Chapter 4

Strategic Pigouvian Taxation and Optimal Pricing of Polluting Non-Renewable Resources

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4.1 Introduction

Recently, several papers have been published on the intertemporal properties of a carbon tax. Among them we can quote Hoel (1992, 1993), Sinclair (1992, 1994), Ulph and Ulph (1994), Wirl (1994), Wirl and Dockner (1995), Tahvonen (1995, 1996), Farzin (1996), Farzin and Tahvonen (1996), and Hoel and Kverndokk (1996). These papers can be classified in two groups depending on the approach followed by the authors. A first group formed by Hoel, Sinclair, Ulph and Ulph, Farzin, Farzin and Tahvonen, and Hoel and Kverndokk have focused on the optimal pricing of a non-renewable resource with environmental stock externalities[1]. If the different results obtained in

[1] Within this group we could differentiate Hoel's approach from the one followed by the rest of the authors. Hoel uses a dynamic pollution game with N countries and defines the

these papers are analysed, it appears that the optimal time path of the carbon tax depends critically on the specification of the carbon accumulation process, and in particular on the irreversibility of CO_2 emissions. Thus, if the emissions are partially irreversible, as in Farzin and Tahvonen's (1996) paper, or if reversibility is costly, as in Farzin's (1996) paper, the optimal carbon tax may *increase* monotonically or have a U-shaped form. However, if reversibility is costless, i.e. if a constant rate of decay of the cumulative emissions is assumed, as happens in the Ulph and Ulph (1994) and Hoel and Kverndokk (1996) papers, the tax should initially increase when the initial stock of cumulative emissions is small, but fall later on when the stock of oil nears exhaustion. This is quite evident when the Farzin and Tahvonen and Hoel and Kverndokk papers are compared, since these two papers only differ essentially in the specification of the cumulative emission dynamics and give different temporal paths for the carbon tax.

The second group of authors follows a somewhat different approach. They have tried to capture the strategic features of the global warming problem, developing a model of long-term bilateral strategic interaction between a resource-exporting cartel and a coalition of resource-importing governments[2]. In this framework, they have studied the strategic taxation of CO_2 emissions by the governments of the importing countries. Their model is a global warming differential game with irreversible emissions where the coalition of governments chooses the carbon tax and the cartel the price[3]. Wirl (1994) and Wirl and Dockner (1995) have shown, for the case of zero extraction cost, that the tax *increases* monotonically up to the choke price, whereas the price declines monotonically to zero when a Markov-perfect Nash equilibrium in linear strategies is computed. In Tahvonen (1995, 1996) the monopolistic extraction is computed as a feedback Stackelberg equilibrium assuming that extraction costs are independent of the resource level. When his results are compared with those of Wirl (1994) and Wirl and Dockner (1995) it turns out that the intertemporal properties of the carbon tax and price are the

optimal carbon tax as the pigouvian tax that reproduces the social optimum. We can also include Forster's (1980) paper in this group, although he does not draw out consequences of his model for the temporal path of a pollution tax.

[2] In fact, their model considers a simple stock externality, of which carbon dioxide is just the most discussed example.

[3] In section 4 of Tahvonen's (1996) paper, the case of reversible pollution with depletion effects is studied. But the difficulty of deriving an analytical solution leads the author to compute numerical examples.

same irrespective of whether we have a Nash or Stackelberg equilibrium.

In this chapter we propose a revision of these two approaches that consists in the introduction of *depletion effects* into the analysis. We assume that the extraction costs depend positively on the extraction rate and cumulative extractions. In this way, we extend and complete the analysis of the strategic taxation of CO_2 emissions and present new results on the optimal pricing of a polluting non-renewable resource[4].

Our results show how the depletion effects affect the temporal path of the carbon tax and what the distributive effects of strategic taxation are, making more precise the results obtained by the previous authors. We find that the tax can be *decreasing* and the price increasing if the environmental damage is not very high, or that the tax and producer price can both be increasing. With depletion effects the dynamics of the tax depends critically on the effect a variation in cumulative extractions has on marginal environmental damage. Nevertheless, if the marginal damage is high enough, the producer price should be decreasing, whereas the tax should be increasing. Furthermore, we find that the tax defined by the Nash equilibrium is a *neutral pigouvian tax,* in the sense that the tax only corrects the market inefficiency caused by the stock externality, and not the inefficiency associated with the market power of the resource cartel. When the efficient solution is computed, we find the same kind of results for the user cost or shadow price of the resource. The dynamics of the shadow price also depends on the environmental damage parameter value so that an increasing user cost must only appear when the environmental damages are high enough. For this solution we get a simple expression for the critical value of this parameter that leads us to conclude that the shadow price would have to be increasing only when the pollution damage is high with respect to extraction costs. Moreover, we find that the two equilibria converge asymptotically to the same values as in Wirl (1994). However, we clarify this result showing that the aggregate welfare of the market equilibrium is lower than the aggregate welfare for the efficient solution. This means that the strategic taxation of CO_2 emissions does not allow us to re-establish the efficiency of the market since the tax only corrects the inefficiency caused by the stock externality as we have just mentioned above, but does not eliminate the market power of the producers. For this reason, we find that the market equilibrium is more conservationist than

[4] See Rubio and Escriche (1998) for another extension that consists in the computation of the feedback Stackelberg equilibria.

the efficient equilibrium because the producers use their monopoly power for reducing, for their own profit, the rate of extraction.

Our chapter is organised as follows: we present the global warming differential game with depletion effects in section 4.2; in section 4.3 we compute the Markov-perfect Nash equilibrium, and in section 4.4 the optimal pricing of polluting non-renewable resources and compare the two equilibria. Section 4.5 summarises the conclusions and suggests directions for additional research.

4.2 The Model

In this chapter we extend Wirl and Dockner's (1995) model[5]. We begin by describing the demand side of the market, assuming that the consumers of the importing countries act as price-taker agents. Under this assumption, we can write the consumers' net welfare as: $\int_0^\infty e^{-\delta t}\{aq(t)-(1/2)q(t)^2-[p(t)+\psi(t)]q(t)+R(t)-dz(t)^2\}dt$, where $aq(t)-(1/2)q(t)^2$ is the consumers' gross surplus, $q(t)$ is the amount of the resource bought by the importing countries, $p(t)$ is the producer price, $\psi(t)$ is the tax fixed by the importing country governments, $R(t)$ is an income transfer that the consumers receive from the government, and $dz(t)^2$ is the environmental or pollution damage function, where $z(t)$ is the cumulative emissions and d is a positive parameter. If we consider that global warming is a clear example of a *stock externality* we have to establish that consumers take as a given not only the price of the resource but also the evolution of the accumulated emissions and, moreover, the income transfer, since this is controlled by the governments, so that, finally, the resource demand only depends on consumer price: $q(t)=a-p(t)-\psi(t)$. On the other hand, as $\psi(t)$ represents the tax fixed by the importing country's government, we are implicitly assuming that there exists a coalition or some kind of *cooperation* among the importing countries' governments which allows us to represent the resource market as a model of long-term bilateral strategic interaction between a resource exporting cartel (OPEC) and a coalition of resource importing countries' governments (the West).

The governments are supposed to tax emissions in order to maximise the

[5]See that paper for more details. Our version of the game is also closely related to the one developed in section 3 of Tahvonen's (1996) paper. The novelty of our approach in the specification of the model, with respect to these two papers, is that we suppose that average extraction costs depend on cumulative extractions.

discounted present value of the net consumers' surplus. We also assume that tax receipts, $\psi(t)q(t)$, are completely refunded to the consumers through the transfer $R(t)$. As a result the optimal time path for the tax is given by the solution of the following problem[6]:

$$\max_{\{\psi(t)\}}\int_0^\infty e^{-\delta t}\Big\{a(a-p(t)-\psi(t))-\frac{1}{2}(a-p(t)-\psi(t))^2 \\ -p(t)(a-p(t)-\psi(t))-dz(t)^2\Big\}\,dt, \tag{4.1}$$

where δ is the discount factor.

The dynamics of cumulative resource consumption determines simultaneously the dynamics of the CO_2 concentration in the atmosphere:

$$\dot{z}(t)=a-p(t)-\psi(t),\quad z(0)=z_0\geq 0. \tag{4.2}$$

Following Wirl and Dockner's and Tahvonen's approach, we suppose that the identity between resource consumption and CO_2 emissions is not crucial as long as we can measure oil in terms of that unit that releases one ton of carbon into the atmosphere. This simplified version of the cumulative emission dynamics has also been used by Hoel (1992, 1993)[7].

Let us turn to the other side of the market. We assume that extraction costs depend linearly on the rate of extraction and on the cumulative extractions, $C(q(t),z(t))=[cz(t)]q(t)$, and that the objective of cartelised producers is to define a price strategy that maximises the discounted present value of profits[8]:

$$\max_{\{p(t)\}}\int_0^\infty e^{-\delta t}\{(p(t)-cz(t))(a-p(t)-\psi(t))\}\,dt. \tag{4.3}$$

Although we incorporate depletion effects into the analysis, we consider that the stock externality is largely irrelevant to the welfare of exporting countries,

[6]In Wirl and Dockner (1995) a study is made of how the Leviathan motive of the governments modifies the temporal path of the tax in a global warming differential game without extraction costs.

[7]Given this linear relationship between resource consumption and emissions, ψ could be interpreted as well as a resource import tariff, and the chapter as a study on import tariffs and non-renewable resources with stock externalities.

[8]Because in our model there is no uncertainty, we can establish that in the equilibrium market resource consumption is equal to extraction rate and, consequently, cumulative emissions are equal to cumulative extractions.

and that the cumulative extractions are not constrained by the resource in the ground but by its negative impact on extraction costs and climate. Moreover, following Karp (1984), we assume that the producers get no utility from consuming the resource. This assumption is not too great a departure from reality since most major resource exporters consume a negligible portion of their production. Thus, we represent the strategic interactions in the resource market as a *differential game* between a coalition of importing countries' governments and cartelised exporters of oil, where the coalition of governments chooses the tax and the cartel the price.

4.3 A Neutral Pigouvian Tax

In this section we obtain the solution to the game through the computation of a Markov-perfect Nash equilibrium. We use Markov strategies because these kinds of strategies capture essential strategic interactions, provide a dynamically consistent, subgame perfect equilibrium and are analytically tractable.

Markov strategies have to satisfy the following system of Bellman equations[9]:

$$\begin{aligned}\delta W_1 &= \max_{\{\psi\}} \{a(a-p-\psi) - \frac{1}{2}(a-p-\psi)^2 \\ &\quad - p(a-p-\psi) - dz^2 + W_1'(a-p-\psi)\}, \qquad (4.4)\\ \delta W_2 &= \max_{\{p\}} \{(p-cz)(a-p-\psi) + W_2'(a-p-\psi)\}. \qquad (4.5)\end{aligned}$$

From the first order conditions for the maximisation of the r.h.s. of the Bellman equations we get the reaction functions of the governments and producers:

$$\begin{aligned}\psi^N &= -W_1', \qquad (4.6)\\ p^N &= \frac{1}{2}(a + cz - W_2' - \psi^N), \qquad (4.7)\end{aligned}$$

where superscript N stands for the Markov-perfect Nash equilibrium. These results establish that the optimal tax is independent of the price fixed by the producers, and that the price and tax are strategic substitutes for the producers. Thus, for the governments of the importing countries the optimal policy consists, as we show below, of defining a *neutral pigouvian tax*

[9] Time arguments will be eliminated when no confusion arises.

equal to the user cost or shadow price of cumulative emissions. This means that when there is no strategic advantage, i.e. when the two players move simultaneously, the importing countries cannot use the tax for reducing the market power of the cartel, since the optimal tax *only* corrects the market inefficiency caused by the stock externality. For this reason we *define* this tax as a *neutral pigouvian tax* that does not correct the inefficiency associated with the market structure. Notice also that the tax is positive. This differs from the well known proposition, see Buchanan (1969), that establishes that the pigouvian instrument under a monopoly should be a subsidy. The explanation of this divergence is, as Wirl and Dockner (1995) have already pointed out, that the resource market is divided into exporting and importing countries, and the latter do not take into account the producers' surplus in their objective function.

Applying standard techniques of optimal control we get (see Kamien and Schwartz (1991, section 23)):

$$\left|W_1'\right| = \int_t^{\infty} e^{-\delta(\tau-t)} 2dz \, d\tau - \int_t^{\infty} e^{-\delta(\tau-t)} p \frac{\partial p}{\partial z} \, d\tau.$$

This expression allows us to present an economic interpretation of the user cost or shadow price for the importing countries of an increment in one unit of the cumulative emissions at any time t. The first component appears in different papers on the optimal pricing of a non-renewable resource with environmental stock externalities; see, for instance, Farzin (1996) and Farzin and Tahvonen (1996); and it is equal to the discounted present value of the increment in future and present environmental damage caused by an increment at time t of the cumulative emissions. However, the second component only appears when the interdependence between the exporting and importing countries is taken into account; see in this case Tahvonen (1996); and it is equal to the discounted present value of the effect on future and present consumers' welfare caused by the reaction of the exporting countries to a variation of cumulative emissions at time t. Notice that the sign of this effect can be positive or negative, depending on the optimal policy or strategy adopted by the cartel.

By substitution of (4.6) in (4.7) we get the solution of the price as a function of the first derivatives of the value functions: $p^N = \frac{1}{2}(a + cz + W_1' - W_2')$. Next, incorporating the optimal strategies into the Bellman equations (4.4) and (4.5) we eliminate the maximisation and obtain, after some calculations, a pair of non-linear differential equations:

$$\delta W_1 = \frac{1}{8}(a - cz + W_1' + W_2')^2 - dz^2, \tag{4.8}$$

$$\delta W_2 = \frac{1}{4}(a - cz + W_1' + W_2')^2. \tag{4.9}$$

Notice that both value functions depend on $W_1' + W_2'$, and so does the consumer price, $\pi = p + \psi$, and the rate of extraction:

$$\pi^N = \frac{1}{2}[a + cz - (W_1' + W_2')], \tag{4.10}$$

$$q^N = \frac{1}{2}(a - cz + W_1' + W_2'). \tag{4.11}$$

This regular occurrence of the term $W_1' + W_2'$ simplifies the solution of the differential equation system (4.8) and (4.9) and allows, as happens in Wirl and Dockner's model, a complete analysis of the asymmetric game defined in section 4.2.

Before presenting the Markov-perfect Nash equilibrium we want to establish and demonstrate the result we have mentioned above.

Proposition 4.1 *The Markov-perfect Nash equilibrium of the game defines a neutral pigouvian tax.*

Proof. This proof is quite obvious if one realises, firstly, that in the market there exist two kinds of inefficiencies, one caused by the *stock externality* and the other by the *market power* of the producers, and, secondly, that the optimal tax is equal to the user cost of the cumulative emissions. The strategy of the proof is simple: we compute the monopolistic equilibrium without intervention of importing countries' governments, assuming that consumers take into account the damage caused by the cumulative CO_2 emissions, and then we check that this equilibrium is identical to the Nash equilibrium. The monopolistic equilibrium is calculated in two stages. In the first stage, price-taker consumers determine the demand function, and, in the second, the cartel decides the price. Then the extraction rate is determined by the demand function.

The Bellman equation for the consumers, if they *internalise* external costs, is:

$$\delta W_1 = \max_{\{q\}} \{aq - \frac{1}{2}q^2 - pq - dz^2 + W_1' q\}. \tag{4.12}$$

The maximisation of the right-hand side gives us the resource demand function, $q = a - p - W_1'$, and substitution in the producers' profit function yields:

$$\delta W_2' = \max_{\{p\}} \{(p - cz)(a - p - W_1') + W_2'(a - p - W_1')\}. \tag{4.13}$$

From the maximisation of the right-hand side we obtain the same optimal strategy as in the Nash equilibrium, $p^M = p^N = \frac{1}{2}(a + cz + W_1' - W_2')$, where the superscript M stands for the monopolistic equilibrium without the inefficiency caused by the stock externality. Then by substitution of the control variables into the Bellman equations, we get the same system of differential equations (4.8) and (4.9) and, consequently, the same solution. Therefore, the monopolistic equilibrium, without stock externality, is *identical* to the Nash equilibrium and we can conclude that the optimal tax defined by the Nash equilibrium is a *neutral pigouvian tax* in the sense that it only corrects the inefficiency caused by the stock externality and leaves the cartel with its monopolistic profits.

The solution to the differential equation system (4.8) and (4.9) allows us to calculate the linear Markov-perfect Nash equilibrium strategies.

Proposition 4.2 *Let*

$$q^N(z) = \begin{cases} 0, & z_\infty^N \le z \\ \frac{1}{2}[a + y^N - (c - x^N)z], & z \le z_\infty^N, \end{cases} \tag{4.14}$$

where

$$x^N = \frac{1}{2}\left\{2c + \frac{4}{3}\delta - \left[\frac{16}{3}\left(c\delta + \frac{\delta^2}{3} + 2d\right)\right]^{1/2}\right\}, \tag{4.15}$$

$$y^N = -\frac{3a(c - x^N)}{4\delta + 3(c - x^N)} < 0, \ a + y^N > 0 \ \textit{and} \ c - x^N > 0 \tag{4.16}$$

and

$$z^N = (z_0 - z_\infty^N)\exp\{-1/2(c - x^N)t\} + z_\infty^N, \tag{4.17}$$

where

$$z_\infty^N = \frac{a\delta}{c\delta + 2d}. \tag{4.18}$$

Then $q^N(z)$ constitutes a global asymptotically stable Markov-perfect Nash equilibrium (MPNE) for the infinite horizon differential game under consideration, where z^N_∞ is the cumulative emission long-run equilibrium.

Proof. See Appendix A (section 4.7).

As we have just seen in Appendix A, Proposition 4.2 permits us to calculate the optimal dynamics of the rate of extraction, the producer price, the tax and the consumer price and the discounted present value for the two players, providing a complete analytical characterisation of the solution of the game. The long-run equilibrium value for cumulative emissions has been computed as a particular solution of the differential equation that defines the dynamics restriction of the problem. However, this value can be derived directly using more straightforward economic arguments. The producers exploit the resource until the value function is zero. This implies from (4.5) that $p - cz = -W_2'$. On the other hand, the first order condition which gives the reaction function of the producers can also be written as $p-cz-(a-p-\psi) = -W_2'$, so it follows immediately that $a-p-\psi = q_\infty = 0$ and $a - p = -W_1'$, using the reaction function of the governments. With $q_\infty = 0$ the consumers' value function is $\delta W_1 = -dz_\infty^2$ and $-W_1' = 2dz_\infty/\delta$ and by equalisation we get $p = a - (2dz_\infty/\delta)$. Finally, if we assume that extraction of the resource continues until the marginal profit is zero we get $p = cz$, and then we obtain $z_\infty = a\delta/(c\delta+2d)$. This means that the exploitation of the resource must end for a zero marginal profit and value function.

The solution includes the pay-offs of the players, which are given by the value function for the initial value of the state variable, z_0,

$$W_1^N(z_0) = \frac{1}{2}\alpha_1^N z_0^2 + \beta_1^N z_0 + \mu_1^N, \tag{4.19}$$

where

$$\alpha_1^N = \frac{1}{\delta}\left[\frac{1}{4}(c - x^N)^2 - 2d\right], \tag{4.20}$$

$$\beta_1^N = -\frac{a(c - x^N)}{4\delta + 3(c - x^N)}, \tag{4.21}$$

$$\mu_1^N = \frac{1}{8\delta}(a + y^N)^2. \tag{4.22}$$

Thus, for the consumers and for the producers,

$$W_2^N(z_0) = \frac{1}{2}\alpha_2^N z_0^2 + \beta_2^N z_0 + \mu_2^N, \tag{4.23}$$

where

$$\alpha_2^N = \frac{1}{2\delta}(c - x^N)^2, \tag{4.24}$$

$$\beta_2^N = -\frac{2a(c - x^N)}{4\delta + 3(c - x^N)}, \tag{4.25}$$

$$\mu_2^N = \frac{1}{4\delta}(a + y^N)^2. \tag{4.26}$$

From these expressions, the following corollary holds.

Corollary 4.3 *If the initial cumulative emissions are zero, the discounted present values of the net consumers' welfare and profits are positive and equal to (4.22) and (4.26).*

Proof. Straight from (4.19) and (4.23). If we make $z_0 = 0$ in the value functions we have $W_1(0) = \frac{1}{8\delta}(a + y^N)^2$ and $W_2 = \frac{1}{4\delta}(a + y^N)^2$.

However, we cannot extrapolate this result for z_0 in the interval $(0, z_\infty)$ because, as we have shown above, the value function for the consumers is strictly negative for z_∞. This means that the extraction of the resource will be profitable only if the initial value for the cumulative extractions is not very high. In particular, the exploitation of the resource will take place if the initial value is in the interval $[0, \bar{z}]$, where $\bar{z}$ is defined by the positive root of the equation $W_1(z) = 0$. For the producers, the extraction gives a positive pay-off provided that the initial value of the state variable is in the interval $[0, z_\infty)$. However, as long as the consumers only demand a positive quantity of the resource when the cumulative emissions are below the upper bound, $\bar{z}$, the exploitation of the resource will occur only when the initial value of cumulative emissions is below this critical value. From now on, we will assume that $z_0 = 0$. This simplifies the analysis enormously and helps us to reduce the length of the chapter. Nevertheless, we want to point out that the results obtained in the rest of the chapter can be generalised for z_0 in the interval $(0, \bar{z})$.[10]

[10] The generalisation of the results for $z_0 \in (0, \bar{z})$ is available from the authors.

Finally, we compute the dynamics of the rate of extraction, the producer price, the emission tax and the consumer price. To get the temporal paths of these variables we substitute W_1' and W_2' in the linear strategies for q, p, ψ and π by the coefficients of the value functions we have calculated in Appendix A, and then we rearrange the terms and eliminate z, using (4.58):

$$q^N = 1/2(a+y^N)\exp\{-1/2(c-x^N)t\}, \tag{4.27}$$

$$p^N = \frac{ac\delta}{c\delta+2d} - \frac{1}{2}(c+\alpha_1^N-\alpha_2^N)z_\infty^N \exp\{-1/2(c-x^N)t\}, \tag{4.28}$$

$$\psi^N = \frac{2ad}{c\delta+2d} + \alpha_1^N z_\infty^N \exp\{-1/2(c-x^N)t\}, \tag{4.29}$$

$$\pi^N = a - 1/2(a+y^N)\exp\{-1/2(c-x^N)t\}. \tag{4.30}$$

We can now summarise the dynamics of the variables and the long-run equilibrium of the game as follows.

Proposition 4.4 *Along the equilibrium path the rate of extraction decreases while the consumer price increases. The producer price is increasing (decreasing) if $c+\alpha_1^N-\alpha_2^N$ is positive (negative) and the emission tax is increasing (decreasing) if α_1^N is negative (positive). Moreover, the market equilibrium approaches a long-run equilibrium characterised by: $q_\infty = 0$, $\pi_\infty = a$, $p_\infty = \frac{ac\delta}{c\delta+2d}$ and $\psi_\infty = \frac{2ad}{c\delta+2d}$.*

If we focus on the tax dynamics, we have just seen that this depends on the sign of coefficient α_1^N, which is given by (4.20). This expression allows us to study the relationship between this coefficient and the damage parameter, d, and hence the relationship between the pollution damage and the optimal temporal path of the tax. We know that α_1^N is positive when $d=0$; see (4.20). Now, using (4.15) it is easy to establish that α_1^N is decreasing with respect to d and that there exists a positive value, that we name as $\underline{d}_\psi^N$, for which the coefficient of the value function W_1 is zero. Thus, we get that, when d is lower than $\underline{d}_\psi^N$, the emission tax is decreasing, and, when d is higher than $\underline{d}_\psi^N$, it is increasing. Or, in other words, if the pollution damage is high with respect to extraction costs the optimal tax would have to be increasing.

The interpretation of this result is quite intuitive if one realises that the differential game under consideration integrates characteristics of two models with different properties. Making $c=0$ we have Wirl and Dockner's (1995) model, and for $d=0$ we have a version of Karp's (1984) model, where ψ

must be interpreted as an import tariff and the issue addressed is whether it is advantageous for the importing countries to fix a tariff on the resource. In the first case, we can check that x^N and α_1^N are negative for any positive value of parameter d and that the tax is always increasing[11]. For $c=0$, (4.29) is written as $\psi^N = a + \alpha_1^N z_\infty^N \exp\{x^N t/2\}$ and then $d\psi^N/dt > 0$. This means that if there are *no depletion effects,* the optimal tax for the importing countries must rise. In the second case, it is evident that α_1^N is positive and the tax is decreasing. For $d=0$, (4.29) is written as $\psi^N = \alpha_1^N z_\infty^N \exp\{-1/2(c-x^N)t\}$ and then $d\psi^N/dt < 0$, since the sign of $c-x^N$ does not change (this is shown in the next paragraph). We obtain, in this case, that, when the *environmental damage* is *zero* and the depletion effects are positive, the optimal policy for the importing countries is a decreasing import tariff. Thus, we have two trends of opposite sign acting in our game, and we find that when the pollution damage is high with respect to extraction costs, the increasing trend dominates, and the tax is increasing. However, if on the contrary the pollution damage is low, the decreasing trend dominates and the tax is decreasing.

If we focus now on the temporal path of the producer price we get the same kind of results. The dynamics of the variable depends on the sign of the following expression:

$$c+\alpha_1^N-\alpha_2^N = c - \frac{1}{4\delta}(c-x^N)^2 - \frac{2d}{\delta}. \tag{4.31}$$

It is easy to show that for d equal to zero this expression is positive. As $c+\alpha_1^N-\alpha_2^N$ can be written as $c-x^N+2\alpha_1^N$, we can use this last expression for determining the sign of the former. For $d=0$ we have found that α_1^N is positive; then $c+\alpha_1^N-\alpha_2^N$ is positive if $c-x^N$ is positive for $d=0$. To calculate $c-x^N$ we use (4.15), yielding:

$$c-x^N = \frac{1}{2}\left\{-\frac{4}{3}\delta + \left[\frac{16}{3}\left(c\delta+\frac{\delta^2}{3}\right)\right]^{1/2}\right\}. \tag{4.32}$$

If we suppose that $c-x^N$ is negative or zero, the following must be satisfied:

$$\left[\frac{16}{3}\left(c\delta+\frac{\delta^2}{3}\right)\right]^{1/2} \le \frac{4}{3}\delta.$$

[11] In fact, the tax is also increasing when the extraction costs are quadratic but independent of the cumulative extractions, as has been showed by Tahvonen (1996).

Taking the square of this inequality we get $16/3(c\delta) \leq 0$, which is a contradiction. As a result we have to accept that $c - x^N$ is positive and conclude that $c + \alpha_1^N - \alpha_2^N$ is positive as well. Now, applying calculus to (4.31) and using (4.15) we can establish that $c + \alpha_1^N - \alpha_2^N$ is decreasing with respect to d and that there exists a positive value, that we represent by $\underline{d}_p^N$, for which the producer price is constant. Thus, we get that when d is lower that $\underline{d}_p^N$ the price is increasing, whereas it is decreasing if d is higher than $\underline{d}_p^N$. Or, in other words, if the pollution damage is high with respect to extraction costs the optimal producer price must be decreasing. This result is also justified by the two opposite trends we have found in our model. For $c = 0$ we know that the producer price is decreasing, but for $d = 0$ it is increasing. Consequently, when these two parameters are positive we can obtain both types of dynamics depending on the values of the parameters. On the other hand, the compatibility between a decreasing quantity and price can be explained by resorting to the reaction function (4.7). According to this function the producer price and the tax are strategic substitutes since the tax reduces the marginal revenue of the cartel. Moreover, the reaction function establishes that the price increases, *ceteris paribus*, with the *complete* marginal cost of the resource, defined by the marginal extraction cost plus the user cost, $cz - W_2^{'}$. Then as the tax increases and the complete marginal cost decreases along the equilibrium path, when the pollution damage is high enough, we find that the dynamics of the producer price have to be decreasing. Obviously, as the extraction rate is decreasing, the negative effect of an increase of the tax on the quantity must be higher than the positive effect of a reduction of the complete marginal cost on the extraction rate.

Finally, we can describe the different temporal paths that the tax and producer price can follow, depending on the environmental damage. First, we define the existing relationship between $\underline{d}_\psi^N$ and $\underline{d}_p^N$. As $c + \alpha_1^N - \alpha_2^N$ is equal to $c - x^N + 2\alpha_1^N$, we get that when $d = \underline{d}_\psi^N$, $c + \alpha_1^N - \alpha_2^N = c - x^N$, which is positive for any positive value of d, as is established in (4.16). Thus, $c + \alpha_1^N - \alpha_2^N$ is zero for a higher value than $\underline{d}_\psi^N$, and then we can conclude that $\underline{d}_\psi^N$ is lower than $\underline{d}_p^N$. Now, we are able to present the different temporal trajectories depending on the value of parameter d. *Given* c and δ, if d is lower than $\underline{d}_\psi^N$ the price is increasing and the tax is decreasing; if d is in the interval $(\underline{d}_\psi^N, \underline{d}_p^N)$ the price and tax are increasing; and, finally, if d is higher than $\underline{d}_p^N$ the price is decreasing but the tax is increasing. This last relationship can also be presented as follows.

Corollary 4.5 *If an increase in cumulative emissions has an effect on marginal damage higher than* $2\underline{d}_p^N$, *then the optimal producer price is decreasing whereas the optimal tax is increasing.*

Notice that the effect of an increase in cumulative emissions on marginal damage is given by $\partial^2 D/\partial z^2$ which is equal to $2d$, so that it is sufficient with $\partial^2 D/\partial z^2$ higher than $2\underline{d}_p^N$ to have an increasing tax with a decreasing price.

This result already appears in Wirl and Dockner (1995) and Tahvonen's (1996) papers, but as long as they do not take into account the depletion effects on the extraction of the resource, the pollution tax is increasing and the producer price is decreasing for all d. In this chapter we complete their analysis showing that the tax can be decreasing and the price increasing if the pollution damage is not very high, or that the tax and producer price can both be increasing.

4.4 Optimal Pricing of Polluting Resources

In order to get a welfare evaluation of the MPNE we compute in this section the Pareto efficient solution of the game. To obtain this solution we maximise the addition of the consumers' welfare and profits taking into account the evolution of the cumulative emissions. Then the efficient strategy of extraction has to satisfy the Bellman equation:

$$\delta V = \max_{\{q\}} \{aq - \frac{1}{2}q^2 - dz^2 - czq + V^{'}q\}. \tag{4.33}$$

From the first order condition for the maximisation of the r.h.s. of the Bellman equation we get the optimality condition: marginal utility (price) equal to marginal cost, which includes two components, the marginal extraction cost and the efficient shadow price or user cost of the resource:

$$a - q = cz - V^{'}, \tag{4.34}$$

so that the efficient extraction strategy is given by:

$$q^E = a - cz + V^{'}, \tag{4.35}$$

where superscript E stands for the efficient solution. Next, incorporating the efficient strategy into the Bellman equation (4.33) we eliminate the maximisation and obtain, after some calculations, the following non-linear differential equation:

$$\delta V = \frac{1}{2}(a - cz + V')^2 - dz^2. \tag{4.36}$$

The solution to this differential equation allows us to calculate the linear efficient strategy.

Proposition 4.6 *Let*

$$q^E(z) = \begin{cases} 0, & z_\infty^E \le z \\ a + \beta^E - (c - \alpha^E)z, & z \le z_\infty^E, \end{cases} \tag{4.37}$$

where

$$\alpha^E = \frac{1}{2}\left\{2c + \delta - \left(4c\delta + \delta^2 + 8d\right)^{1/2}\right\}, \tag{4.38}$$

$$\beta^E = -\frac{a(c - \alpha^E)}{\delta + c - \alpha^E} < 0,\ a + \beta^E > 0 \text{ and } c - \alpha^E > 0, \tag{4.39}$$

and

$$z^E = (z_0 - z_\infty^E)\exp\{-(c - \alpha^E)t\} + z_\infty^E, \tag{4.40}$$

where

$$z_\infty^E = \frac{a\delta}{c\delta + 2d}. \tag{4.41}$$

Then $q^E(z)$ constitutes a global asymptotically stable efficient equilibrium (EE) for the infinite horizon differential game under consideration, where z_∞^E is the cumulative emission long-run equilibrium.

Proof. The proof follows that of Proposition 4.2.

The long-run equilibrium value for cumulative emissions has been calculated as a particular solution of the differential equation that defines the dynamic constraint of the model. However, this value can be derived directly using more straightforward arguments. First, we establish that the exploitation of the resource continues until the marginal cost is equal to the maximum price consumers are willing to pay. This occurs for a zero extraction rate ($q_\infty^E = 0$), which implies, according to the optimality condition (4.34), that $a = cz_\infty - V'_\infty$, where a is the maximum price consumers are willing to pay. On the other hand, the value function for a zero extraction rate is $V_\infty = -d(z_\infty^E)^2/\delta$ and the user cost or shadow price is $V'_\infty = -2dz_\infty^E/\delta$; then by substitution in the optimality condition we get (4.41).

The solution includes the pay-offs of the agents, which are given by the value function for the initial value of the state variable, z_0,

$$V^E(z_0) = \frac{1}{2}\alpha^E z_0^2 + \beta^E z_0 + \mu^E, \tag{4.42}$$

where α^E and β^E are given by (4.38) and (4.39) and μ^E is $\frac{1}{2\delta}(a + \beta^E)^2$. So, we can conclude the following.

Corollary 4.7 *If the initial cumulative emissions are zero, the discounted present value of the addition of the net consumers' welfare and profits is $V(0) = \frac{1}{2\delta}(a + \beta^E)^2$.*

Finally, we compute the dynamics of the extraction rate and resource shadow price. To get the temporal paths of these variables first we eliminate z from (4.37) using (4.40), and then using condition (4.34) we obtain the dynamics of the shadow price[12]:

$$q^E = (a + \beta^E)\ \exp\{-(c - \alpha^E)t\}, \tag{4.43}$$

$$-V' = \frac{2ad}{c\delta + 2d} + \alpha^E z_\infty^E\ \exp\{-(c - \alpha^E)t\}. \tag{4.44}$$

We can now summarise the dynamics of the variables and the long-run equilibrium of the efficient solution as follows.

Proposition 4.8 *Along the efficient path the rate of extraction decreases. The shadow price of the resource is increasing (decreasing) if α^E is negative (positive). Moreover, the efficient path approaches a long-run steady state characterised by $q_\infty = 0$ and $-V' = \frac{2ad}{c\delta+2d}$.*

This proposition establishes that the dynamics of the shadow price depends on the sign of coefficient α^E, which is given by (4.38). This expression allows us to study the relationship between this coefficient and the damage parameter, d, and hence the relationship between the pollution damage and the efficient temporal path of the shadow price. We know that α^E is positive when $d = 0$. Suppose that

$$2c + \delta - (4c\delta + \delta^2)^{1/2} \le 0,$$

[12] We assume that the initial cumulative emissions are zero.

then

$$2c + \delta \leq (4c\delta + \delta^2)^{1/2},$$

and

$$(2c + \delta)^2 \leq 4c\delta + \delta^2,$$

resulting in a contradiction: $4c^2 \leq 0$.

On the other hand, α^E is decreasing with respect to d and there exists a positive value, that we name as $\underline{d}^E$, for which the coefficient of the value function V is zero. In fact, it is easy to check that $\underline{d}^E = c^2/2$. Thus, we get that when d is lower than $\underline{d}^E$ the shadow price of cumulative emissions is decreasing, and when d is higher than $\underline{d}^E$ it is increasing. Or, in other words, if the pollution damage is high with respect to extraction costs the shadow price would have to be increasing. The interpretation of this result follows the intuition presented in section 4.3 to explain the dynamics of the emission tax.

This relationship between the coefficient α^E and the damage parameter allows us to establish the following.

Corollary 4.9 *If an increase in cumulative emissions has an effect on marginal damage higher than c^2, the efficient shadow price is increasing.*

Notice that the effect of an increase in cumulative emissions on marginal damage is given by $\partial^2 D/\partial z^2$ which is equal to $2d$, so that it is sufficient with $\partial^2 D/\partial z^2$ higher than c^2 to have an increasing shadow price, since $\underline{d}^E = c^2/2$.

Finally, we use this solution to evaluate the efficiency of the equilibrium market. If one compares the two long-run equilibria one immediately realises that the market equilibrium converges to the efficient values: $z_\infty^N = z_\infty^E = a\delta/(c\delta+2d)$, $q_\infty^N = q_\infty^E = 0$ and $\psi_\infty^N = -V_\infty' = 2ad/(c\delta+2d)$. This property is explained because the optimality conditions that characterise the two equilibria, (4.7) and (4.34), are identical in the long-run. Condition (4.7) can be rewritten as

$$a - 2q = cz - (W_1' + W_2'), \tag{4.45}$$

using the inverse demand function to eliminate the price. Using this expression we have that when the rate of extraction is zero the marginal revenue is equal to average revenue or the maximum price consumers are willing to pay, a, and the l.h.s. of the two optimality conditions coincide. On the other hand, for a zero extraction rate the value function of the efficient solution, (4.33), and the value function of the importing countries for the market equilibrium, (4.4), present the same expression, $V = W_1 = -dz^2/\delta$, so that $V' = W_1'$. Moreover, it is well known that with depletion effects the user cost or shadow price the producers associated to the resource, $-W_2'$, tends to zero, giving as a result that the r.h.s for the two optimality conditions is the same and the two conditions define the same long-run equilibrium.

This property has been used by Wirl (1994, p. 11) to conclude that from an *environmentalist's* point of view, cooperation between consumers and producers is not necessary as long as non-cooperation implies the same stationary stock of pollution. This is right strictly from an environmentalist's point of view but is questionable from an *economic* point of view since the equilibrium market is not efficient and, in that case, the cooperation could increase the pay-offs of the players. To show this argument we compare the aggregate pay-offs of the two equilibria:

$$V^E(0) = \mu^E = \frac{1}{2\delta}(a + \beta^E)^2, \tag{4.46}$$

and

$$W^N(0) = \mu_1^N + \mu_2^N = \frac{3}{8\delta}(a + y^N)^2. \tag{4.47}$$

Let us suppose that $V^E(0) \leq W^N(0)$; then

$$8(a + \beta^E)^2 \leq 6(a + y^N)^2;$$

using (4.39) to calculate $a + \beta^E$, and (4.16) to calculate $a + y^N$, we can eliminate them by substitution and get

$$\left(4\delta + 3(c - x^N)\right)^2 \leq 12\left(\delta + c - \alpha^E\right)^2.$$

Developing the squares yields

$$\begin{aligned} & 4\delta^2 + 24\delta(c - x^N) + 9(c - x^N)^2 \\ \leq \;& 24\delta(c - \alpha^E) + 12(c - \alpha^E)^2. \end{aligned}$$

Using (4.38) to calculate $c - \alpha^E$, and (4.15) to obtain $c - x^N$, we can eliminate them by substitution yielding

$$\delta + 3\left[\frac{16}{3}\left(c\delta + \frac{\delta^2}{3} + 2d\right)\right]^{1/2} \leq 3(4c\delta + \delta^2 + 8d)^{1/2}.$$

Finally, raising to a square and simplifying terms, we obtain the following contradiction:

$$12c\delta + 8\delta^2 + 24d + 6\delta \left[\frac{16}{3}\left(c\delta + \frac{\delta^2}{3} + 2d\right)\right]^{1/2} \leq 0.$$

The sign of this inequality leads us to conclude that the aggregate pay-offs of the efficient equilibrium are higher than the aggregate pay-offs of the equilibrium market and, consequently, that the cooperation can increase the welfare of the economic agents.

In the last part of this section we compare the temporal paths of the variables of the game. First, we try to establish the relationship between the optimal linear strategies. We can summarise our findings as follows.

Proposition 4.10 *The MPNE is more conservative than the EE, i.e., given any resource stock level, the efficient extraction exceeds the Nash extraction rate. Moreover, the efficient initial value for the extraction rate is higher than in the MPNE.*

Proof. See Appendix B (section 4.8).

These results are independent of the parameter values. Besides, they are consistent with the finding obtained by Hotelling that establishes that the monopolist is the conservationist's best friend. In section 4.3, Proposition 4.1, we have concluded that the MPNE defines a neutral pigouvian tax because it only corrects the inefficiency caused by the stock externality but it does not affect the market power of the producers. This means that the inefficiency of the MPNE is caused by the monopoly power of the cartel that reduces in its own benefit the rate of extraction. Thus, as the inefficiency caused by the stock externality is corrected by the tax the importing countries' governments fix, the rate of extraction would have to increase to approach the resource efficient intertemporal allocation. For this reason the extraction efficient strategy is less conservationist than the strategy defined by the market equilibrium.

From this result we can compare the temporal path of the cumulative emissions and the rate of extraction.

Proposition 4.11 *The cumulative extractions for the EE are higher than for the MPNE for all $t \in (0, \infty)$. However, this variable converges asymptotically to the same value in both cases. The rate of extraction for the EE is first above but later below the MPNE rate of extraction.*

Proof. See Appendix C (section 4.9).

This result establishes clearly the difference between the two equilibria although they converge asymptotically to the same values. The equilibrium market is more conservationist throughout the period of exploitation of the resource, resulting in lower cumulative emissions. However, for the rate of extraction the temporal paths intersect once since the cumulative extractions are the same in the long-run. Nevertheless, even with a second phase for which the efficient rate of extraction is more conservationist the discounted present value of the aggregate welfare is higher for the efficient solution as we have just shown above. This is better understood if it is remembered that the discount effect gives greater weight to the pay-offs closer to the present.

4.5 Conclusions

We have examined the strategic pigouvian taxation of CO_2 emissions in the framework of a global warming differential game with depletion effects between a resource-exporting cartel and a coalition of resource-importing countries' governments. We have determined the intertemporal properties of the carbon tax showing that these depend on the importance of environmental damage in comparison with depletion effects. Nevertheless, we have found that if *environmental damage* is high enough the tax should be *increasing* and the producer price decreasing. Besides, we have shown that the pigouvian tax only corrects the market inefficiency caused by the stock externality and that, in that case, the strategic taxation of emissions does not affect the monopolistic power of the cartel. For this reason we find that the market equilibrium is more conservationist than the efficient equilibrium because the producers use their monopoly power for reducing the rate of extraction and increasing their profits. From the efficient equilibrium we are also able to characterise the optimal pricing of the resource and show that if the pollution damage is high with respect to extraction costs the shadow price would have to increase.

The scope of our results is limited by the specification of the game and the irreversibility assumption for the emissions. However, this approach seems to us, for the moment, inevitable to make the analysis tractable[13]. Obviously,

[13]Notice that the irreversibility of emissions allows us to work with a unique state variable. See Tahvonen (1996), section 4, to get an idea of the difficulties that appear when two state variables are considered.

these limitations point out directions for additional research, although in the framework of the model developed in this chapter some additional extensions could be considered. We have supposed that the stock externality is largely irrelevant to the welfare of exporting countries. However, this reduces the *global* character of the greenhouse effect. For this reason it would be interesting to introduce into the analysis environmental damage along with domestic energy consumption in the exporting countries, and to study the issue of the unilateral taxation of CO_2 emissions. Another extension could be to increase the number of the importing countries to analyse the issue of cooperation among the importing countries to control the global warming problem. Finally, cooperative game theory could be applied when the importing countries' governments have some strategic advantage, since in this case cooperation could increase the pay-offs of the two players.

4.6 Acknowledgement

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4.7 Appendix A: Derivation of Linear Markov-Perfect Nash Equilibrium Strategies

The linear strategies can be determined by proposing quadratic solutions for the value functions:

$$W_1(z) = \frac{1}{2}\alpha_1^N z^2 + \beta_1^N z + \mu_1^N,$$
$$W_2(z) = \frac{1}{2}\alpha_2^N z^2 + \beta_2^N z + \mu_2^N.$$

Substituting the value functions and their first derivatives into the Bellman equations and equating coefficients yields the following system of equations (we omit the superscript N when no ambiguity arises):

$$\frac{\delta}{2}\alpha_1 = \frac{1}{8}(c-\alpha_1-\alpha_2)^2 - d, \tag{4.48}$$
$$\delta\beta_1 = -\frac{1}{4}(a+\beta_1+\beta_2)(c-\alpha_1-\alpha_2), \tag{4.49}$$
$$\delta\mu_1 = \frac{1}{8}(a+\beta_1+\beta_2)^2, \tag{4.50}$$
$$\frac{\delta}{2}\alpha_2 = \frac{1}{4}(c-\alpha_1-\alpha_2)^2, \tag{4.51}$$
$$\delta\beta_2 = -\frac{1}{2}(a+\beta_1+\beta_2)(c-\alpha_1-\alpha_2), \tag{4.52}$$
$$\delta\mu_2 = \frac{1}{4}(a+\beta_1+\beta_2)^2. \tag{4.53}$$

Even though this system of equations presents a recursive structure its solution is quite long and complex. However, a simple transformation enormously simplifies its solution. We define $x = \alpha_1 + \alpha_2$ and $y = \beta_1 + \beta_2$ and add equations (4.48) and (4.51) and equations (4.49) and (4.52), obtaining a simplified system of equations in the new variables:

$$\delta x = \frac{3}{4}(c-x)^2 - 2d, \tag{4.54}$$
$$\delta y = -\frac{3}{4}(a+y)(c-x). \tag{4.55}$$

In the light of these two equations and the differential equations (4.8) and (4.9), it appears that the solution corresponds to an aggregate value function

$V = W_1 + W_2 = \frac{1}{2}xz^2 + yz + w$ whose coefficients must satisfy equation: $\delta V = 3/8(a - cz + V')^2 - dz^2$. Equation (4.54) has two real roots. We choose the one which satisfies the stability condition: $d\dot{z}/dz < 0$. To apply this condition, we write the rate of extraction (4.11) in terms of coefficients x and y, resulting in $q = 1/2[a + y - (c - x)z]$, and then as $\dot{z} = q$ we have that $d\dot{z}/dz = -1/2(c - x) < 0$, which requires that $c - x > 0$. Expression (4.15) is the root that satisfies this condition. Given the value of x, (4.55) yields the value of the coefficient y; see (4.16). Knowledge of these two coefficients is sufficient for the computation of the rate of extraction, as can be seen above, and the consumer price π. To obtain the producer price and the tax strategies, we need to solve (4.48), (4.49), (4.51) and (4.52), using (4.54) and (4.55), to obtain $\alpha_1, \beta_1, \alpha_2$ and β_2.

Finally, we solve the first order differential equation $\dot{z} = 1/2[a + y - (c - x)z]$ to obtain the long-run cumulative extraction equilibrium. The solution to this equation is:

$$z = C\exp\{-1/2(c - x)t\} + \frac{a + y}{c - x}, \tag{4.56}$$

where $(a + y)/(c - x)$ is the particular solution $\dot{z} = 0$ and C is an integration constant. Then, as $c - x$ is positive, the long-run equilibrium is the particular solution. If we substitute $a + y$ in the long-run equilibrium value we get:

$$z_\infty = \frac{4a\delta}{4c\delta - 4\delta x + 3(c - x)^2},$$

which can be rewritten as:

$$z_\infty = \frac{4a\delta}{4c\delta + 8d} = \frac{a\delta}{c\delta + 2d}, \tag{4.57}$$

taking into account that $-4\delta x + 3(c - x)^2 = 8d$, according to (4.54). Then using the initial condition, z_0, to eliminate the integration constant we get the optimal dynamics of the state variable of the game:

$$z = (z_0 - z_\infty)\exp\{-1/2(c - x)t\} + z_\infty, \tag{4.58}$$

and by substitution in the linear strategies the dynamics of the rest of the variables of the model, achieving a complete analytical characterisation of the Markov-perfect Nash equilibrium.

4.8 Appendix B: Proof of Proposition 4.10

If we take into account that the strategies are linear, see (4.14) and (4.37), and define the same rate of extraction for the same long-run equilibrium value of the cumulative emissions, it is sufficient to know the relative position of the independent terms to make the comparison. Let us suppose that $a + \beta^E \leq \frac{1}{2}(a + y^N)$. Using (4.39) to calculate $a + \beta^E$, and (4.16) to calculate $a + y^N$, we can eliminate them by substitution and get

$$2\delta + 3(c - x^N) \leq 2(c - \alpha^E). \tag{4.59}$$

Resorting to (4.38) to calculate $c - \alpha^E$, and (4.15) to obtain $c - x^N$, we can eliminate them by substitution yielding

$$\delta + \frac{3}{2}\left[\frac{16}{3}\left(c\delta + \frac{\delta^2}{3} + 2d\right)\right]^{1/2} \leq (4c\delta + \delta^2 + 8d)^{1/2}.$$

Finally, raising to a square and simplifying terms, we obtain the following contradiction:

$$4\delta^2 + 8c\delta + 16d + 3\delta\left[\frac{16}{3}\left(c\delta + \frac{\delta^2}{3} + 2d\right)\right]^{1/2} \leq 0.$$

The sign of this inequality leads us to conclude that $a + \beta^E > \frac{1}{2}(a + y^N)$ which determines the relative position of the linear strategies and allows us to establish the comparison between the slopes. As long as $q^N_\infty = q^E_\infty$ and $z^N_\infty = z^E_\infty = z_\infty$ we have that

$$a + \beta^E - (c - \alpha^E)z_\infty = \frac{1}{2}(a + y^N) - \frac{1}{2}(c - x^N)z_\infty,$$

which can be rewritten as

$$a + \beta^E - \frac{1}{2}(a + y^N) = \left((c - \alpha^E) - \frac{1}{2}(c - x^N)\right) z_\infty.$$

Then, as the difference in the l.h.s. is positive, we have that

$$c - \alpha^E > \frac{1}{2}(c - x^N). \tag{4.60}$$

The comparison between the initial values for the rate of extraction is immediate since $q^N(0) = \frac{1}{2}(a + y^N)$ and $q^E(0) = a + \beta^E$.

4.9 Appendix C: Proof of Proposition 4.11

We begin comparing the temporal paths of cumulative extractions. For the MPNE the dynamics of this variable is given by (4.58), which, for $z_0 = 0$, can be written as $z^N = z_\infty \left(1 - \exp\{-1/2(c - x^N)t\}\right)$. For the EE using (4.40) we get $z^E = z_\infty \left(1 - \exp\{-(c - \alpha^E)t\}\right)$, so that the difference between the temporal paths is

$$z^N - z^E = z_\infty \left[\exp\{-(c - \alpha^E)t\} - \exp\{-1/2(c - x^N)t\}\right],$$

which is negative for all $t \in (0, \infty)$ since $c - \alpha^E > 1/2\ (c - x^N)$, as we have just shown in the proof of Proposition 4.6.

For the comparison of the extraction rate temporal paths we use (4.27) and (4.43). In this case the difference between the two temporal paths is given by the following expression

$$\begin{aligned} q^N - q^E &= 1/2(a + y^N)\exp\{-1/2(c - x^N)t\} \\ &\quad -(a + \beta^E)\exp\{-(c - \alpha^E)t\}. \end{aligned}$$

For $t = 0$ we know that the difference $q^N(0) - q^E(0)$ is negative, since $1/2(a + y^N) < a + \beta^E$, as we have just established in the comparison of the linear strategies of the two equilibria; see also proof of Proposition 4.6. For $t \neq 0$ we can find the number of intersection points from the equation $q^N - q^E = 0$. This equation can be written as

$$\frac{1/2(a + y^N)}{a + \beta^E} = \exp\{1/2(c - x^N)t - (c - \alpha^E)t\}, \tag{4.61}$$

where the l.h.s. is a positive constant less than one and the r.h.s. is a decreasing and convex function which takes the value one for $t = 0$, and tends to zero when t tends to infinity. This shows us that the temporal paths cut each other once in the interval $[0,\infty)$, and, consequently, we can conclude that for $0 \leq t < t'$, where t' is the solution to equation (4.61), the MPNE extraction rate is lower than the EE extraction rate, whereas for $t' < t$ the relationship between the two temporal trajectories is the contrary.

References

1. Buchanan, J.M. (1969), 'External diseconomies and corrective taxes and market structure', *American Economic Review*, vol. 59, 174-177.

2. Farzin, Y.H. (1996), 'Optimal pricing of environmental and natural resource use with stock externalities', *Journal of Public Economics*, vol. 62, 31-57.

3. Farzin, Y.H. and Tahvonen, O. (1996), 'Global carbon cycle and the optimal time path of a carbon tax', *Oxford Economic Papers*, vol. 48, 515-536.

4. Forster, B.A. (1980), 'Optimal energy use in a polluted environment', *Journal of Environmental Economics and Management*, vol. 7, 321-333.

5. Hoel, M. (1992), 'Emission taxes in a dynamic international game of CO_2 emissions', in Pethig, R. (ed.), *Conflicts and cooperation in managing environmental resources*, Berlin: Springer-Verlag.

6. Hoel, M. (1993), 'Intertemporal properties of an international carbon tax', *Resource and Energy Economics* , vol. 15, 51-70.

7. Hoel, M. and Kverndokk, S. (1996), 'Depletion of fossil fuels and the impacts of global warming', *Resource and Energy Economics*, vol. 18, 115-136.

8. Kamien, M.I. and Schwartz, N. (1991), *Dynamic optimisation. The calculus of variations and optimal control in economics and management*, 2nd ed., Amsterdam: North-Holland.

9. Karp, L. (1984), 'Optimality and consistency in a differential game with non-renewable resources', *Journal of Economic Dynamics and Control*, vol. 8, 73-97.

10. Rubio, S.J. and Escriche, L. (1998), 'Strategic pigouvian taxation, stock externalities and polluting non-renewable resources', Working Paper, Instituto Valenciano de Investigaciones Económicas, Valencia.

11. Sinclair, P. (1992), 'High does nothing and rising is worse: Carbon taxes should keep declining to cut harmful emissions', *Manchester School*, vol. 60, 41-52.

12. Sinclair, P. (1994), 'On the trend of fossil fuel taxation', *Oxford Economic Papers*, vol. 46, 869-877.

13. Tahvonen, O. (1995), 'International CO_2 taxation and the dynamics of fossil fuel markets', *International Tax and Public Finance*, vol. 2, 261-278.

14. Tahvonen, O. (1996), 'Trade with polluting nonrenewable resources', *Journal of Environmental Economics and Management*, vol. 30, 1-17.

15. Ulph, A. and Ulph, D. (1994), 'The optimal time path of a carbon tax', *Oxford Economic Papers*, vol. 46, 857-868.

16. Wirl, F. (1994), 'Pigouvian taxation of energy for flow and stock externalities and strategic, noncompetitive energy pricing', *Journal of Environmental Economics and Management*, vol. 26, 1-18.

17. Wirl, F. and Dockner, E. (1995), 'Leviathan governments and carbon taxes: Costs and potential benefits', *European Economic Review*, vol. 39, 1215-1236.

Chapter 5

Environmental and Economic Policy in Monopolistically Competitive Markets

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5.1 Introduction

Most of the dynamic phenomena observed in modern economies are due to a rapidly growing volume of innovations aimed at increasing the efficiency of firms' responses to the needs of the market. An important share of the aforementioned innovations aim at installing or improving the infrastructure which is required for the transportation of economic goods from the place of production to the place of consumption. Both the state and private investors are involved in such an effort. For example, a highway may be the result of public investment, but firms have to invest in their own transportation infrastructure if they want to use the highway. Communication networks