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Fractional p-Laplacian evolution equations

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ABSTRACT

In this paper we study the fractional p-Laplacian evolution equation given by

$$u_t(t,x) = \int_A \frac{1}{|x-y|^{N+sp}} |u(t,y)-u(t,x)|^{p-2}(u(t,y)-u(t,x))dy \quad \text{for } x \in \Omega,\ t>0,$$

$0 < s < 1$, $p \geq 1$. In a bounded domain Ω we deal with the Dirichlet problem by taking $A = \mathbb{R}^N$ and $u = 0$ in $\mathbb{R}^N \setminus \Omega$, and the Neumann problem by taking $A = \Omega$. We include here the limit case $p = 1$ that has the extra difficulty of giving a meaning to $\frac{u(y)-u(x)}{|u(y)-u(x)|}$ when $u(y) = u(x)$. We also consider the Cauchy problem in the whole $\mathbb{R}^N$ by taking $A = \Omega = \mathbb{R}^N$. We find existence and uniqueness of strong solutions for each of the above mentioned problems. We also study the asymptotic behaviour of these solutions as $t \to \infty$. Finally, we recover the local p-Laplacian evolution equation with Dirichlet or Neumann boundary conditions, and for the Cauchy problem, by taking the limit as $s \to 1$ in the nonlocal problems multiplied by a suitable scaling constant.

RÉSUMÉ

Dans cet article, on étudie l'équation d'évolution suivante :

$$u_t(t,x) = \int_A \frac{1}{|x-y|^{N+sp}} |u(t,y)-u(t,x)|^{p-2}(u(t,y)-u(t,x))dy \quad \text{for } x \in \Omega,\ t>0,$$

pour $x \in \Omega$, $t > 0$, $0 < s < 1$ et $p \geq 1$. On considére trois situations, correspondant au cas d'une condition de bord du type Dirichlet, $A = \mathbb{R}^N$ et $u = 0$ sur $\mathbb{R}^N \setminus \Omega$, le cas d'une condition de Neumann, $A = \Omega$, et le problème de Cauchy, $A = \Omega = \mathbb{R}^N$. Le cas limite $p = 1$ est aussi étudié. Dans cette situation on donne sens au quotient $\frac{u(y)-u(x)}{|u(y)-u(x)|}$ sur l'ensemble $\{u(y) = u(x)\}$. On démontre l'existence et l'unicité de la solution dans tous les cas évoqués. On établit aussi leur comportement asymptotique

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