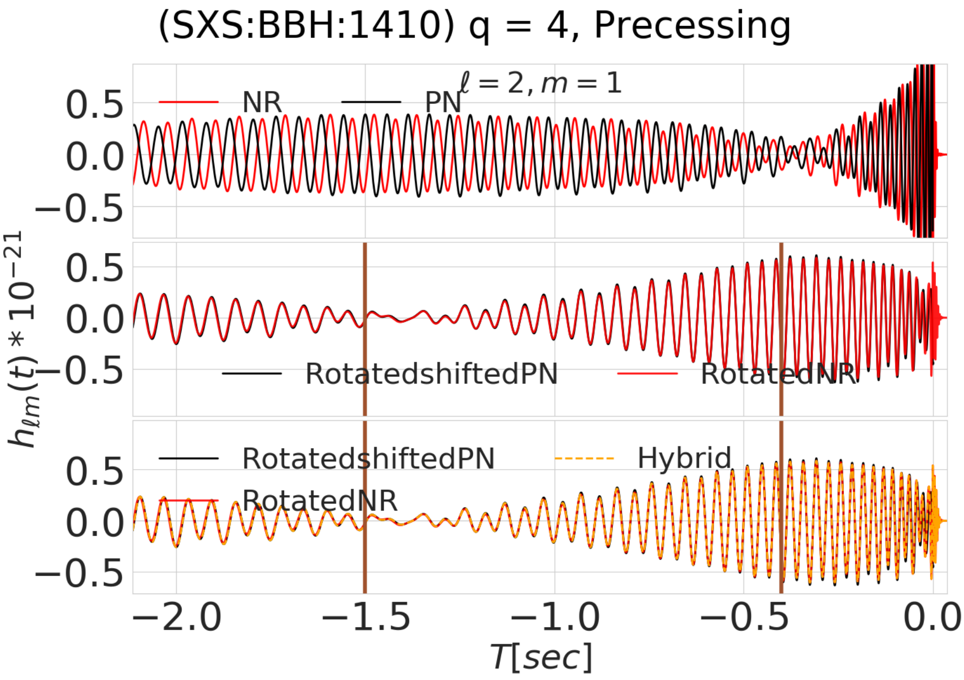
$$\Delta = min_{t_0,\phi_0,\Psi} \int_{t_1}^{t_2} \sum_{l,m} |H_{lm}^{\mathrm{NR}}(t) - H_{lm}^{\mathrm{PN}}(t - t_0) e^{i(m\phi_0 + 2\Psi)}| \mathrm{dt}$$

- Construct the Hybrid
$$h_{lm}^{\mathrm{hyb}} = \tau(t) H_{lm}^{\mathrm{NR}}(t) + [1 - \tau(t)] H_{lm}^{\mathrm{PN}}(t - t_0') e^{i(m\phi_0' + 2\Psi')}$$

where $\tau(t)$ is function that smoothly goes from 0 to 1 in hybrid interval

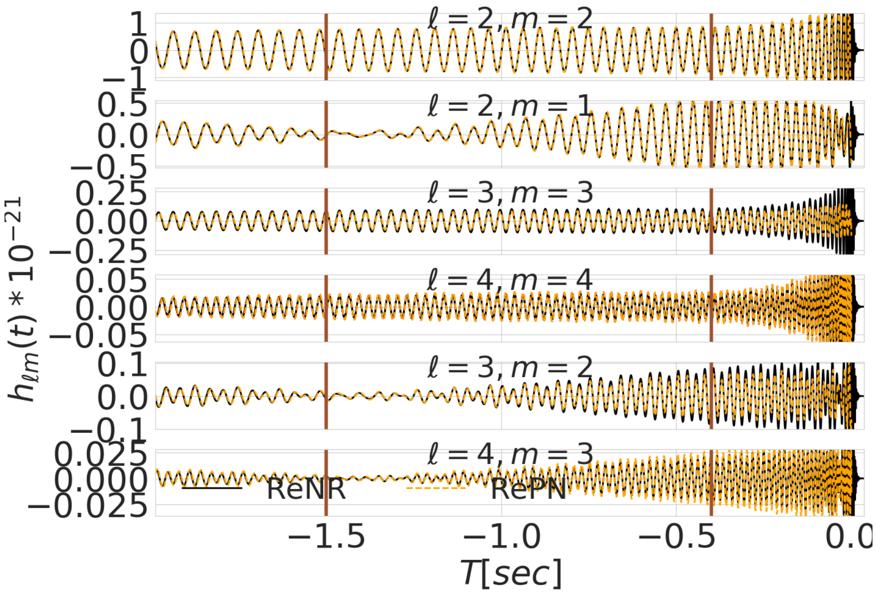
Complete Hybridization

- $\chi_1 = (0.239, 0.318, 0.244)$, $\chi_2 = (-0.361, 0.039, 0.289)$, $M_{tot} = 70M_{\odot}$ [7, 8, 9]



Hybridization of Precessing Binary Waveform

- Precessing case **SXS:BBH:1410**, $q = 4$, $\chi_1 = (0.239, 0.318, 0.244)$, $\chi_2 = (-0.361, 0.039, 0.289)$, $M_{tot} = 70M_{\odot}$



Quality of Hybrid Waveforms

- Compute mode by mode mismatch using advanced LIGO power spectral density

$$\langle h_1|h_2\rangle = 2\int_{-\infty}^{\infty} \frac{h_1^*(f)h_2(f)}{S_n(f)}\mathrm{df}$$

$$\mathcal{O} = \frac{\langle h_1|h_2\rangle}{\sqrt{\langle h_1|h_1\rangle\,\langle h_2|h_2\rangle}}$$

$$\mathcal{M} = 1 - \mathcal{O}$$

- Biased characterization at SNR 10: $\mathcal{M} > 0.5\%$

Quality of Hybrid Waveforms

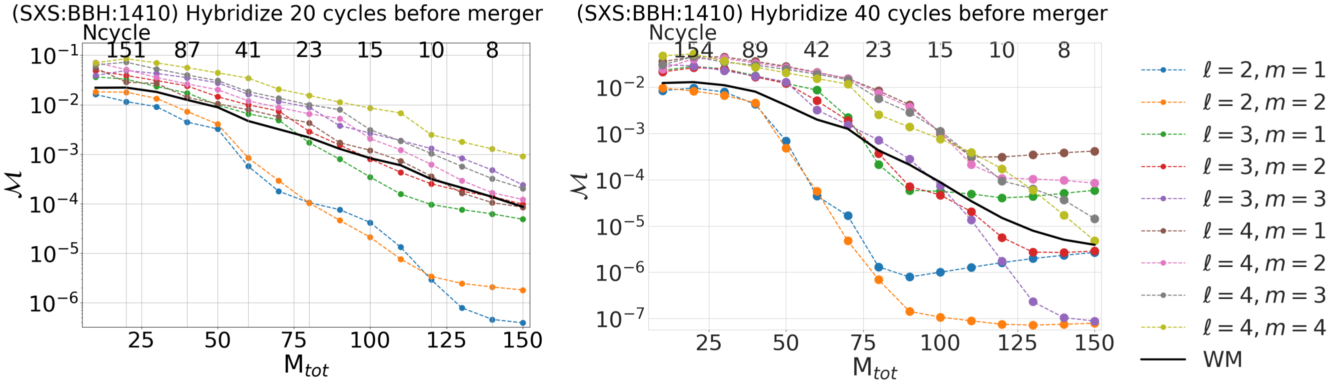
- Mode weighted Mismatch

$$W[\mathcal{M}] = \frac{\sum_{lm} \rho_{lm}^2 \mathcal{M}_{lm}}{\sum_{lm} \rho_{lm}^2}$$

where $\mathcal{M}_{lm}$ are the mode-by-mode, time-and-phase-maximized mismatches and $\rho_{lm}^2 = h_{lm} h_{lm}$.

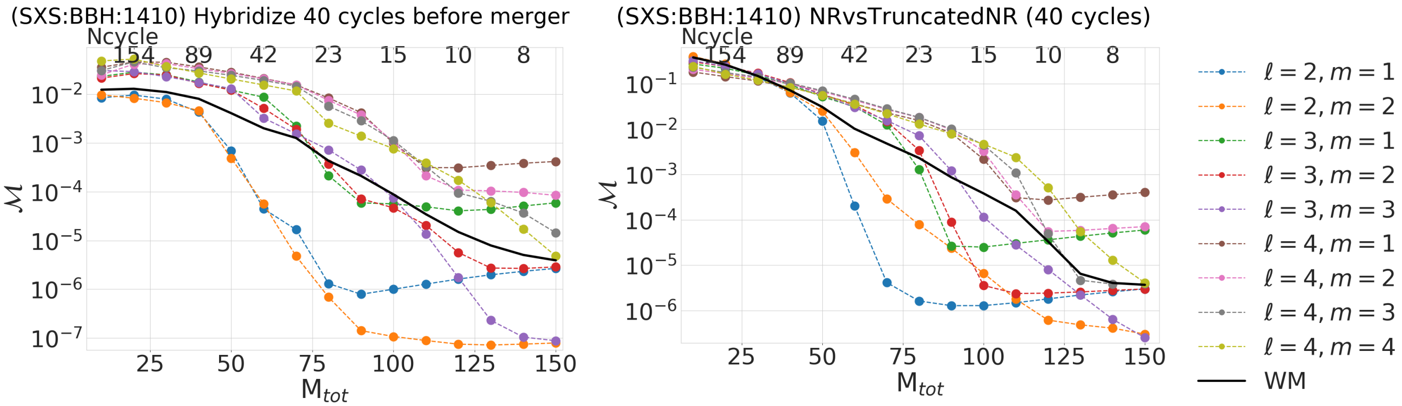
Hybrid Quality: Length of NR Waveform in Hybrid

- Precessing case **SXS:BBH:1410** [7, 8]
- PN-NR hybrid versus Full-NR
- Longer NR waveform in hybrid is better



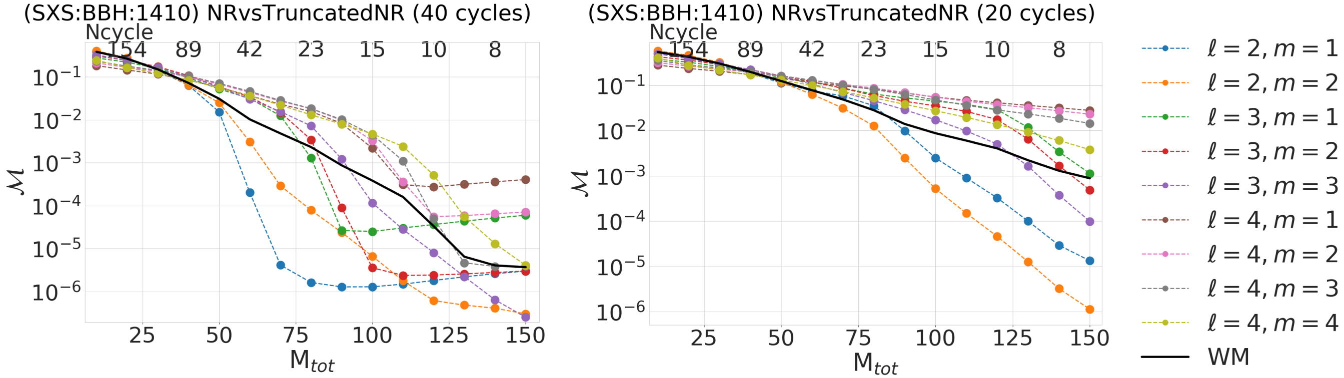
Hybrid Quality: Short-NR versus PN-NR hybrid

- Precessing case **SXS:BBH:1410**
- PN-NR hybrid versus Full-NR and short-NR versus Full-NR
- PN-NR hybrid are better than shorter NR waveforms



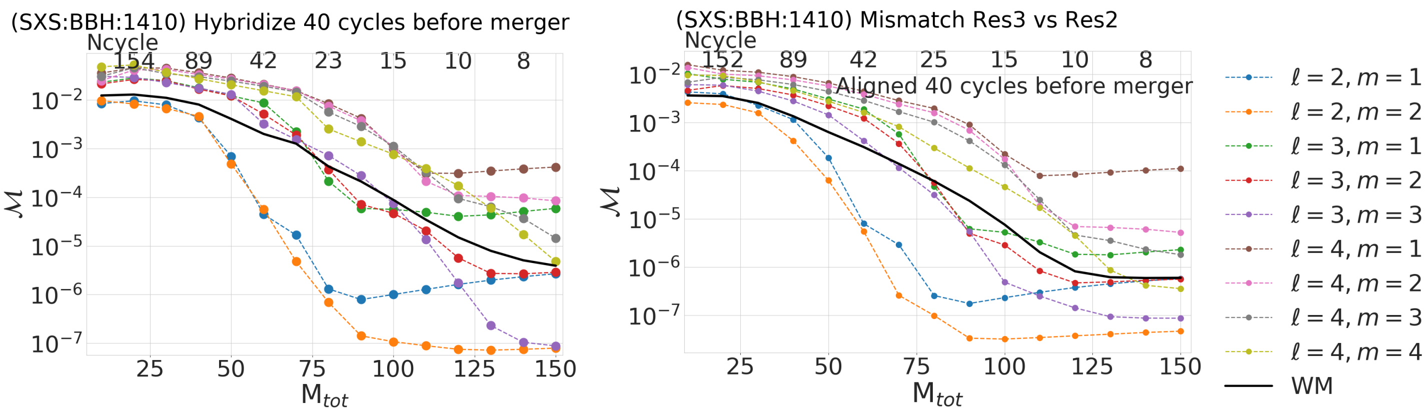
Hybrid Quality: The Length of NR Waveform

- Precessing case **SXS:BBH:1410**
- Longer NR waveforms are better than shorter ones



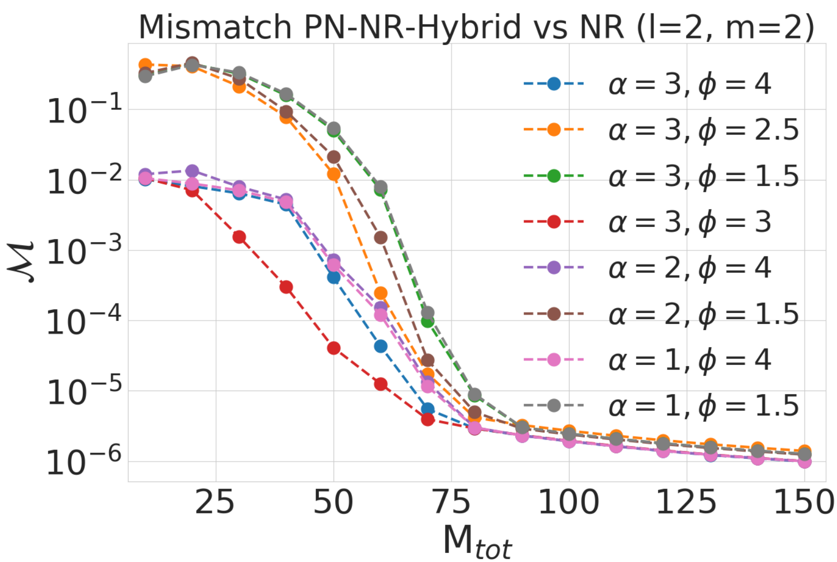
Hybrid Quality: Hybrid versus Low Resolution NR Waveform

- Precessing case **SXS:BBH:1410**
- PN-NR hybrid and Low-High resolution NR hybrid versus Full-NR
- Low resolution NR in hybridization perform better than PN



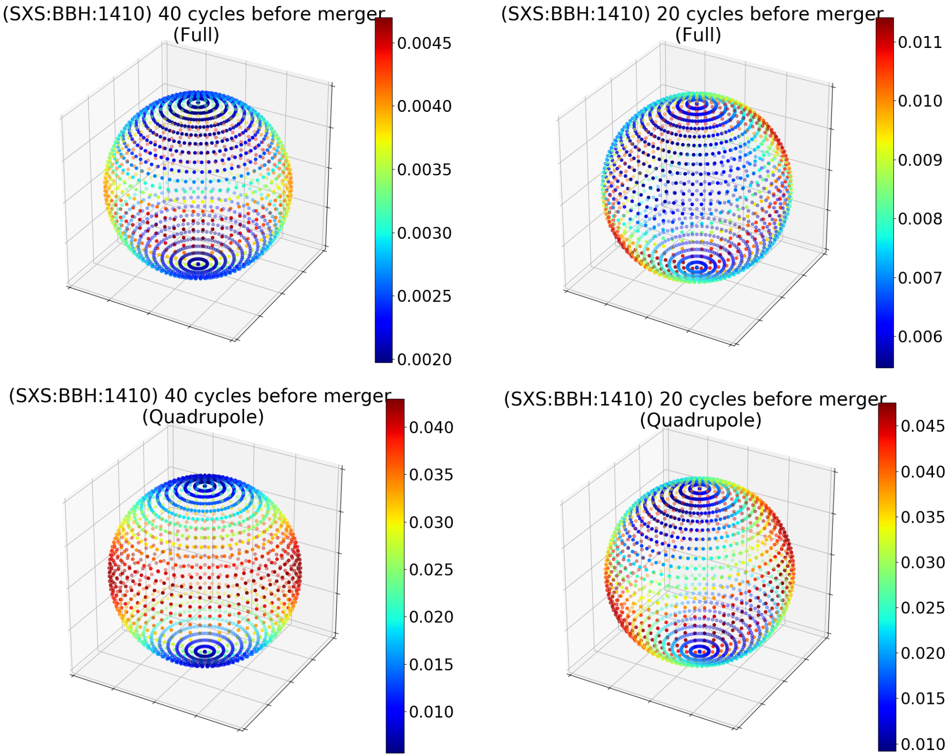
Hybrid Quality: PN Errors

- In general higher order PN approximant is better. We found phase ϕ order with 3 PN order terms works better than available higher orders in phase with the maximum available amplitude α order.



Mismatch as Function of Angles

- $h(t) = \sum_{lm} h_{lm}\,{}^{-2}Y_{lm}(\theta,\phi),\ M_{tot} = 40M_{\odot}$



- Higher order modes are important

Conclusion and Future

- We introduce an automated method to hybridize precessing binary waveforms
- We tested the accuracy of our hybridization procedure using mismatch
- We tested effects of the length of the numerical waveforms, hybrid versus different numerical resolutions as well as post-Newtonian errors on the hybrid.
- All presented work is published in [10]
- **Future:**
  - Testing length of hybrid interval and how it affects the mismatch.
  - Analyzing other hybridization errors (truncation and numerical) for many different waveform models

Thank You

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Jam Sadiq, Yosef Zlochower, Richard O'Shaughnessy, and Jacob Lange.
Hybrid waveforms for generic precessing binaries for gravitational-wave data analysis.
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Co-precessing Frame

$$
\begin{aligned}
I_2 &\equiv \frac{1}{2}(h|L_+L_+|h) \\
&= \frac{1}{2}\sum_{lm} c_{lm}c_{l,m+1}h^*_{l,m+2}h_{lm} \\
I_1 &\equiv (h|L_+(L_z+1/2)|h) \\
&= \sum_{lm} c_{lm}(m+1/2)h^*_{l,m+1}h_{lm} \\
I_0 &\equiv \frac{1}{2}\left(h|L^2-L_z^2|h\right) \\
I_0 &= \frac{1}{2}\sum_{lm}[l(l+1)-m^2]|h_{lm}|^2 \\
I_{zz} &\equiv (h|L_zL_z|h) = \sum_{lm} m^2|h_{lm}|^2
\end{aligned}
$$

where $c_{lm} = \sqrt{l(l+1)-m(m+1)}$.

Wigner Matrix

$d^{\ell}_{mm'}(\beta)$ given by

$$d^{l}_{m'm}(\beta) = \sqrt{(l+m)!\,(l-m)!\,(l+m')!\,(l-m')!}$$
$$\times \sum_{k} \frac{(-1)^{k+m'-m}}{k!\,(l+m-k)!\,(l-m'-k)!\,(m'-m+k)!}$$
$$\times \left(\sin\frac{\beta}{2}\right)^{2k+m'-m} \left(\cos\frac{\beta}{2}\right)^{2l-2k-m'+m} . \qquad (1)$$

Hybrid Quality: IMRPhenomXHM-NR hybrid versus PN-NR hybrid

- Precessing case **SXS:BBH:1410**
- PN-NR hybrid versus Full-NR and IMR-NR hybrid versus Full-NR
- PN-NR hybrid are better than IMR-NR hybrid