

**ADVANCED MICROECONOMICS II**  
**June, 12, 2006**

Surname:.....First name:.....

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1. State the First Welfare Theorem. Give an example arguing why it may not hold. (1.25 points).

2. Consider a two-agent exchange economy with utility functions  $U_i(G, x_i) = G^\alpha x_i^{1-\alpha}$ , where  $x_i$ ,  $i = 1, 2$  is the amount of money for the private good. Each agent has an initial endowment of the private good (money) equal to  $w_i$ , and the amount of public good is just the sum of the two agents' contributions. Find the efficient allocations of the public good  $G$ . (1.25 points).

3. As the number of agents increases, show what happens with the core allocations of an economy which are not the Walrasian equilibria of such economy. (2.5 points).

4. Consider a two-agent exchange economy with utility functions  $U_1 = x_{11}^{1/2} x_{12}^{1/2}$ ,  $U_2 = \min\{x_{21}, x_{22}\}$ , and initial endowments  $w_1 = (1, 1)$ ,  $w_2 = (1, 1)$ . Compute the Walrasian equilibrium and the core. Verify that the Walrasian equilibrium utility levels belong to the utility frontier of the economy. Exhibit graphically all the results. (2.5 points).

5. Define both the utility frontier and the social optima of an economy. Consider a social optimum allocation, is it efficient? is every Pareto efficient allocation a social optimum? Argue and exhibit graphically your answer. (1.25 points).

6. Consider a two-agent exchange economy with utility functions  $U_1 = x_{11} + x_{12}$ ,  $U_2 = \min\{x_{21}, x_{22}\}$  and initial endowments,  $w_1 = (1,0)$ ,  $w_2 = (0,1)$ . Suppose the following allocations,  $a_1 = [(1/3, 2/3), (2/3, 2/3)]$ ,  $a_2 = [(1/4, 3/4), (3/4, 1/4)]$ ,  $a_3 = [(1/3, 1/3), (2/3, 2/3)]$ ,  $a_4 = [(1,1), (0,0)]$ . Argue whether they belong to the core of the economy. (1.25 points).