Gravitational wave signature of proto-neutron star convection

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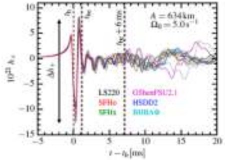
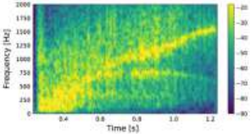
GW signal in CCSNe

Richers et al 2017



Bounce signal:
- only fast rotating models
- Δt ~ 5 ms
- f ~ 600-900 Hz
- h ~10^{-21} @ 10 kpc

Torres-Forné et al 2019



Post-bounce "SN" signal:
- g-modes, SASI, convection
- Δt ~ 0.1-1- s
- f ~ 50-2000 Hz
- h ~10^{-23}-10^{-22}@ 10 kpc

PNS convection signal:
- Computationally expensive
- Δt ~ 10-50- s
- f ~ ?
- h ~ ?

-100 ms — Onset of collapse

0 — Bounce and shock formation

0.1 – 1 s — Onset of explosion

10-50 s — End of the convective phase

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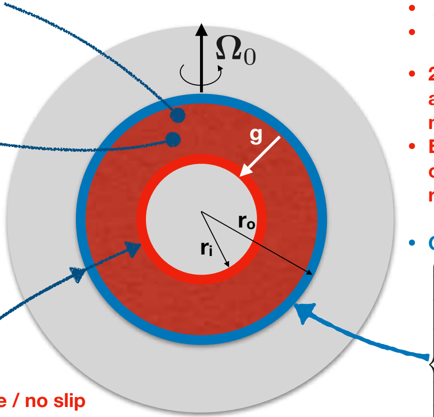
Modelling the PNS convective zone

Input:
- Temperature profile
- Density profile

Transport coefficients:
- Kinematic viscosity ν
- Thermal diffusivity κ
- Magnetic diffusivity η



Hypothesis:
- Spherical geometry
- Adiabatic stratification
- Low Mach convection

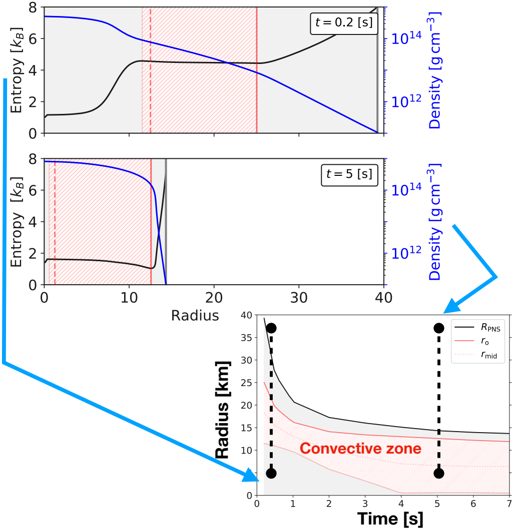
- 2nd order diffusion approximation for the neutrino transport
- Electrical conductivity of degenerate, relativistic electrons

- Orders of magnitude

$$\begin{cases} \Phi_o \sim 10^{52}\,\mathrm{erg/s} \\ r_o \sim 25\,\mathrm{km} \\ T_o \sim 10^{11}\,\mathrm{K} \\ \varrho_o \sim 10^{13}\,\mathrm{g/cm^3} \\ \nu_o \sim 10^{10}\,\mathrm{cm^2/s} \\ \kappa_o \sim 10^{12}\,\mathrm{cm^2/s} \\ \eta_o \sim 10^{-3}\,\mathrm{cm^2/s} \end{cases}$$

Boundary conditions:
- **Mechanical:** stress-free / no slip
- **Thermal:** fixed entropy flux
- **Magnetic:** perfect conductor ($B_{//}$) / pseudo-vacuum ($B_{\perp}$)

Early and late time background models



Source

Lorenz Hüdepohl's PhD thesis

Prometheus-Vertex code

1D model + MLT

LS220 EoS

27 $M_\odot$ progenitor

PNS baryonic mass 1.78 $M_\odot$

Method

1. stability determined according to the Schwarzschild criterion
2. deduce the shell geometry
3. fit the background profile $(\tilde{\varrho}, \tilde{T})$

The MHD anelastic equations

Braginsky+95, Lantz+99, Jones+[11,14]

$$[d] = r_o - r_i, \quad [t] = d^2/\nu_o, \quad [S] = d\ \partial S/\partial r|_{r_o}, \quad [p] = \Omega \varrho_o \nu_o, \quad [B] = \sqrt{\Omega \varrho_o \mu_0 \eta_o}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{u} \times \mathbf{B})}_{\text{Induction}} - \underbrace{\frac{1}{Pm} \nabla \times (\eta \nabla \times \mathbf{B})}_{\text{Dissipation}}$$

$$0 = \nabla \cdot (\tilde{\varrho} \mathbf{u})$$

$$\frac{D\mathbf{u}}{Dt} = -\overbrace{\boldsymbol{\nabla}\left(\frac{p}{E\tilde{\varrho}}\right)}^{\text{Pressure}} - \overbrace{\frac{2}{E}\mathbf{e}_z \times \mathbf{u}}^{\text{Coriolis}} - \overbrace{\frac{Ra}{Pr}\frac{d\tilde{T}}{dr} S \mathbf{e}_r}^{\text{Buoyancy}} + \overbrace{\mathbf{F}_\nu}^{\text{Viscosity}} + \overbrace{\frac{1}{EPm}\frac{1}{\tilde{\varrho}}(\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B}}^{\text{Lorentz}}$$

$$\frac{DS}{Dt} = \frac{1}{Pr\tilde{\varrho}\tilde{T}} \underbrace{\nabla \cdot \left(\kappa \tilde{\varrho} \tilde{T} \boldsymbol{\nabla} S\right)}_{\text{Heat flux}} + \frac{Pr}{Ra\tilde{\varrho}\tilde{T}}\left(\underbrace{\frac{\eta}{Pm^2 E}(\nabla \times \mathbf{B})^2}_{\text{Ohmic heating}} + \underbrace{Q_\nu}_{\text{Viscous heating}}\right)$$

3D MHD direct numerical simulations

Control parameters

Prandtl number	$Pr = \nu_o / \kappa_o$
magnetic Prandtl number	$Pm = \nu_o / \eta_o$
Ekman number	$E = \dfrac{\nu_o}{\Omega d^2}$
Rayleigh number	$Ra = \dfrac{\tilde{T}_o d^3 \left. \frac{\partial S}{\partial r} \right\|_{r_o}}{\nu_o \kappa_o}$

Input

$Ra/Ra_c \sim 10$

$\mathrm{Pr} = 0.1$

$Pm \sim 5 \quad (\ll 10^{14})$

$E \equiv P_{\mathrm{rot}} \in [1\,\mathrm{ms}, 10^2\,\mathrm{ms}]$

magic-sph.github.io

Output

Gravitationnal signal computed with the quadrupole approximation

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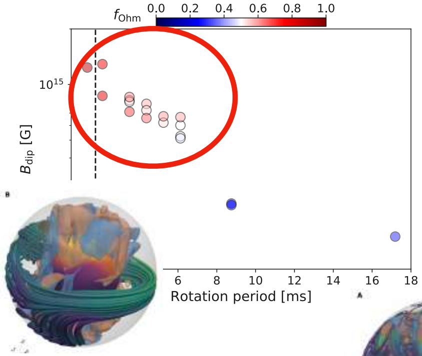
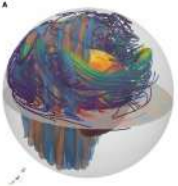
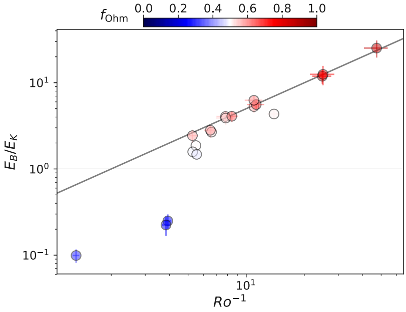
PNS convective dynamos and magnetar formation

Dipole field strength





Strong field scaling



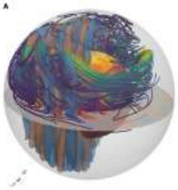
Magnetostrophic balance : Lorentz-Coriolis

$$\frac{E_B}{E_k} \propto Ro^{-1} \qquad Ro = \frac{u}{d\Omega}$$

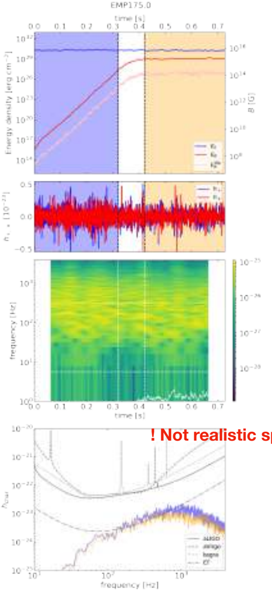
Raynaud et al. 2020

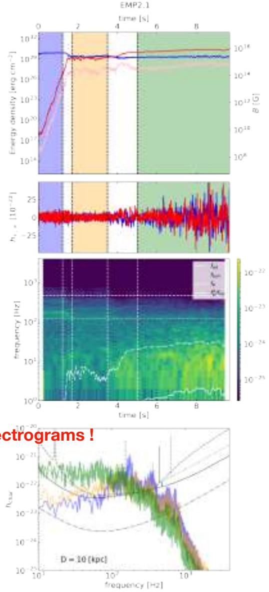
Typical cases: slow versus fast rotation

P=175 ms

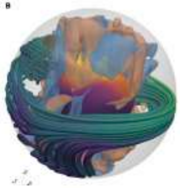


Alpha-omega dynamo





P=2.1 ms



Strong field dynamo

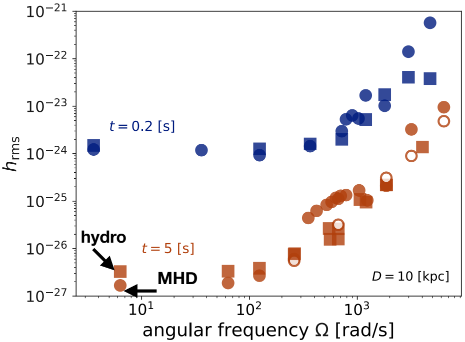
Magnetar formation

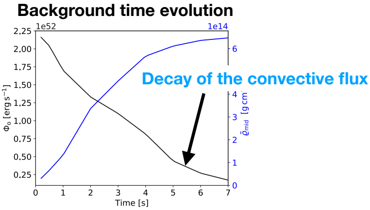
(Raynaud et al . 2020)

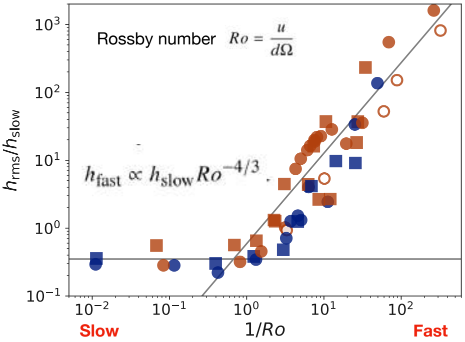
NB: fixed background !

! Not realistic spectrograms !

Amplitude scaling





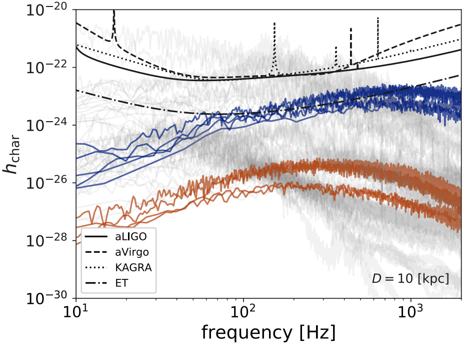


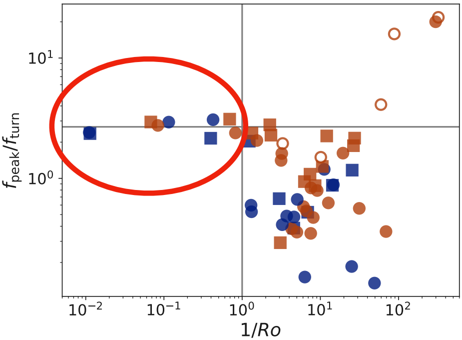
Scaling relation for slow/fast regimes

$$h_{\rm slow} \propto \frac{2G}{Dc^4} r_{\rm mid}^2 M_{\rm conv} \left(\frac{\gamma}{2\alpha_{\rm mlt}} + 1\right) \frac{d^{-2/3}}{c_s^2} \left(|\partial_r \ln \tilde{T}| \frac{\Phi_{\rm o}}{4\pi r_{\rm mid}^2 \tilde{\varrho}}\right)^{4/3}, \quad (33)$$

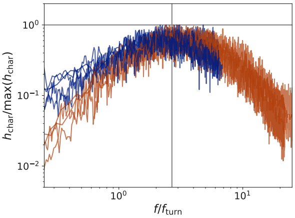
$$h_{\rm fast} \propto \frac{2G}{Dc^4} r_{\rm mid}^2 M_{\rm conv} \left(\frac{\gamma}{2\alpha_{\rm mlt}} + 1\right) \frac{d^{2/5}}{c_s^2} \left(|\partial_r \ln \tilde{T}| \frac{\Phi_{\rm o}}{4\pi r_{\rm mid}^2 \tilde{\varrho}}\right)^{4/5} \Omega^{8/5}$$

Frequency scaling: slow rotation





Peak frequency scales with the turnover frequency U/d

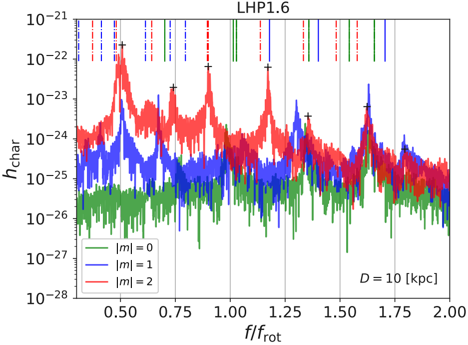


Broad spectrum due to convection

Fast rotation: spectra

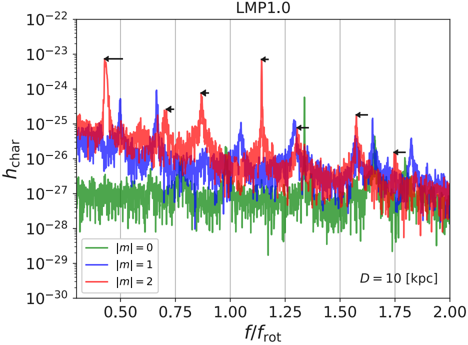
Hydro

P = 1.6 [ms]



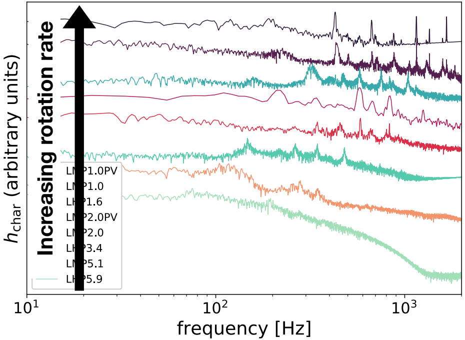
MHD

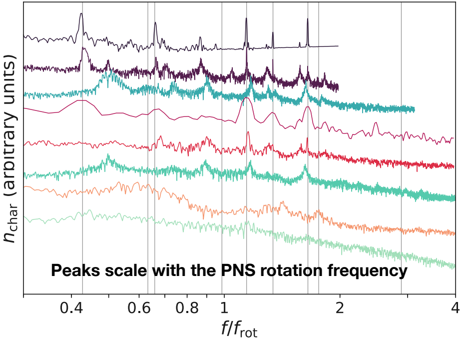
P = 1.0 [ms]



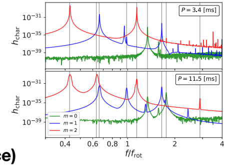
Fast rotation: frequency scaling

t = 5 s post bounce **1 ms < Period < 6 ms**



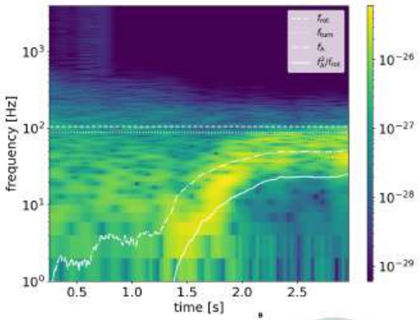


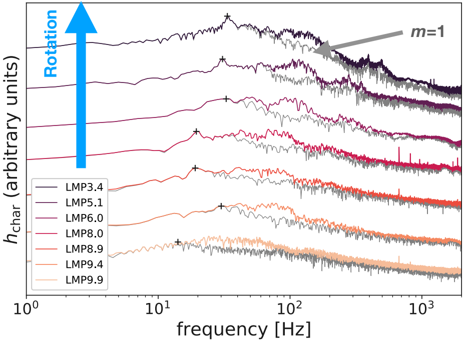
Signature of inertial modes

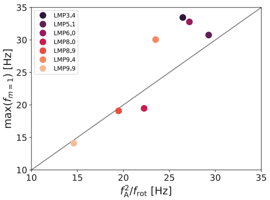


Ra=0 models
(Decaying turbulence)

Strong field dynamo signature ?







Hypothesis: *m*=1 Rossby mode modified by toroidal magnetic field ?

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Conclusions

Raynaud et al. (arXiv:2103.12445)

Slow rotation ($Ro \gg 1$)

- broad spectrum
- peak scales with f_{turn}
- weak impact of B field

Fast rotation ($Ro \ll 1$)

- h_{rms} strongly increases
- complex spectra
- peaks scale with f_{rot}
- inertial modes
- low frequency signature of strong field dynamo

Limitations

- consider only one background model
- no continuous evolution of the PNS cooling (no realistic GW template)
- convective zone only (no g-modes)

Perspectives: detectability

- use amplitude/frequency scalings to rescale the signal as a function of the background evolution