Anisotropic inflationary loop quantum cosmology: primordial gravitational waves and predictions for the CMB

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11 Iberian GW Meeting (9/06/21)

Overview

- Most of the models of the early universe are homogeneous and isotropic. Planck observations (2018) did not confirm with strong evidence any departure.

- However there is consensus that some anomalies at large scales (dipolar, quadrupolar, etc.) are present, indicating new (pre-)inflationary physics (L. Shamir (2020) reported that a distribution of galaxy spin directions show a quadrupolar-like alignment at more than 5σ).

- We will focus on the influence of anisotropies in the pre-inflationary universe (with special attention to tensor modes).

- Cosmological perturbation theory on inflationary Bianchi I spacetimes has been studied in great detail (Pereira, Pitrou, Uzan, 2007-2008).

- They discuss that anisotropies "break" scale invariance, isotropy (inducing high-order multipoles) and introduce scalar-tensor and tensor-tensor cross-correlations.

Overview

- But in classical GR, anisotropies can be large at the onset of inflation (and before). There is no well-posed initial value problem for perturbations.
- However, in bouncing inflationary cosmologies, this issue is alleviated (anisotropies are arbitrarily small in the far past).
- We complete a Fock quantization for perturbations (with anisotropies treated non perturbatively), and compute their power spectra at the end of inflation.
- We find upper bounds on the anisotropies (shear) via constraints on the quadrupolar anomaly reported by Planck Collaboration and discuss new observational effects (generation of *TB* and *EB* correlation functions).

Bianchi I spacetimes in LQC

- We consider LQC anisotropic bouncing models. Here, in the far past and future spacetime becomes isotropic.

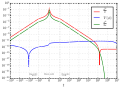
- The effective dynamics is determined by (Ashtekar, Wilson-Ewing, Mena-Marugán, Martín-Benito, ...)

$$G_{\mu\nu} = 8\pi G\,(T_{\mu\nu} + T_{\mu\nu}^{\mathrm{LQC}}), \tag{1}$$

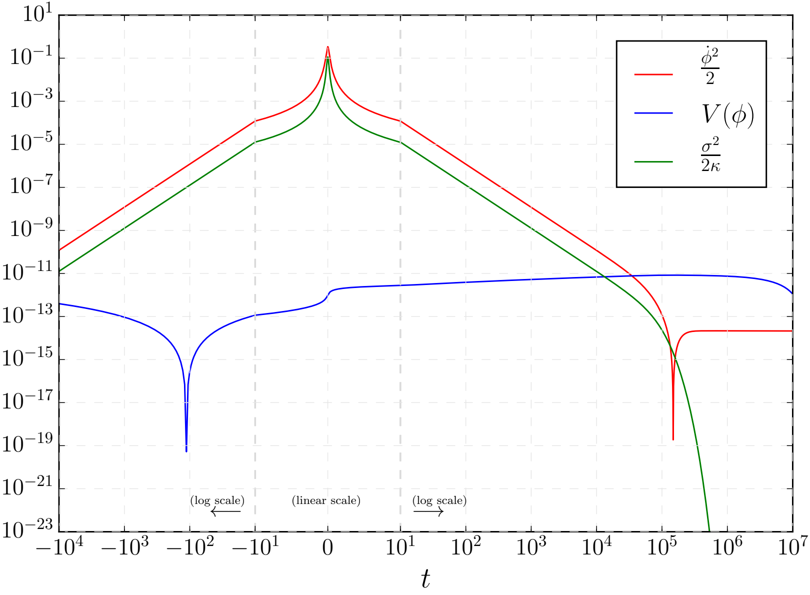
- The energy density, mean Hubble parameter and shear are bounded above (Gupt, Singh, 2012-2013)

$$\rho_{\max} = 0.41\rho_{\mathrm{Pl}}, \quad H_{\max} = \frac{8.34}{\ell_{\mathrm{Pl}}}, \quad \sigma^2_{\max} = \frac{11.57}{\ell^2_{\mathrm{Pl}}}.$$

- The background is determined by initial conditions when the mean scale factor $a(t)$ bounces at $t = t_B$ (i.e. $H(t_B) = 0$). There we fix $\sigma^2(t_B)$, $\Psi(t_B)$, $\phi(t_B)$ and the choice for the scale factors $a(t_B) = 1$ and $a_1(t_{\mathrm{end}}) = a_2(t_{\mathrm{end}}) = a_3(t_{\mathrm{end}})$.



Bianchi I spacetimes in LQC



Bianchi I: gauge invariant perturbations

- The EOMs of each mode is now given by

$$\ddot{\Gamma}_\mu + 3\,H\,\dot{\Gamma}_\mu + \frac{k^2}{a^2}\,\Gamma_\mu + \frac{1}{a^2}\sum_{\mu'=0}^{2}\mathcal{U}_{\mu\mu'}\,(\hat{k})\Gamma_{\mu'} = 0\,, \tag{2}$$

  with $k^2/a^2 = (k_1^2/a_1^2 + k_2^2/a_2^2 + k_3^2/a_3^2)$. Besides, Γ_0 refers to the scalar mode, Γ_1 and Γ_2 to the two tensor (transverse and traceless) polarizations ($+$ and $\times$).

- It is more convenient to express the Fourier modes of tensor pertubations in the helicity basis (circular polarization)

$$\Gamma_{\pm 2}(\vec{k}) = \frac{1}{\sqrt{2}}\left(\Gamma_1(\vec{k}) \mp i\Gamma_2(\vec{k})\right)\,. \tag{3}$$

- Then, we express $\Gamma_s(\vec{k})$ as a linear combination of the elements of the (orthonormal) basis of complex solutions normalized to

$$\sum_{s=0,\pm 2}\bar{v}_s^{(\lambda)}(\vec{k})\dot{v}_s^{(\lambda')}(\vec{k}) - \dot{\bar{v}}_s^{(\lambda)}(\vec{k})v_s^{(\lambda')}(\vec{k}) = -i\frac{4\kappa}{a^3\mathcal{V}_0}\delta^{\lambda\lambda'}\,. \tag{4}$$

Bianchi I: gauge invariant perturbations

- Quantum fields are given by $\hat{\Gamma}_s(\vec{k}) = \sum_{\mu=0}^{2} v_s^{(\mu)}(\vec{k})\hat{a}_\mu(\vec{k}) + \bar{v}_s^{(\mu)}(-\vec{k})\hat{a}_\mu^\dagger(-\vec{k})$,

$$[\hat{a}_\mu(\vec{k}), \hat{a}^\dagger_{\mu'}(\vec{k}')] = \delta_{\mu\mu'}\,\delta_{\vec{k},\vec{k}'}\,, \quad \hat{a}_\mu(\vec{k})|0\rangle = 0. \tag{5}$$

- For perturbations, we consider the 0th order adiabatic (also known as massless Minkowski) vacuum state for perturbations at 10^3 Planck secs. before the bounce

$$\begin{aligned}
v^{(1)}(\vec{k}) &= \sqrt{\frac{4\kappa}{a^2\mathcal{V}_0}}\frac{1}{\sqrt{2k}}(1,0,0), & \dot{v}^{(1)}(\vec{k}) &= \sqrt{\frac{4\kappa}{\mathcal{V}_0}}\frac{1}{a^2}\frac{-ik}{\sqrt{2k}}(1,0,0),\\
v^{(2)}(\vec{k}) &= \sqrt{\frac{4\kappa}{a^2\mathcal{V}_0}}\frac{1}{\sqrt{2k}}(0,1,0), & \dot{v}^{(2)}(\vec{k}) &= \sqrt{\frac{4\kappa}{\mathcal{V}_0}}\frac{1}{a^2}\frac{-ik}{\sqrt{2k}}(0,1,0),\\
v^{(3)}(\vec{k}) &= \sqrt{\frac{4\kappa}{a^2\mathcal{V}_0}}\frac{1}{\sqrt{2k}}(0,0,1), & \dot{v}^{(3)}(\vec{k}) &= \sqrt{\frac{4\kappa}{\mathcal{V}_0}}\frac{1}{a^2}\frac{-ik}{\sqrt{2k}}(0,0,1).
\end{aligned} \tag{6}$$

Bianchi I: Fock quantization of perturbations

- The relevant observables are the power spectra

$$\langle 0|\hat{\Gamma}_{(s}(\vec{k})\hat{\Gamma}_{s')}(\vec{k}')|0\rangle = \mathcal{V}_0^{-1}\,\frac{2\pi^2}{k^3}\,\mathcal{P}_{ss'}(\vec{k})\,\delta_{\vec{k},-\vec{k}}\,, \quad \mathcal{P}_{ss'}(\vec{k}) = \mathcal{V}_0\,\frac{k^3}{2\pi^2}\,\sum_{\mu}\left[v_s^{(\mu)}(\vec{k})\,\bar{v}_{s'}^{(\mu)}(\vec{k})\right]$$

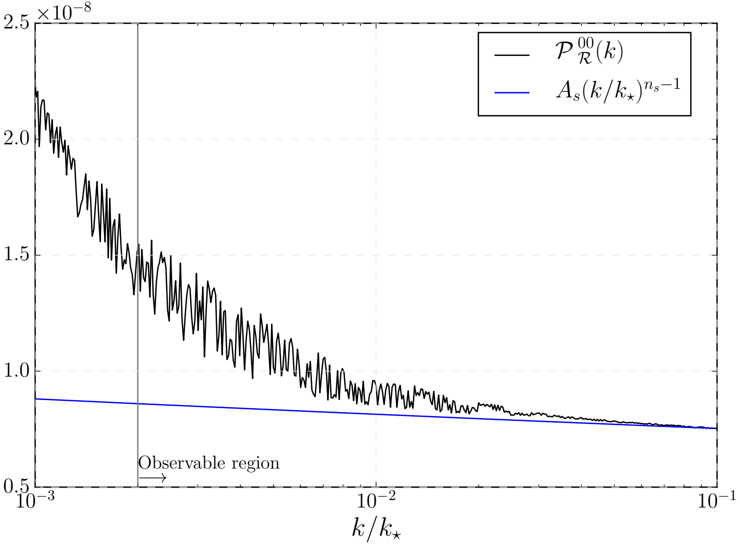
- Power spectra satisfy: $\mathcal{P}_{ss'}(\vec{k})$ are real and positive if $s = s'$, otherwise they are complex; $\bar{\mathcal{P}}_{ss'}(\vec{k}) = \mathcal{P}_{ss'}(-\vec{k})$ (reality conditions); $\mathcal{P}_{ss'}(\vec{k}) = \mathcal{P}_{s's}(-\vec{k})$ (commutation relations). A parity-invariant vacuum state implies $\mathcal{P}_{ss'}(\vec{k}) = \mathcal{P}_{-s-s'}(-\vec{k})$ ($\mathring{h}_{ij}$ is parity invariant)

- We compute the power spectra $\mathcal{P}_{ss'}(\vec{k})$ at the end of inflation. For convenience

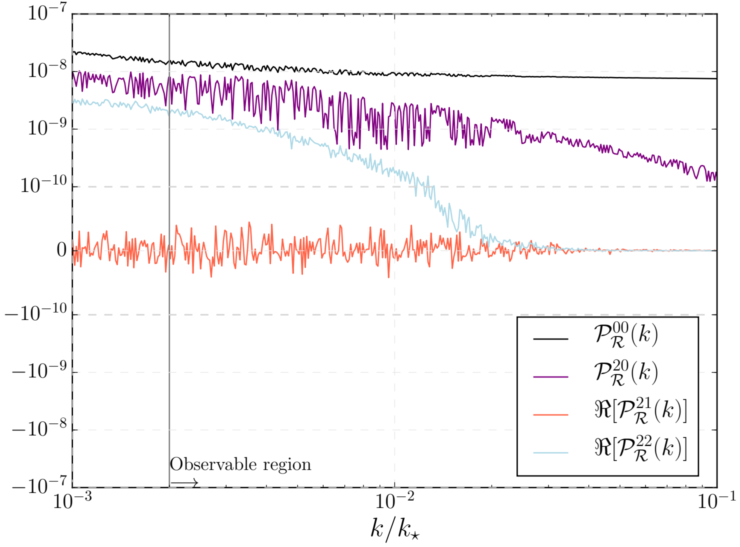
$$\mathcal{P}_{ss'}(\vec{k}) = \sum_{L=|s-s'|}^{\infty}\,\sum_{M=-L}^{L}\mathcal{P}_{ss'}^{LM}(k)\,{}_{s-s'}Y_{LM}(\hat{k}). \tag{7}$$

with ${}_sY_{LM}(\hat{k})$ the usual spin-weighted spherical harmonics. They are zero when $L < |s|$ (Therefore, $P_{ss'}^{LM}(k) = 0$ for $L < |s-s'|$, i.e. only $\mathcal{P}_{00}$ and $\mathcal{P}_{22} = \mathcal{P}_{-2-2}$ will contribute when $L = 0$).

Scalar power spectrum

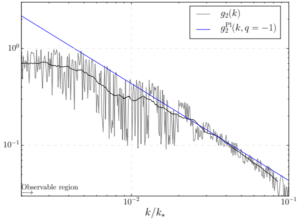


Scalar power spectrum



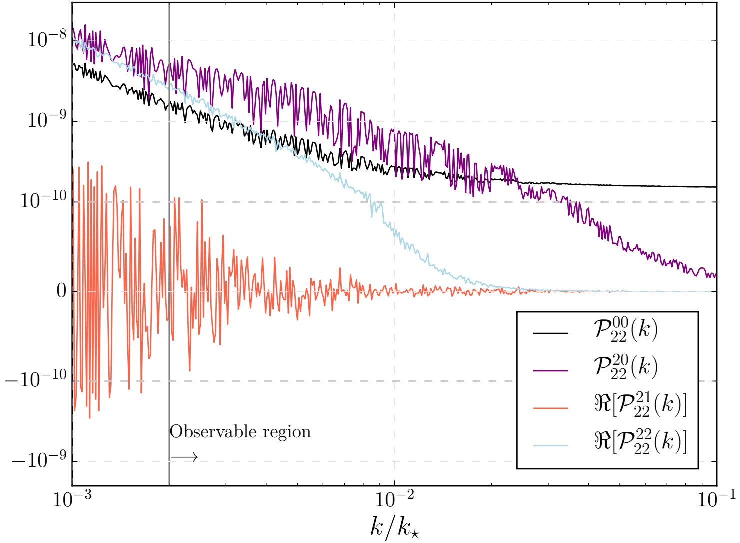
Quadrupole of $P_{00}(\vec{k})$: constraints on the shear

- Planck collaboration provides constraints on g_2 associated to the quadrupolar moments $\mathcal{P}_{\mathcal{R}}^{2M}(k)$, where $\mathcal{P}_{\mathcal{R}}(\vec{k}) \propto \mathcal{P}_{00}(\vec{k})$.
- We can constraint the background parameter space, namely $\sigma^2(t_B)$, $\Psi(t_B)$, and $\phi(t_B)$.
- We find that the minimum allowed value of $\phi(t_B)$ (number of e-folds) grows with $\sigma^2(t_B)$ (amount of anisotropies), but it does not strongly depends on $\Psi(t_B)$ (distribution of anisotropies).

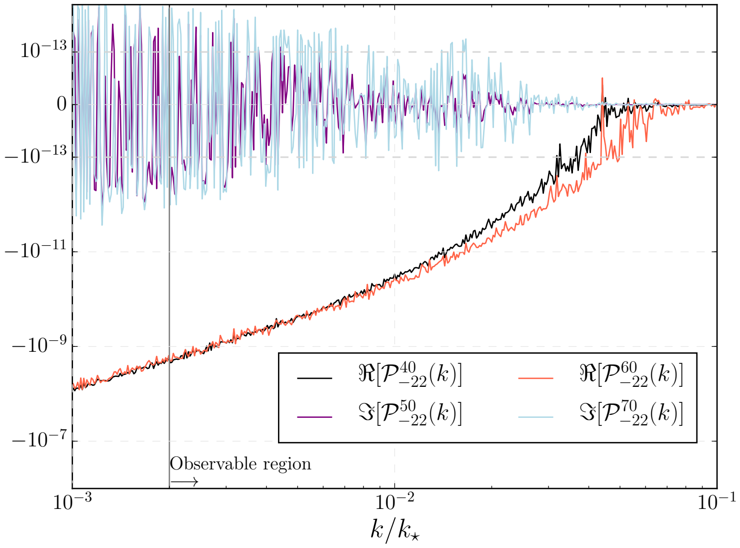


$$\sigma^2(t_B) = 5.45, \quad \Psi(t_B) = 0.0, \quad \phi(t_B) = 1.1.$$

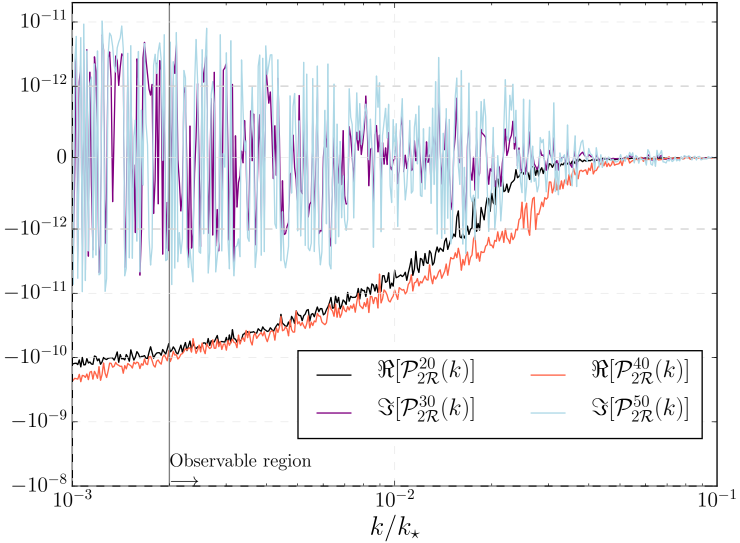
Tensor power spectrum



Tensor-tensor cross-correlations

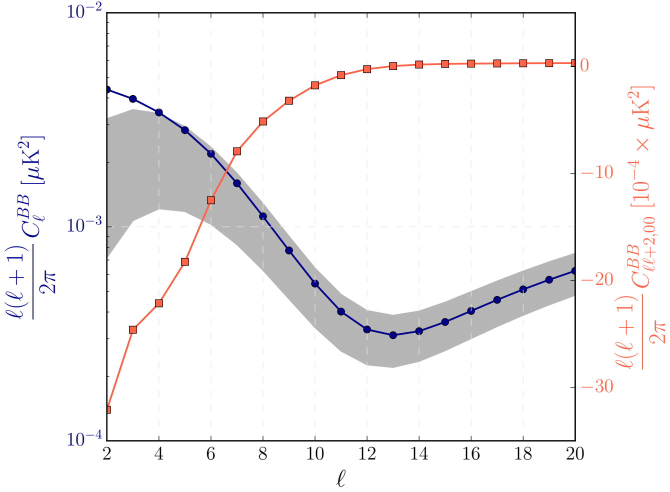


Scalar-tensor cross-correlations



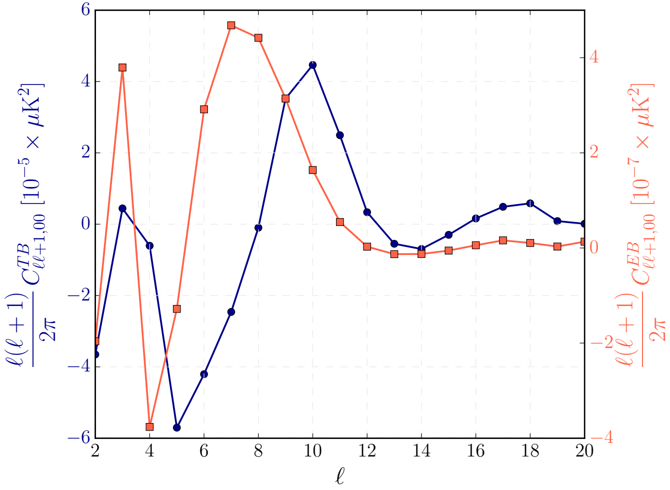
BB angular correlation functions

In the case of the correlation functions *TT*, *EE*, *BB* and *TE* one has $C^{XX'}_{\ell\ell',mm'} = 0$ if $\ell + \ell'$ is odd.



TB-*EB* angular correlation functions

In the case of the *TB* and *EB* correlation functions $C^{BY'}_{\ell\ell',mm'} = 0$ if $\ell + \ell'$ is even.



Summary

- We study quantum gauge-invariant cosmological perturbations for anisotropic inflationary spacetimes.
- We compute the power spectra within a concrete bouncing inflationary scenario. Here, tensor perturbations show a stronger coupling to anisotropies (enhanced particle production at large scales).
- We find upper bounds on anisotropies thanks to the constraints on the quadrupolar anomaly given by Planck Collaboration.
- Given the constraints above, we see that BB correlation function shows higher power at low multipoles than the isotropic standard scenario (as a consequence of the enhancement of power of tensor modes at large scales).
- Moreover, anisotropies generate angular (TB and EB) correlation functions, which would identically vanish in the isotropic limit.