Topological Strings and Donaldson-Thomas invariants

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work in progress with A. Sinkovics and R.J. Szabo

Topological Strings on Calabi–Yau Manifolds

- Topological Strings play a very important role in modern mathematical physics
- They compute F–terms in supersymmetric theories and capture microscopic properties of Black Holes.
- Mathematically they count enumerative invariants of Calabi-Yau geometries such as the Gromov–Witten, the Gopakumar–Vafa and the recently introduced Donaldson–Thomas.
- Roughly, the physical interpretation of the Donaldson–Thomas invariant $D_{n,\beta}$ is that it gives the number of bound states of n D0 branes and D2 branes wrapping the cycle β with a *single* D6 brane wrapping the full Calabi–Yau
- They are the underlying objects in the "melting crystal" formulation of topological strings

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Topological Field Theories and Equivariant Localization

- We will focus on local toric Calabi–Yaus
- In this framework the DT problem can be rephrased in terms of an auxiliary topological Yang–Mills theory that lives on the D6 branes worldvolume
- DT invariants are "generalized instantons" of this theory an can be computed by the techniques of equivariant cohomology and localization used by Nekrasov in the context of Seiberg–Witten theory
- They are classified by 3d Young tableaux: the generalized instantons reproduce the states of the melting crystal

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Non Abelian Invariants

- This is a generalization of the DT invariants with an arbitrary number of D6 branes. We are using two independent techniques to compute the non abelian invariants on any toric geometry.
- A noncommutative deformation of the theory resolves the singularities of the moduli space. The invariants can be computed by direct localization of the path integral.
- We can introduce an auxiliary topological matrix quantum mechanics directly on the moduli space. This reformulates the problem in an "ADHM–like" fashion.
- But many physical open problems and potential developments further: OSV conjecture, S–duality in topological strings, mirror symmetry, wallcrossings...

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Orbifold Invariants

- In the recent years it has been proven possible to introduce Gromov–Witten invariants for orbifolds
- What about DT invariants? Open problem: we choose a pragmatic approach and try to compute them directly as an instanton problem.
- We define the invariants in terms of "colored" partition: each box of the 3D Young tableau has a different color according to the transformation properties of the instanton under the orbifold action
- Comparison with orbifold Gromov–Witten invariants is technically challenging: not yet understood!

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