Unemployment, Taxation and Public Expenditure in OECD Economies
Technical Appendix

Rafael Doménech and José Ramón García
University of Valencia
January, 2007.

1. Decentralized economy with distortionary taxation.

The representative household chooses an optimal consumption path that maximizes the utility function

$$U(0) = \int_0^{\infty} e^{-\rho t} \frac{(c_t)^{1-\theta}}{1-\theta} dt \tag{1}$$

subject to the budget constraint (2) given by

$$\begin{aligned} & l_t(w_t(1-\tau^l) + tr_t) + r_t(1-\tau^k)k_t + (1-l_t)s_t + a_t \\ = \; & (1+\tau^c)c_t + \dot{k}_t + \delta k_t \end{aligned} \tag{2}$$

Therefore, the Hamiltonian of the optimization problem in a decentralized economy corresponds to

$$\begin{aligned} \mathcal{H}_t \;=\; & e^{-\rho t} \frac{(c_t)^{1-\theta}}{1-\theta} + \lambda_t [r_t(1-\tau_t^k)\, k_{pt} + l_t(w_t(1-\tau_t^l) + tr_t) \\ & +(1-l_t)s_t + a_t - (1+\tau_t^c)c_t - \delta k_t] \end{aligned} \tag{3}$$

where λ_t is the shadow price of physical capital.

The first-order conditions for the representative consumer taking $\tau_t^k, \tau_t^l, tr_t, \tau_t^c, a_t$, and l_t as given are determined by the following expressions

$$e^{-\rho t}(c_t)^{-\theta} = \lambda_t(1+\tau^c), \tag{4}$$

$$-\frac{\dot{\lambda}_t}{\lambda_t} = r_t(1-\tau^k) - \delta, \tag{5}$$

which are interpreted in the usual way.

1.1 Imperfect competition in product markets

We consider and economy characterized by monopolistic competition in the good markets, where firm's production function is given by

$$y_{it} = A_t^{1-\alpha-\varepsilon} k_{pt}^{\alpha} k_{gt}^{\varepsilon} (l_t^d)^{1-\alpha} \tag{6}$$

whereas their demand functions are given by[1]

$$y_{it} = \left(\frac{p_{it}}{p_t} \right)^{-\xi} y_t \tag{7}$$

where p_{it} is the price set by firm i, p_t is the aggregate price level, y_t aggregate demand (taken as given by the firm) and $1/\xi$ represents the market power of the firm. When $\xi = \infty$ the firm takes the price as given.

In the general case, labour demand is determined by.

$$l_{it}^d = \left[\frac{(1-\frac{1}{\xi})(1-\alpha)\left[A_t^{1-\alpha-\varepsilon} k_{pt}^{\alpha} k_{gt}^{\varepsilon}\right]^{(1-\frac{1}{\xi})} y_t^{\frac{1}{\xi}}}{w_{it}} \right]^{\frac{1}{\frac{1-\alpha}{\xi}+\alpha}} \tag{8}$$

which is a decreasing function of $1/\xi$ and where w_{it} is the real wage. Notice that when goods markets are competitive ($\xi = \infty$) then

$$l_{it}^d = \left[\frac{(1-\alpha) A_t^{1-\alpha-\varepsilon} k_{pt}^{\alpha} k_{gt}^{\varepsilon}}{w_{it}} \right]^{1/\alpha} \tag{9}$$

1.2 Wage bargaining under right to manage.

In the case in which the bargaining power of the union is $0 < \varrho < 1$, the wage bargained is given by:

$$w_{it} = \frac{s_t}{\left(1 - \frac{1}{(\frac{1-\varrho}{\varrho})(1-\alpha)(\xi-1)+\frac{1}{\frac{1-\alpha}{\xi}+\alpha}}\right)(1-\tau^l+\beta_t\sigma_{tr})} \tag{10}$$

whereas in the case of a monopolistic union ($\varrho = 1$)

$$w_{it} = \frac{s_t}{(1-\frac{1-\alpha}{\xi}-\alpha)(1-\tau^l+\beta_t\sigma_{tr})} \tag{11}$$

[1] For more details about this type of models or example, Layard, Nickell and Jackman (1991).

As we can see, the wage is a increasing function of the bargaining power of the union (ϱ) and the firm market power (ξ). The higher wage level implies a lower employment given τ^l and $\beta\sigma_{tr}$. Notice that when goods markets are competitive ($\xi = \infty$) the wage is given by

$$w_{it} = \frac{s_t}{(1-\alpha)(1-\tau^l+\beta_t\sigma_{tr})} \tag{12}$$

1.3 Employment and wage bargaining

As in Cahuc and Zylberberg (2004) or Ooghe, Schokkaert and J. Flecher (2003), in the general case in which the firm and the trade union bargain over employment and wages, they maximize the following surplus

$$\max_{\{w,\,l\}} \; l_{it}^{\varrho} \left[w_{it}(1-\tau^l+\beta_t\sigma_{tr}) - s_t \right]^{\varrho} \left[\Pi(w_{it}, l_{it}) \right]^{1-\varrho} \tag{13}$$

where $\Pi(w_{it}, l_{it})$ is the profit function and the parameter ϱ indicates the relative bargaining strength of the trade union.

Differentiating the preceding expression with respecto to w_{it} and l_{it} we obtain the following first-orden conditions:

$$\varrho \frac{(1-\tau^l+\beta_t\sigma_{tr})}{w_{it}(1-\tau^l+\beta_t\sigma_{tr}) - s_t} - (1-\varrho)\frac{l_{it}}{\Pi} = 0 \tag{14}$$

$$\varrho \frac{1}{l_{it}} + (1-\varrho)\frac{\left[(1-\frac{1}{\eta}) y_{it}^{-\frac{1}{\eta}} y_t^{\frac{1}{\xi}} \frac{\partial y_{it}}{\partial l_{it}} - w_{it} \right]}{\Pi} = 0 \tag{15}$$

where $(1-\frac{1}{\eta}) y_{it}^{-\frac{1}{\eta}} y_t^{\frac{1}{\xi}} \frac{\partial y_{it}}{\partial l_{it}}$ is the marginal revenue function

Combining (14) and (15) we obtaing the contract curve

$$(1-\frac{1}{\eta}) y_{it}^{-\frac{1}{\eta}} y_t^{\frac{1}{\xi}} \frac{\partial y_{it}}{\partial l_{it}} = \frac{s_t}{(1-\tau^l+\beta_t\sigma_{tr})} \tag{16}$$