

# Utrecht University Network Node

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- Quantum Gravity
- Particle Physics/QCD
- Cosmology
- Super-symmetric black holes
- Physics of instantons
- Integrability on  $AdS/CFT$  and related issues
  - Integrability of physical strings on  $AdS_5 \times S^5$
  - $S$  matrix of strings on  $AdS_5 \times S^5$ , finite size corrections, ect
  - Operators with large spin, BES equation
  - Scattering amplitudes

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- Integrability on  $AdS/CFT$  and related issues
  - Integrability of strings on  $AdS_5 \times S^5$
  - $AdS/CFT$   $S$  matrix, finite size corrections.
  - Operators with large spin, BES equation ( today )
  - Scattering amplitudes ( Friday )

Consider a massless gauge theory

Twist two operators

$$\mathcal{O}_S = \text{Tr}(\phi D^S \phi), \quad S \gg 1 \quad \Rightarrow \quad \Delta_{\mathcal{O}_S} = f(g) \log S + \mathcal{O}(1)$$

$f(g)$ : Cusp anomalous dimension, scaling function, etc.

- Anomalous dimension of large Lorentz spin operators.
- Expectation value of Wilson loops with a cusp.
- Scattering amplitudes.

It is hard to compute  $f(g)$  for QCD, we can try computing it for a simpler theory, as for instance, planar  $\mathcal{N} = 4$  SYM. Many simplifications, in particular

- Planar  $\mathcal{N} = 4$  SYM is supposed to be integrable, so we can hope to apply the tools of integrable systems to compute the cusp anomalous dimension. Besides, integrability serves as a strong moral support.
- By using the *AdS/CFT* duality, we can get results from the strong coupling side.

Integrability: The anomalous dimension of certain operators are computed by using some Bethe ansatz.



An integral equation for  $f(g)$  has been proposed

BES equation ( Beisert, Eden, Staudacher )

$$f(g) = 8g^2 - \frac{8}{3}\pi^2 g^4 + \dots$$

- Explicit computations confirm the BES equation up to four loops!! Bern, Czakon, Dixon, Kosover, Smirnov, Moch, Vermaseren, Vogt,...

## *AdS/CFT* duality

Four dimensional  
maximally SUSY Yang-Mills  $\Leftrightarrow$  Type IIB string theory  
on  $AdS_5 \times S^5$ .

- Compute  $f(g)$  at strong coupling by considering strings spinning on  $AdS_5$

## $\sigma$ -model computations Gubser, Klebanov, Polyakov, Frolov, Tseytlin, Roiban, Tirziu

$$f(g) = 4g - \frac{3 \log 2}{\pi} + \frac{K}{4\pi^2} \frac{1}{g} + \dots$$

Very recently people learnt how to deal with the BES equation at strong coupling, and found perfect agreement with the strong coupling result!! (Basso, Korchemsky, Kotanski)

- $f(g)$  is a non BPS quantity, of definite physical relevance, that can be extrapolated from weak to strong coupling, providing a highly non trivial test of many ideas: *AdS/CFT*, integrability, etc.

Future directions?

Operators with large spin  $S$  and large  $SO(6)_R$  charge  $J$

$$\mathcal{O} = \text{Tr}(\phi^k D^S \phi^{J-k}), \quad S, J \gg 1, \quad J/\log S = j$$

$$\Downarrow$$

$$\Delta_{\mathcal{O}} = (f(g) + \epsilon(g, j)) \log S$$

- Most of  $\epsilon(g, j)$  is given by an  $O(6)$  bosonic  $\sigma$ -model! , in particular, there is a prediction for string theory at all loops ( L.F.A., J. Maldacena ).