In *Progress in Economics Research*
Edited by Albert Tavidze, pp. 123-146
ISBN 1-59033-310-1

Chapter 6

SUSTAINABLE ENDOGENOUS GROWTH AND PIGOUVIAN TAXATION*

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Abstract

An AK model of ecologically sustainable endogenous growth in which environmental quality has a positive influence on individual welfare and on the productivity of capital is presented. The effect of different environmental policies on the long-run growth of the economy is studied in the framework of this model. The results establish that an optimal policy which taxes production and subsidizes pollution abatement has a favorable effect on environmental quality, and could increase the growth rate if the positive external effects of the environment on the productivity are important. Furthermore, it is shown that this kind of environmental policy is *neutral* in budgetary terms, i.e. tax receipts are equal to subsidies. Finally, it is found that tighter standards have a positive effect on the growth rate only if the previous standards were not very far from the level of environmental quality corresponding to the market equilibrium.

Key words:sustainable growth, external effects, Pigouvian taxes, emissions control.
JEL Classification: H23, O41, Q28.

1 Introduction

A great many papers have been written on the relationship between economic growth and environmental preservation since R.C. d'Arge published his *Essay on economic*

*This paper was presented at the *Tenth Annual Conference* of the *European Association of Environmental and Resource Economists*, Rethymnon, Greece, June 30-July 2, 2000 and at the 15^{th} *Annual Congress of the European Economic Association*, Bozen-Bolzano, Italy, August 30-September 2, 2000. We would like to thank the participants in these meetings for their interest and, in particular, professor Y. Hossein Farzin for his comments. Financial support from the *Instituto Valenciano de Investigaciones Económicas* and from the *Ministerio de Ciencia y Tecnología* under grant BEC2000-1432 is gratefully acknowledged.

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growth and environmental quality in 1971. The principal questions occupying economists have been: Is long-run economic growth compatible with environmental preservation? That is to say, is sustainable growth viable? Under what conditions? What would be the effect of greater concern for the environment over economic growth? How do environmental externalities influence growth rate, and thus, what is the effect of environmental policy on economic growth?

These questions have been analyzed in many of these papers in the framework of stationary models and exogenous growth models.[1] In this type of models, either there is no long-run growth or if there is long-run growth it is exogenously determined, so that environmental quality may have a negative effect on capital accumulation or no effect on the growth rate. In addition, in this literature the emphasis has been put on analysis of the efficient growth path without paying much attention to growth based on market equilibrium.

However, since the appearance of the new theory of growth at the end of the eighties and the start of the nineties, a series of papers has been published in which these questions are addressed in the framework of endogenous growth models.[2] Among these, we would like to draw particular attention to those of Gradus and Smulders (1993), Ligthart and van der Ploeg (1994), Huang and Cai (1994), Hung, Chang and Blackburn (1994), Ewijk and Wijnbergen (1995), Bovenberg and Smulders (1995) and (1996), Michel and Rotillon (1995), Smulders and Gradus (1996), Mohtadi (1996), Rosendahl (1996), Stokey (1998), Aghion and Howitt (1998) and Grimaud (1999). These contributions can be classified into two groups. The first, comprised of the work of Ligthart and van der Ploeg (1994), Huang and Cai (1994), Hung, Chang and Blackburn (1994), Michel and Rotillon (1995) and Mohtadi (1996), would include models which predict *ecologically unsustainable* growth. In these models growth is achieved at the cost of continuous deterioration in environmental quality. In the second group, which contains the remaining papers, appears models which predict *ecologically sustainable* growth. According to these models, under determined conditions, the economy could follow a path of balanced growth with stable emissions, and therefore a constant level of environmental quality, and in some cases growth could even be compatible with decreasing emissions as in Aghion and Howitt's (1998) model.[3] Within this latter group we can discern three sub-groups according to the effect on growth rate of a greater concern of the individuals for the environment or the effect of a tighter environmental policy. Using an extension

[1]See Rubio and Fisher (1994) for a review of this literature.

[2]See Smulders (1999) for a quick review of the literature on endogenous growth and the environment.

[3]In Stokey's (1998) paper, stabilization of the emissions has a negative effect on the rate of return of capital and makes long-run sustainable growth inviable. Reference can also be made to the papers of John and Pecchenino (1994), John et al. (1995) and Elbasha and Rose (1996). In John and Pecchenino (1994) and John et al. (1995), the relationship between environmental quality and growth is analyzed in the framework of an overlapping-generations model, and this subject is addressed with the inclusion of technological change and international trade in Elbasha and Rose (1996).

of Rebelo's (1991) basic model, Gradus and Smulders (1993) show that a higher environmental quality *reduces* the rate of growth when environmental quality only affects consumers' welfare. Grimaud (1999), who has derived an implementation of the optimal path in the Aghion-Howitt model using subsidies and pollution permits, also finds that there exists a trade-off between growth and environmental quality in a Schumpeterian model that distinguishes between tangible and intellectual capital and where the intensity of pollution affects the productivity of both sorts of capital. Again Gradus and Smulders (1993) and Ewijk and Wijnbergen (1995) conclude that the effect is *positive* using different versions of Lucas' (1988) model, even when environmental quality affects utility.[4] This result is based on the fact that environmental quality has a *positive* effect on the accumulation of human capital. Finally, Bovenberg and Smulders (1995) and (1996), and Smulders and Gradus (1996) show that the effect is *ambiguous* and that it depends on the relative importance of environmental quality as *amenity value* or as *productive value*, so that only if the productive value of environmental quality is higher than the amenity value will the environmental policy have positive effects, not only on environmental quality but also on economic growth.

Our contribution continues along the line of the papers published by Mohtadi (1996) and Smulders and Gradus (1996). We are particularly interested in the design of optimal environmental policy, as set out by Mohtadi (1996), but in the framework of a model of ecologically sustainable growth.[5] For this purpose we present a model of endogenous growth *à la Rebelo*, in which the productivity of capital depends positively on environmental quality but is constant for a given level of environmental quality, and we assume that firms can devote resources to pollution abatement but that the productivity of these resources is *decreasing*.[6] Our main purpose is to study the effect of environmental policy on the growth rate, focusing on two types of policy. First, we are interested in an environmental policy based on Pigouvian taxes and subsidies, the objective of which is to restore the allocative efficiency of the market, which we can define as a *pricing policy*. In the second place, we focus on a sub-optimal policy of pollution control, that is, a policy based on *standards*, and we analyze the effect on the growth rate of a tighter environmental policy, consisting of requiring a lower emission level from firms.[7] The paper presents two parts: in the first part, we assume that the environment only enters into the utility function of households; in the second part, we take into consideration that pollution can have a

[4]In Rosendahl's (1996) paper the effect is *null* because environmental quality only affects the production of consumer goods and not the accumulation of human capital.

[5]See Bovenberg and de Mooij (1997) and Hettich (1998) for a complementary approach. In these papers is studied how an environmental tax reform affects pollution and economic growth in endogenous growth models with pre-existing tax distortions.

[6]This assumption differentiates our paper from Mohtadi's (1996), which does not take into consideration that firms can allocate resources to pollution abatement.

[7]In line with Mohtadi (1996), this paper is solely concerned with long-term effects as long as this kind of AK models of endogenous growth has no transitional dynamics.

negative effect on productivity. In this way we can clarify what effects environmental quality has on the growth path of the economy through productivity.[8]

Our results indicate that if the environment does not affect the productivity of the economy, the environmental policy has a *negative* effect on the growth rate and it is not possible to reconcile growth with environmental preservation in the long-run. If this is not the case and the environment has a productive value, the optimal environmental policy, which consists of establishing a tax on production and a subsidy on abatement pollution, could have a positive effect on growth if the external effects in the production are sufficiently large. In other words, only if the environmental externalities in the production are enough high it may be expected that the optimal environmental taxation boosts growth. Moreover, we show that the optimal policy is *self-financing*, in the sense that tax receipts are sufficient to pay for subsidies needed to restore the allocative efficiency. This result is of interest since one of the limitations of Pigouvian taxes (subsidies) is that lump sum subsidies (taxes) are required to distribute the gains resulting from the restoration of the efficiency. Finally, we conclude that a tightening of environmental policy based on emission control, starting from the emission level corresponding to the market equilibrium, has a positive effect on growth even if the external effects in the production are low provided that the environmental policy be not very restrictive., i.e. if small reductions in the emission level are proposed.

The paper is structures as follows: in Section 2 we present the model of endogenous growth and environmental quality; and in Section 3 we develop a first approach to the issues which interest us, on the assumption that environmental quality only affects utility. This approach allows us to evaluate in Section 4, by comparison, the importance of the productive value of environmental quality. This Section includes four sub-sections. In the first the efficient balanced growth path is studied, in the second the market equilibrium path, in the third the Pigouvian taxes are calculated, and finally in the fourth policy based on emissions control is analyzed. Last come the conclusions.

2 The model

Let us consider an endogenous growth model for a closed economy with a rational representative consumer and two goods: a private good, $C(t)$, and a public

[8]Smulders and Gradus (1996) present a general equilibrium one-sector model in which environment is essential for production and welfare. Their model is not strictly an AK model but it can be obtained as a particular case. In our paper, focusing on an AK model, we mainly complete the analysis of the environmental policy developed by these authors. In particular, we compute the instruments which allow to implement the optimal path, we calculate the budgetary balance resulting from the application of the optimal environmental policy and we show that for this policy the decentralized solution exactly reproduces the efficient allocation. Moreover, we extend the analysis to cover a sub-optimal policy based in standards. Finally, our paper allows to evaluate how the effects of environmental policy depend on the productive value of the environment.

good, $E(t)$, which represents the level of environmental quality. The preferences are described by the following utility function

$$U(C(t), E(t)) = \frac{1}{1-\sigma}\left[(C(t)E(t)^{\alpha})^{1-\sigma} - 1\right], \tag{1}$$

where α and σ are positive constants. The social welfare is given by

$$W = \int_0^{\infty} \frac{1}{1-\sigma}\left[(C(t)E(t)^{\alpha})^{1-\sigma} - 1\right] e^{-\rho t} dt, \tag{2}$$

where ρ is the discount rate. For this specification the elasticities of marginal utility, both that with respect to consumption and that with respect to environmental quality, are constant and the intratemporal elasticity of substitution between $C(t)$ and $E(t)$ in utility is unity.[9] In this function the parameter α lets us characterize the individual preferences with respect to the two goods, such that the greater is the value of α the greater the importance of environmental quality is in the preferences. Keep in mind that for a given combination of $(C(t), E(t))$, α determines the willingness to pay for environmental quality which is defined by the Marginal Rate of Substitution, $\text{MRS}_{CE} = \alpha C(t)/E(t)$, so that an increment in α increases the willingness to pay for environmental quality.[10]

Next we establish the relationship between the environment and economic activity. We assume that the environmental quality is related negatively to the capital stock, $K(t)$, and positively to the pollution abatement, $A(t)$[11]. Given the linear dependency we are going to establish between production and capital stock, the negative effect of accumulation of capital on environmental quality represents the environmental damage caused by the productive activity. In this paper we adopt the pollution function proposed by Gradus and Smulders (1993, page 31), but we interpret it in the terms appearing in Mohtadi (1996).

$$E(t) = \left(\frac{A(t)}{K(t)}\right)^{\eta}, \quad 0 < \eta < 1. \tag{3}$$

The principal characteristic of this function is that it shows *decreasing returns* for pollution abatement, $E_{AA} < 0$.

The technology is linear with respect to $K(t)$,but concave with respect to $E(t)$,

$$Y = f(K(t), E(t)) = BK(t)E(t)^{\beta}, \quad 0 \leq \beta < 1. \tag{4}$$

So that environmental quality is considered, although indirectly, to be a production factor, in the sense that a higher level of environmental quality positively affects

[9] Smulders and Gradus (1996) have shown that these conditions are required to obtain a balanced growth. This kind of specification is also used by Huang and Cai (1994) and Mohtadi (1996), and it is based in the utility function proposed by King, Plosser and Rebelo (1988).

[10] In Section 3.1 we investigate which are the conditions that the parameters α and σ must satisfy to obtain an ecologically sustainable balanced growth path.

[11] In this paper we exclude the pollution associated with consumption activity.

the productivity of the direct production factors, in the Rebelo's model, the capital stock.

Defined the production function, the capital stock dynamics is given by the following differential equation

$$\dot{K} = BK(t)\,E(t)^{\beta} - C(t) - A(t). \tag{5}$$

We do not take into account the depreciation of capital stock, although incorporating it into the analysis through a constant depreciation rate would not modify the qualitative results of the paper.

3 The sustainable balanced growth rate and the optimal environmental policy.

In this section we are interested in determining the conditions which make economic growth compatible with environmental preservation, and in the Pigouvian taxes which lead the economy to the efficient growth path when the environment only affects consumers' welfare. The results of this section will enable us to clarify what effects environmental quality has on the growth path of the economy through productivity. To eliminate the productive value of the environment we make the parameter β equal to zero in (4).

3.1 The efficient path.

First we calculate the efficient growth path which we use later both to evaluate the market equilibrium path and to calculate the Pigouvian taxes which correct the allocative distortions caused by the external effects associated with the environment. The efficient path is found by internalizing the external effects associated with consumption, which is done by maximizing for $C(t)$ and $E(t)$ the social welfare function (2) subject to the restrictions (3) and (5) for a given initial value of capital stock. Eliminating $E(t)$ using pollution or environmental quality function (3), the control variables of the optimal control problem are $C(t)$ and $A(t)$ and the current Hamiltonian is[12]

$$H(K,\lambda,C,A,t) = \frac{1}{1-\sigma}\left[C^{1-\sigma}\left(\frac{A}{K}\right)^{\eta\alpha(1-\sigma)} - 1\right] + \lambda\left[BK - C - A\right], \tag{6}$$

where λ is the co-state variable.

[12]Without loss of generality, the time reference of the variables will be omitted, provided it is not required for correct interpretation of the expressions.

The first-order conditions establish that

$$C^{-\sigma}\left(\frac{A}{K}\right)^{\eta\alpha(1-\sigma)} = \lambda, \tag{7}$$

$$\eta\alpha\frac{C^{1-\sigma}}{K}\left(\frac{A}{K}\right)^{\eta\alpha(1-\sigma)-1} = \lambda. \tag{8}$$

On the margin, the marginal utility of consumption must be equal to the marginal valuation of pollution abatement which is given by the product of the marginal utility of environmental quality and the marginal effect of the pollution abatement on environmental quality. The second-order condition is satisfied if the utility function, $U(C, E(A, K)) = U(C, E, K)$, is strictly concave with respect to control variables which implies that the marginal utility of consumption and pollution abatement are decreasing and, moreover, that $\sigma + \eta\alpha(\sigma - 1)$ is positive. This inequality defines a lower bound, less than unity, on the elasticity of marginal utility of consumption, $\eta\alpha/(1+\eta\alpha) < \sigma$.

Moreover, the Euler equation determines the dynamic optimality of the allocation:

$$\dot{\lambda} = \lambda\left(\rho - B\right) + \eta\alpha\frac{C^{1-\sigma}}{K}\left(\frac{A}{K}\right)^{\eta\alpha(1-\sigma)}. \tag{9}$$

Thus, equations (5) and (7)-(9), together with the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda K = 0$ implicitly describe the efficient path for any given initial capital stock value. The easiest way to characterize the optimal paths is to look for the *sustainable balanced growth* paths which we define as a solution $\{K, \lambda, C, A\}$ to the optimization problem for an initial condition $K(0) = K_0$, such that the growth rates of K, λ, C, A and the ratios Y/K, C/K and A/K are constant. We call this sustainable growth because the growth is compatible with preservation of the environment; that is, with a constant level of environmental quality. Remember that environmental quality is a function of the A/K ratio.

Let g_C be the growth rate of consumption, $\dot{C}/C$. Then from (7), by differentiation, we get $\dot{\lambda}/\lambda = -\sigma g_C$, where σ is the elasticity of marginal utility of consumption. In the process of differentiating (7), we treat the A/K ratio as a constant, just as we have established above in the definition of sustainable balanced growth. Then using (9) we get

$$B = \rho + \sigma g_C + \frac{\eta\alpha}{\lambda}\frac{C^{1-\sigma}}{K}\left(\frac{A}{K}\right)^{\eta\alpha(1-\sigma)}. \tag{10}$$

Thus, along the path of balanced growth, the marginal product of capital is equal to its opportunity cost, which in our model includes an additional term in comparison with the standard growth model which represents the negative effect that capital stock has on utility through its influence on environmental quality

$$-U_E E_K = \eta\alpha \frac{C^{1-\sigma}}{K}\left(\frac{A}{K}\right)^{\eta\alpha(1-\sigma)}. \tag{11}$$

To have an homogeneous expression this term is divided in (10) by the price of consumption good defined by the marginal utility of consumption, $U_C = \lambda$.

Using (8) to eliminate λ from the right-hand side of (10) and reordering terms, we obtain

$$g_C = \frac{1}{\sigma}\left[B - \rho - \frac{A}{K}\right] \tag{12}$$

which, interpreting (10), we name the *asset market equilibrium condition.*

From (7) and (8) we have that $C = A/\eta\alpha$, then eliminating C by substitution in the dynamic restriction (5), adding terms and dividing by K we get

$$g_K = B - \left(\frac{1+\eta\alpha}{\eta\alpha}\right)\frac{A}{K}, \tag{13}$$

which we name the *goods market equilibrium condition.* Imposing $g_C = g_K$, the system of equations (12) and (13) allows us to calculate the growth rate and the level of environmental quality which correspond with the efficient path of sustainable balanced growth of the economy. If we also keep in mind that $g_A = g_K$, given that the level of environmental quality must remain constant, we conclude that all the variables in our model must grow at the same rate g.

Solving (12) and (13) we obtain

$$\left(\frac{A}{K}\right)_P = \frac{\eta\alpha\left[\rho + B(\sigma-1)\right]}{\sigma + \eta\alpha(\sigma-1)}, \tag{14}$$

$$g_P = \frac{B - \rho(1+\eta\alpha)}{\sigma + \eta\alpha(\sigma-1)}. \tag{15}$$

Where $\sigma + \eta\alpha(\sigma - 1)$ is positive by the strict concavity of the utility function. Then $(A/K)_P$ and g_P have strictly positive values if the following conditions on the parameters hold[13]

$$\frac{B-\rho}{B} < \sigma, \quad \alpha < \frac{B-\rho}{\eta\rho}, \tag{16}$$

where the terms of the inequalities are positive, provided that $B - \rho > 0$. The first condition guarantees that the environmental quality is positive and the second condition that the rate of growth is also positive. These two conditions impose certain restrictions on the preferences to get a sustainable balanced growth for the economy. In particular, the elasticity of marginal utility of consumption cannot be

[13] P denotes the Pareto efficient intertemporal allocation.

very low and the willingness to pay for environmental quality cannot be very high. Nevertheless, our results show that a sustainable balanced growth is compatible with a elasticity of marginal utility of consumption lower than unity since $(B-\rho)/B < 1$. This also means that is not necessary to establish any particular assumption on the sign of the cross-effects on marginal utilities to reach a path of sustainable growth. Finally, we may add that it is easy to prove that if condition $(B-\rho)/B < \sigma$ is met, the transversality condition is also fulfilled.

Next, we evaluate the impacts of variations in the parameters on the quality of the environment and on the growth rate of the economy. Let us consider first the effects of variations in α

$$\frac{\partial \left(\frac{A}{K}\right)_P}{\partial \alpha} = \frac{\eta\sigma\left[\rho + B(\sigma-1)\right]}{(\sigma+\alpha\eta(\sigma-1))^2} > 0, \tag{17}$$

$$\frac{\partial g_P}{\partial \alpha} = -\frac{\eta\left[\rho + B(\sigma-1)\right]}{(\sigma+\alpha\eta(\sigma-1))^2} < 0. \tag{18}$$

The signs of the derivatives establish that an increase in (marginal) willingness to pay for environmental quality (a increase in α) results in an increase in environmental quality and a decrease in the sustainable balanced growth rate.

This result leads us to the conclusion that preferences which imply a higher valuation of the environment are associated with a lower growth rate. In other words, a higher environmental conservation will only be achieved at the cost of a reduction in growth. This result is implicit in conditions (12) and (13), which show an inverse relationship between growth rate and environmental quality, and define a clear trade-off between these two variables. Observe that α determines the C/A ratio and the allocation of output between the investment and the expenditure in consumption and pollution abatement. This is very clear if we rewrite condition (13) as

$$Y = BK = \dot{K} + \frac{1+\eta\alpha}{\eta\alpha}A. \tag{19}$$

Then, for given values of K and A, an increase in α reduces the expenditure and increases the investment resulting in an increase of the growth rate. With $g_C = g_K$, this change causes a disequilibrium in the asset market since the opportunity cost of capital increases. In order to reach again the equilibrium in the asset market, without affecting the equilibrium in the goods market, the rate of growth must decrease and the environmental quality increase. Notice that the adjustment in the asset market cannot be reached by a reduction both the growth rate and the environmental quality because this would cause a disequilibrium in the goods market. Finally, we emphasize the positive effect of an increase in the discount rate on the environmental quality.

3.2 The equilibrium path

In this first part we assume that the environmental quality, which depends on decisions of the firms, does not affect productivity, on account of which the market allocation is seen to be distorted by the presence of unidirectional external effects, from the production activity to the consumption activity. In this framework individuals determine the demand for consumption goods, taking the level of environmental quality as given, and the firms have no incentive to spend in pollution abatement, for which reason we expect that market equilibrium leads to an *ecologically* unsustainable growth, as occurs, for example, in the paper of Michel and Rotillon (1994) and Mohtadi (1996). Hence ecologically sustainable growth will only be possible through public intervention.

Next we show that the efficient intertemporal allocation can be replicated through a *proportional tax on production* if the *public expenditure* allocated to pollution abatement is equal to the efficient level, A_P. Let us assume that the representative consumer maximizes the utility function over an infinite horizon, choosing consumption subject to the usual intertemporal budgetary restriction and taking the level of environmental quality as given. From this maximization problem the well-known Keynes-Ramsey rule is obtained: $r = \rho + \sigma g_C$,where r is the market interest rate. Let us also assume that a great number of firms exist which produce a final homogeneous good under conditions of perfect competition. The firms maximize profits, and the first-order condition for a maximum establishes that $r = (1-\tau)B$, where τ stands for the rate of taxation on production. These two conditions allows us to establish the *asset market equilibrium condition* for a decentralized economy which defines the rate of growth of the market equilibrium as a function of the instrument of environmental policy[14]

$$g_M = \frac{1}{\sigma}\left[(1-\tau)B - \rho\right]. \tag{20}$$

Then the optimal value τ^* is $(A/K)_P/B$, since for this value (12) is equal to (20). In other words, for the optimal value the growth rate of the market equilibrium is equal to the growth rate of the efficient allocation. However, the implementation of this policy has a negative effect on the growth rate of the economy. Notice that for $\tau = 0$, g_M is greater than g_P since in this case the growth rate defined by (12) for the optimal value $(A/K)_P$ is lower than the growth rate defined by (20).

Next we show that the optimal tax does not only ensures that the growth rate corresponding to the equilibrium path is equal to the efficient growth rate but that the decentralized solution exactly reproduces the efficient allocation. If the public expenditure in abatement is A_P, the good market equilibrium condition establishes the following relationship among the initial values

$$C_{M0} = BK_0 - g_P K_0 - A_{P0} = C_{E0}, \tag{21}$$

[14] g_M stands for the sustainable balanced growth rate of market equilibrium.

so that the initial values for consumption, investment and abatement are the same for the two solutions. In this case, if the rate of growth is also the same we have that the levels and the growth rate of the variables are the same and the equilibrium path coincides with the efficient path. Finally, if we look at the budgetary balance resulting from this policy we have that $\bar{T} = \tau^* Y_P - A_P = [(A/K)_P/B]\, BK_P - A_P = 0$, which establishes that tax receipts are exactly sufficient to finance the public expenditure requires to stabilize emissions and achieve the level of environmental quality corresponding to the efficient outcome, so that the resulting public intervention is neutral from a budgetary perspective.

3.3 Environmental standards

In this last subsection we address the study of environmental policy based on emission control. If the government sets the level of environmental quality, using this variable as an instrument of environmental policy, we have that the A/K ratio is exogenously determined, and the firms support the cost of pollution abatement. Then the environmental quality function (3) defines a linear dependence between A and K, $A = \bar{E}^{1/\eta} K$, where $\bar{E}$ is the level of environmental quality set by the regulatory authority. In this case, the firm's profits are given by

$$\pi = BK - rK - A = BK - rK - \bar{E}^{1/\eta} K, \tag{22}$$

and the first order condition for the maximization of profits can be written as $r = B - \bar{E}^{1/\eta}$. From this condition and the Keynes-Ramsey rule we obtain the growth rate of the market equilibrium as a function of the instrument of environmental policy

$$g_M = \frac{1}{\sigma}\left(B - \rho - \bar{E}^{1/\eta}\right). \tag{23}$$

Then the optimal value $\bar{E}^*$ is $(A/K)_P^\eta$, since for this value (12) is equal to (23) and then $g_M = g_P$. In other words, for the optimal value the growth rate of the market equilibrium is equal to the growth rate of the efficient allocation. Again the implementation of this policy reduces the growth rate of the economy. Moreover, this optimal value guarantees that the decentralized solution exactly reproduces the efficient allocation. Using $\bar{E}^*$ we have that the initial value for abatement is $A_{M0} = (\bar{E}^*)^{1/\eta} K_0 = A_{P0}$, so that the equilibrium path coincides with the efficient path. In this case, $C_{M0} = BK_0 - g_M K_0 - A_{M0}$, that for $A_{M0} = A_{P0}$ and $g_M = g_P$ yields $C_{M0} = C_{P0}$ and then the level and the growth rate of variables in the market equilibrium are the same than in the efficient allocation. Finally, we would want to point out that, as was to be expected, we find a negative relationship between the market equilibrium growth rate and the environmental policy instrument. Thus, if the government sets the environmental quality in a suboptimal manner because of the policymakers lack sufficient information on environmental benefits from pollution reduction or/and because of political or social restrictions, the result of any tightening of environmental policy will have a *negative* effect on the growth rate of the economy, independently of the initial level of environmental standards.

4 The environment affects productivity

In this section we show that if the environment affects productivity, there exists a positive relationship between growth rate and environmental quality, provided that the level of environmental quality is not very high. This happens because the increment of production due to an improvement in environmental quality is greater than the increment in resources needed to reduce the pollution and stabilize the environmental quality. For this reason a more restrictive environmental policy could have positive effects on both environmental quality and growth rate.

4.1 The efficient path

To calculate the efficient solution we eliminate $E(t)$ by substitution, using the pollution function (3), which yields the Hamiltonian associated with this problem

$$\begin{aligned} H(K,\lambda,C,A,t) &= \frac{1}{1-\sigma}\left[C^{1-\sigma}\left(\frac{A}{K}\right)^{\eta\alpha(1-\sigma)}-1\right] \\ &\quad +\lambda\left[BK\left(\frac{A}{K}\right)^{\eta\beta}-C-A\right], \end{aligned} \tag{24}$$

and from the first-order conditions we obtain

$$C^{-\sigma}\left(\frac{A}{K}\right)^{\eta\alpha(1-\sigma)} = \lambda, \tag{25}$$

$$\eta\alpha\frac{C^{1-\sigma}}{K}\left(\frac{A}{K}\right)^{\eta\alpha(1-\sigma)-1} + \lambda\eta\beta B\left(\frac{A}{K}\right)^{\eta\beta-1} = \lambda. \tag{26}$$

Again, the valuation of resources on the margin must be the same. However, when the environment affects productivity, the marginal valuation of pollution abatement presents an additional term which measures the effect that pollution abatement has on production through its effect on environmental quality: $f_E E_A$. Thus, condition (26), in terms of the functions of the model, may be written as $U_E E_A + \lambda f_E E_A = \lambda$ or $(U_E E_A/U_C) + f_E E_A = 1$, where 1 is the opportunity cost of pollution abatement in terms of consumption.[15] This condition requires that $f_E E_A < 1$, which implies that

[15] Given the strict concavity of the environmental quality function and of the production function, the strict concavity of the utility function, $U(C, E(A, K)) = U(C, A, K)$, with respect to control variables guarantees the fulfillment of the second-order condition.

$$\eta\beta B\left(\frac{A}{K}\right)^{\eta\beta-1}<1. \tag{27}$$

The dynamic of the state variable is given by the Euler equation

$$\dot{\lambda}=\rho\lambda+\eta\alpha\frac{C^{1-\sigma}}{K}\left(\frac{A}{K}\right)^{\eta\alpha(1-\sigma)}-\lambda B\left(1-\eta\beta\right)\left(\frac{A}{K}\right)^{\eta\beta}. \tag{28}$$

As (25) is equal to (7), from (28) we obtain

$$B\left(1-\eta\beta\right)\left(\frac{A}{K}\right)^{\eta\beta}=\rho+\sigma g_C+\frac{\eta\alpha}{\lambda}\frac{C^{1-\sigma}}{K}\left(\frac{A}{K}\right)^{\eta\alpha(1-\sigma)}, \tag{29}$$

which requires that the net marginal product of capital must be equal to its opportunity cost along the path of sustainable balanced growth. If we compare this condition with the one which we obtained in the previous section (see (10)), we find that they differ only on the left-hand side. When the environment affects productivity, a variation in capital has two effects, one direct and positive for a given level of environmental quality, and the other indirect and negative as a consequence of the dependence of environmental quality on the capital stock. So that the net marginal productivity of capital is the sum of these two effects: $f_K+f_E E_K$, which for the functions in our model gives us the expression that appears on the left-hand side of (29).[16]

Using (25) and (26) to eliminate λ and C from the right-hand side of (29), we obtain the *asset market equilibrium condition*

$$g_C=\frac{1}{\sigma}\left[B\left(\frac{A}{K}\right)^{\eta\beta}-\rho-\frac{A}{K}\right]. \tag{30}$$

Finally, using the relationship between C and A defined by (25) and (26), and the capital stock dynamic restriction, we calculate the *goods market equilibrium condition*

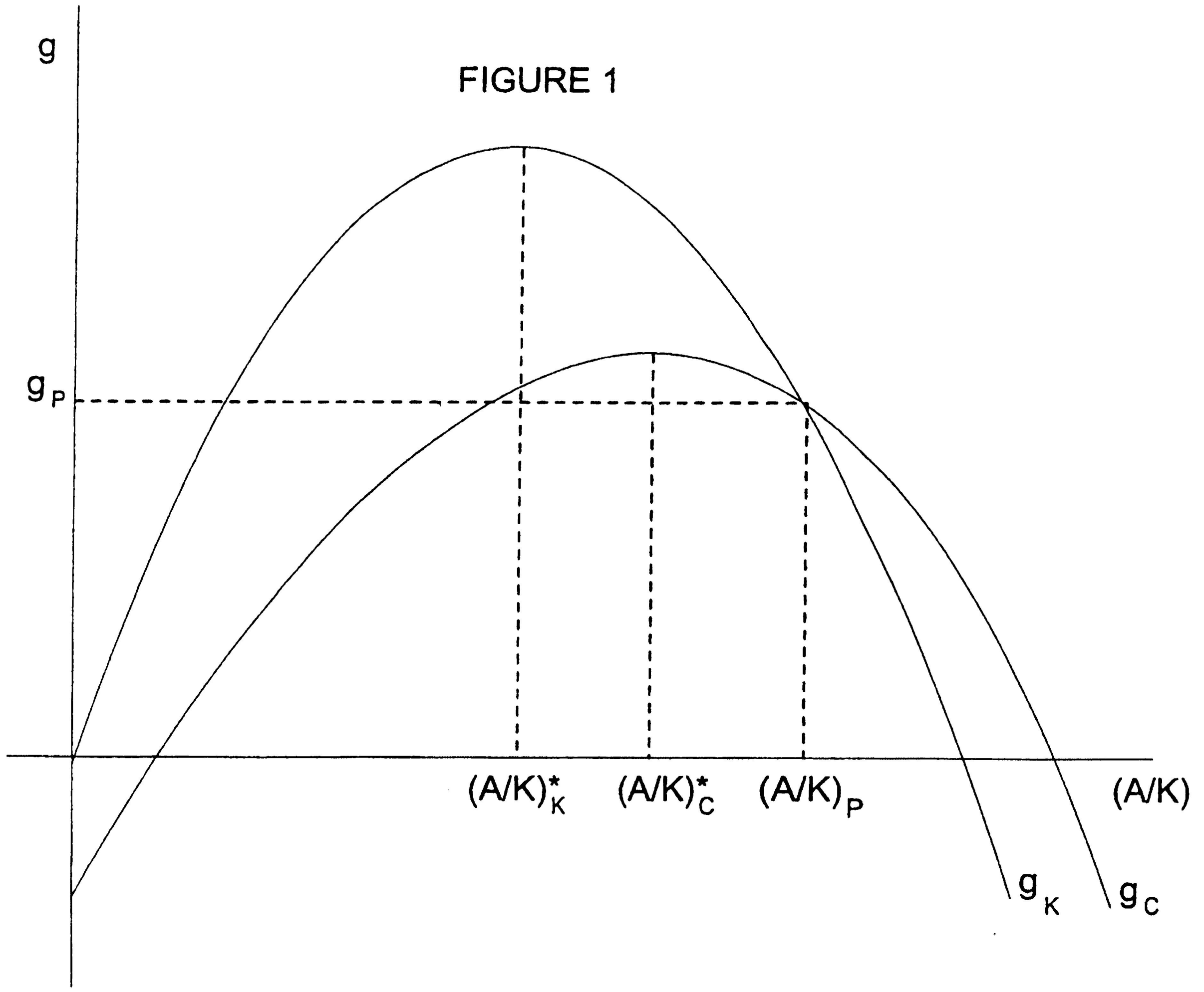
$$g_K=\frac{\alpha+\beta}{\alpha}B\left(\frac{A}{K}\right)^{\eta\beta}-\frac{1+\eta\alpha}{\eta\alpha}\frac{A}{K}. \tag{31}$$

For $g_C=g_K$, the system of equations (30) and (31) determines the growth rate and the level of environmental quality corresponding to the efficient path. Then making (30) equal to (31) we obtain the following equation for the level of environmental quality

$$-\frac{\alpha(\sigma-1)+\sigma\beta}{\alpha}B\left(\frac{A}{K}\right)_P^{\eta\beta}+\frac{\sigma+\eta\alpha(\sigma-1)}{\eta\alpha}\left(\frac{A}{K}\right)_P-\rho=0, \tag{32}$$

[16] Observe that for the assumptions of the model the net marginal productivity is positive, since we have established that $\eta<1$ and $\beta<1$, see (3) and (4).

which has an unique, positive solution if the second-order condition for the maximization of the Hamiltonian is satisfied.[17] Then, given $(A/K)_P$, a enough high value for the productivity parameter, B, guarantees that the rate of sustainable balanced growth is positive.[18] It is also easy to show, see also Appendix A, that (30) and (31) are two concave functions which intersect each other at a point with $0 > g'_C(A/K) > g'_K(A/K)$ for $(A/K)_P$, so that the solution to the system of equations (30) and (31) has the following graphical representation



Next we evaluate the effects of a variation of the willingness to pay for environmental quality on environmental quality and the growth rate of the economy. We will use expression (32) to study the effect on environmental quality. The result is that $\partial(A/K)_P/\partial\alpha > 0$, see Appendix B. The effect on growth rate is given by $\partial g_P/\partial\alpha = \partial g_P/\partial(A/K)_P \partial(A/K)_P/\partial\alpha$, which is negative because $\partial g_P/\partial(A/K)_P$ is negative at the intersection point of the functions g_C and g_K (see Figure 1). Hence, we have the same qualitative results as in Section 3, which confirms the *long-term conflict* existing between the preservation of nature and economic growth even though the positive influence of the environment on productivity is taken into

[17]The second-order condition hold when $\sigma > \alpha\eta/(1+\alpha\eta)$.
[18]This argument is developed in Appendix A.

account in the model. Finally, we find that an increase in the discount rate has a positive effect on the environmental quality and a negative effect on the growth rate of the economy.

4.2 The equilibrium path

When the environment also affects productivity, we find that, because of its nature as a public good, external effects on both consumption and production are present, as a result of which firms' decisions affect both consumer welfare and other firms' profits. In order to take these external effects on production into account we distinguish between an *internal effect* and an *external effect.* We represent the external effect, which the firms consider to be exogenously determined by E_e, and the internal effect, which depends on the firm's decisions by E.[19] In this case the firm's profits are given by

$$\pi = B\left(\frac{A}{K}\right)^{\eta\tilde{\beta}}\left(\frac{A}{K}\right)_e^{\eta(\beta-\tilde{\beta})} K - A - rK,$$

where $E = (A/K)^{\eta}$, $E_e = (A/K)_e^{\eta}$ and $\tilde{\beta} < \beta$.The first-order conditions for the maximization of profits are

$$B(1-\eta\tilde{\beta})\left(\frac{A}{K}\right)^{\eta\tilde{\beta}}\left(\frac{A}{K}\right)_e^{\eta(\beta-\tilde{\beta})} = r, \tag{33}$$

$$\eta\tilde{\beta}B\left(\frac{A}{K}\right)^{\eta\tilde{\beta}-1}\left(\frac{A}{K}\right)_e^{\eta(\beta-\tilde{\beta})} = 1. \tag{34}$$

The first condition establishes that the firm uses capital until its net marginal productivity equals the interest rate, and the second that the firm spends in pollution abatement until its marginal productivity is equal to its opportunity cost.

The market is in equilibrium when the value of E which maximizes profits coincides with the value that the firm consider to be exogenously determined, E_e, such that the expected and the present behavior of this variable are the same. Setting this condition, $E = E_e$, the previous conditions become

$$B(1-\eta\tilde{\beta})\left(\frac{A}{K}\right)^{\eta\beta} = r, \tag{35}$$

$$\eta\tilde{\beta}B\left(\frac{A}{K}\right)^{\eta\beta-1} = 1. \tag{36}$$

Taking the Keynes-Ramsey rule into account, we now obtain from (35) the *equilibrium condition in the asset market*

$$B(1-\eta\tilde{\beta})\left(\frac{A}{K}\right)^{\eta\beta} = \rho + \sigma g_C, \tag{37}$$

[19]This approach was used by Lucas (1998) to analyze the external effects of human capital on the technology of the economy. Here, we adapt this approach to represent the external effects of environmental quality.

which gives us the following expression for the growth rate of consumption

$$g_C = \frac{1}{\sigma}\left[B(1-\eta\tilde{\beta})\left(\frac{A}{K}\right)^{\eta\beta} - \rho\right]. \tag{38}$$

Furthermore, the *goods market equilibrium condition* can be expressed as

$$g_K = B\left(\frac{A}{K}\right)^{\eta\beta} - \frac{C}{K} - \frac{A}{K}, \tag{39}$$

and if we set $g_M = g_K = g_C$, the market equilibrium path is determined by conditions (36), (38) and (39). Specifically, (36) defines the level of environmental quality, (38) the balanced growth rate and (39) the consumption-capital ratio. Thus for the decentralized solution, the level of environmental quality can be obtained in explicit form, $(A/K)_M = (\eta\tilde{\beta}B)^{\frac{1}{1-\eta\beta}}$, and hence the values of the growth rate and the consumption-capital ratio.

If we continue by comparing the two growth paths, we obtain that the comparison between the levels of environmental quality is immediate. Observe that the efficient level is higher than the level $(A/K)^*_C$ defined by expression (50) in Appendix A, see Figure 1, and that this expression is higher than the environmental quality level associated with the equilibrium path, so that we get

$$\left(\frac{A}{K}\right)_M < \left(\frac{A}{K}\right)^*_C < \left(\frac{A}{K}\right)_P. \tag{40}$$

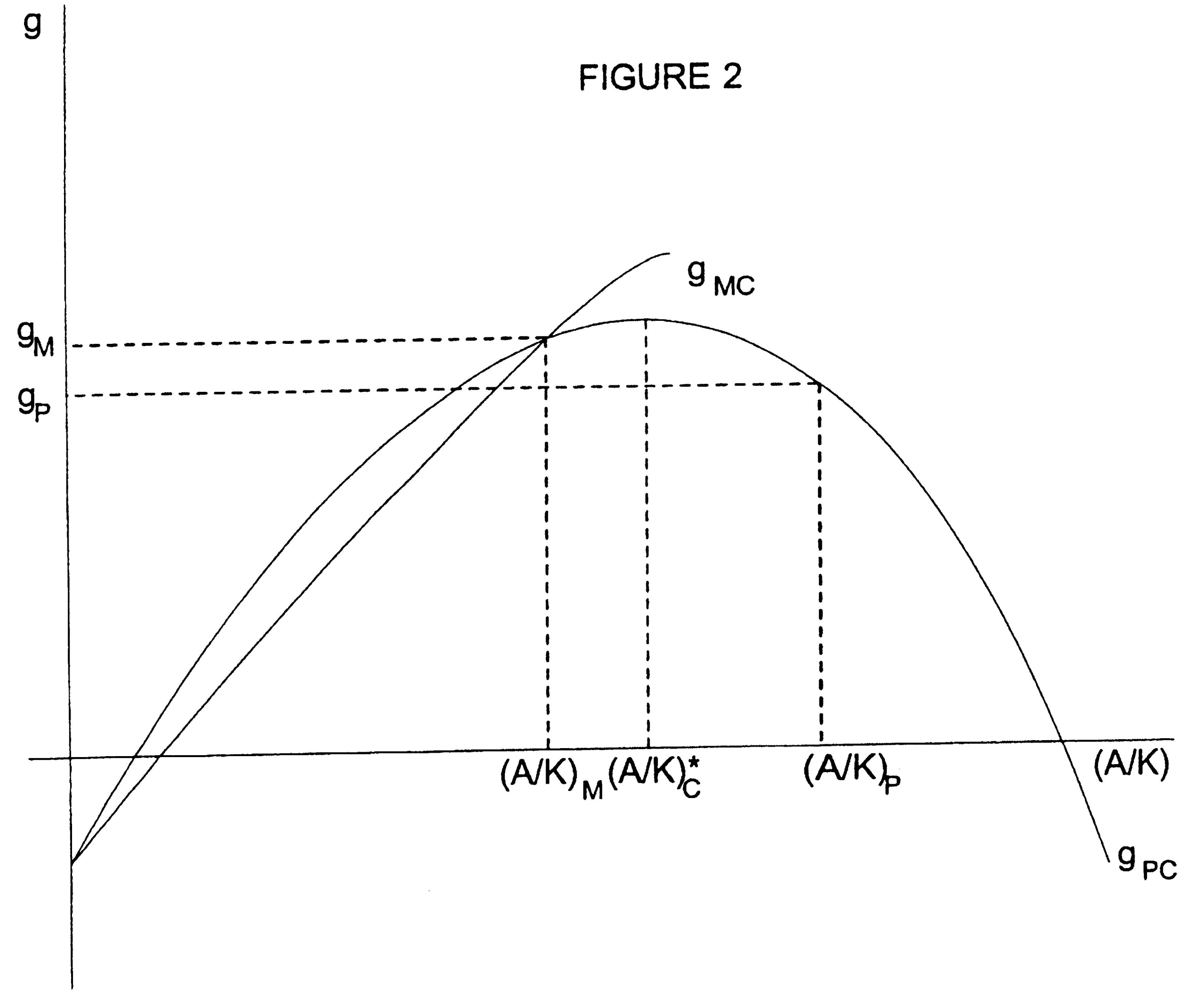
This difference is explained by the *external effects* of environmental quality. Given that pollution is a public bad, when the firms decide on pollution abatement they do not take into account the *positive* external effects of their decisions on consumer welfare and on the productivity of other firms. Thus, the positive effects of abatement are *undervalued* in the decentralized solution, as a result of which less resources are allocated to pollution abatement in comparison with the Pareto efficient allocation. Moreover, when the firms decide on investment, they do not take into account neither the *negative* external effects of their decisions so that the negative effects of capital are *undervalued* in the decentralized solution and the firms keep their capital stock above the efficient level in each moment of time. This explains why the level of pollution is higher for the equilibrium path.

To compare the growth rates of both solutions, we proceed to determine the relative position of the functions $g_C(A/K)$, which are given by expressions (30) and (38). To clarify our notation, we represent the function corresponding to the efficient solution by g_{PC} and that corresponding to the market solution by g_{MC}. If we subtract one function from the other, we obtain

$$g_{PC} - g_{MC} = -\frac{1}{\sigma}\left[\frac{A}{K}\left(1 - \eta\tilde{\beta}B\left(\frac{A}{K}\right)^{\eta\beta-1}\right)\right]. \tag{41}$$

This difference is zero for $A/K = 0$ and $(A/K)_M$, since for the environmental quality level corresponding to the equilibrium path, condition (36) is satisfied, so that the difference is positive for values of A/K lower than $(A/K)_M$ and negative for values greater than $(A/K)_M$. Given these relationships we obtain the following graphical representation

FIGURE 2



In the graph, the equilibrium growth rate is higher than the efficient growth rate. However, we cannot rule out, at least theoretically, a different relationship between these two growth rates, because the results only establish that the intersection point between the two functions must be on the increasing section of the function g_{PC}, which does not exclude that the equilibrium growth rate may be lower than the efficient rate. This will depend on the magnitude of the external effects in the production. In the model the magnitude of the external effects in the production is represented by $\tilde{\beta}$. If the value of this parameter is close to β the magnitude of the external effects is low and the environmental quality level of the equilibrium path is not far from $(A/K)^*_C$, so that we may expect that the market growth rate is above the efficient level. Compare again $(A/K)_M$ with $(\widetilde{A/K})^*_C$ defined by (50) in Appendix A. However, if the external effects are important, $\tilde{\beta}$ is close to zero, and both the environmental quality level and the growth rate are lower than the efficient levels.

4.3 Pigouvian taxes-subsidies

The previous analysis of the equilibrium and efficient paths allows us now to address the design of the environmental policy which lets the economy reach the efficient path as a market equilibrium. Thus, this subsection deals with illustrating how Pigouvian taxes can be used to promote efficiency, which moreover in this case means promoting environmental conservation.

Let us consider the following taxation scenario on firms: a proportional tax on production (τ_Y) combined with a proportional subsidy on pollution abatement (τ_A). In this case the firm's profits are given by

$$\pi = (1-\tau_Y)B\left(\frac{A}{K}\right)^{\eta\tilde{\beta}}\left(\frac{A}{K}\right)_e^{\eta(\beta-\tilde{\beta})}K - (1-\tau_A)A - rK - \bar{T}, \tag{42}$$

where $\bar{T}$ is a lump-sum tax or subsidy, which is exogenously determined by the government, equal to the budgetary balance resulting from the taxation scenario which we have just defined: $\bar{T} = \tau_Y Y - \tau_A A$. In this case, the first order conditions for the maximization of profits (35) and (36) can be written as

$$(1-\tau_Y)B(1-\eta\tilde{\beta})\left(\frac{A}{K}\right)^{\eta\beta} = r, \tag{43}$$

$$(1-\tau_Y)\eta\tilde{\beta}B\left(\frac{A}{K}\right)^{\eta\beta-1} = 1-\tau_A, \tag{44}$$

and the asset market equilibrium condition as

$$g_M(\tau_Y) = \frac{1}{\sigma}\left[(1-\tau_Y)B(1-\eta\tilde{\beta})\left(\frac{A}{K}\right)_M^{\eta\beta} - \rho\right]. \tag{45}$$

Then, we can use this condition to calculate the optimal value for τ_Y. Making $g_M = g_P$ and $(A/K)_M = (A/K)_P$ and using (30) we obtain the optimal value for the tax rate

$$\tau_Y^* = \frac{1}{B\left(1-\eta\tilde{\beta}\right)}\left(\frac{A}{K}\right)_P^{1-\eta\beta}\left[1-\eta\tilde{\beta}B\left(\frac{A}{K}\right)_P^{\eta\beta-1}\right], \tag{46}$$

which can be used in (44) to calculate the optimal value for the subsidy rate

$$\tau_A^* = \frac{1}{1-\eta\tilde{\beta}}\left[1-\eta\tilde{\beta}B\left(\frac{A}{K}\right)_P^{\eta\beta-1}\right]. \tag{47}$$

Observe that the system of equations to calculate the rates is completed with equation (32), which defines implicitly the environmental quality level corresponding to the efficient path. Moreover, it is easy to show that the optimal values for the tax and subsidy rates do not only ensures that the growth rate corresponding to the equilibrium path is socially optimal but that the decentralized solution exactly reproduces the efficient solution. The effect of this policy on growth depends on the magnitude of the external effects in the production, so that only if the environmental externalities in the production are enough high it may be expected that the optimal environmental taxation boosts growth.

In order to evaluate the budgetary balance resulting from this policy we take into account that

$$\tau_Y^* = \tau_A^* \frac{1}{B} \left(\frac{A}{K}\right)_P^{1-\eta\beta}.$$

Then we have that

$$\bar{T} = \tau_Y^* Y_P - \tau_A^* A_P = \tau_A^* \frac{1}{B} \left(\frac{A}{K}\right)_P^{1-\eta\beta} BK_P \left(\frac{A}{K}\right)_P^{\eta\beta} - \tau_A^* A_P = 0,$$

which establishes, as it happened in Section 3.2, that the environmental policy proposed is self-financing and it is not necessary to apply any lump-sum tax or subsidy.

4.4 Environmental standards

Finally, we study what the effects of an environmental policy based on direct control of pollution are. As occurred in Subsection 3.3, this kind of policy establishes a linear relationship between A and K, $A = \bar{E}^{1/\eta} K$, so that the firm's profits are given by

$$\pi = BK\bar{E}^{\beta} - rK - A = BK\bar{E}^{\beta} - rK - \bar{E}^{1/\eta} K, \tag{48}$$

and the first order condition for the maximization of profits can be written as $r = B\bar{E}^{\beta} - \bar{E}^{1/\eta}$. In this case, the equilibrium condition in the asset market defines the balanced growth rate of the economy

$$g_M(\bar{E}) = \frac{1}{\sigma} \left(B\overline{E}^{\beta} - \rho - \bar{E}^{\frac{1}{\eta}} \right), \tag{49}$$

and the optimal value $\bar{E}^*$ is $(A/K)_P^{\eta}$, since for this value the growth rate of the market equilibrium is equal to the growth rate of the efficient allocation. Moreover, this optimal values guarantee that the decentralized solution exactly reproduces the efficient allocation. Again the implementation of the optimal policy may have a positive effect but only if the environmental externalities in the production are enough high.

This function has the same properties of the function g_{PC} represented in Figure 2. Hence that this function presents a unique maximum associated with a positive growth rate, and between the critical values for which $g_M\left(\overline{E}\right) = 0$, the relationship between $\overline{E}$ and g_M is positive if $\overline{E}$ is lower than the maximum and negative when $\overline{E}$ is higher. Based on this relationship we find that a tighter sub-optimal environmental policy, starting from the emission level corresponding to the market equilibrium, has beneficial effects not only on environmental quality but also on growth rate, if this policy is not very restrictive, i.e. if small reductions in the emission level are proposed, even if the growth rate of the equilibrium market is higher than the efficient growth rate. In fact, we can define an interval of values for A/K, $((A/K)_M, (A/K)_C^*)$, such that any tightening of environmental policy inside this interval has a positive effect on growth.[20] See Figure 2. For these values the increment of production due to an improvement in environmental quality is greater than the increment in resources required to reduce emissions and stabilize the environmental quality so that a more restrictive environmental policy has positive effects on both environmental quality and growth rate. However, once the upper limit of the above interval is reached, the environmental policy does not have margin to promote growth.

5 Conclusions

In this paper we have developed a model of endogenous growth *a la Rebelo* in which environmental quality, which depends positively on pollution abatement and negatively on capital stock, has positive effects on the utility of consumers and on the productivity of capital. We have analyzed in the framework of this model the effect of different environmental policies on the growth of the economy.

Our results establish that greater concern of individuals for the environment reduces the long-term growth rate. This is so because, for the optimal values, the marginal effect of environmental quality on the productivity of capital is lower than the marginal effect of environmental quality on the opportunity cost of capital, so that an increase of environmental quality has a negative effect on the growth rate since the marginal cost of capital increases more than its marginal productivity. For this reason the growth rate must decrease in order to recover the equilibrium in the asset market when the willingness to pay for the environmental quality increases.

Furthermore, we show that the level of environmental quality associated with the market equilibrium path is below the efficient level, while the growth rate may be higher or lower depending on the extent of the external environmental effects in the production. In this case, a policy which taxes production and subsidizes pollution abatement would have a favorable effect on environmental quality, and could increase the growth rate of the economy if the external effects are important. In addition, we prove that this policy is *neutral* since it does not affect the budgetary balance of the government because tax receipts are equal to expenditures, for this reason it could be implemented without having to resort to lump-sum taxes/subsidies. Finally, we show that tighter standards have a positive effect on the growth rate only if previous standards were not very far from the level of environmental quality corresponding to the market equilibrium.

[20] Remember that environmental quality is a function of the A/K ratio.

A Relationship between $g_C(A/K)$ and $g_K(A/K)$

If we study functions (30) and (31) to determine whether the intersection point is associated to a positive growth rate, we find two concave functions which present a unique maximum for the following values

$$\left(\frac{A}{K}\right)_C^* = [\eta\beta B]^{\frac{1}{1-\eta\beta}}, \quad \left(\frac{A}{K}\right)_K^* = \left[\eta\beta B \frac{\alpha+\beta}{\alpha+\frac{1}{\eta}}\right]^{\frac{1}{1-\eta\beta}}, \tag{50}$$

which, keeping in mind that $\beta < 1/\eta$, yields $(A/K)_K^* < (A/K)_C^*$. Furthermore, we can establish that $g_C'(A/K)$ is negative at the intersection point. The derivative of this function is

$$g_C'\left(\frac{A}{K}\right) = \frac{1}{\sigma}\left[\eta\beta B\left(\frac{A}{K}\right)^{\eta\beta-1} - 1\right], \tag{51}$$

which is negative if condition (27) is fulfilled. Then, if the productivity parameter, B, is enough large, the maximum for both functions gives a positive value and the intersection point defines a positive value for the growth rate. Notice that for a given value of A/K, functions (30) and (31) are increasing with respect to B. Whether we go further in this point we can see that whichever it is the value for the productivity parameter, equation (32) has an unique, positive solution for $\sigma > \alpha\eta/(1+\alpha\eta)$. Then what happens when the productivity parameter changes is that both curves move simultaneously. It is easy to show that whether the parameter increases both curves and the intersection point move up and left if $\alpha\eta/(1+\alpha\eta) < \sigma < \alpha/(\alpha+\beta)$ or up and right if $\alpha/(\alpha+\beta) < \sigma$. Whether the parameter decreases they move down and right if $\alpha\eta/(1+\alpha\eta) < \sigma < \alpha/(\alpha+\beta)$ or down and left if $\alpha/(\alpha+\beta) < \sigma$. But in any case their relative position does not change and for this reason if the productivity parameter is enough high we have to expect a positive value for the rate growth.

We can also conclude that $g_C'(A/K) > g_K'(A/K)$ for $(A/K)_P$. Let us assume that $g_C'(A/K) \leq g_K'(A/K)$ then it is obtained that

$$\frac{\sigma+\eta\alpha(\sigma-1)}{\eta} \leq (\alpha(\sigma-1)+\sigma\beta)\,\eta\beta B\left(\frac{A}{K}\right)_P^{\eta\beta-1}. \tag{52}$$

On the other hand, we can rewrite (32) as

$$\frac{1}{\alpha}\left(\frac{A}{K}\right)_P\left[-(\alpha(\sigma-1)+\sigma\beta)\,B\left(\frac{A}{K}\right)_P^{\eta\beta-1} + \frac{\sigma+\eta\alpha(\sigma-1)}{\eta}\right] = \rho > 0,$$

which implies that

$$\frac{\sigma+\eta\alpha(\sigma-1)}{\eta} > (\alpha(\sigma-1)+\sigma\beta)\,B\left(\frac{A}{K}\right)_P^{\eta\beta-1}.$$

Then given that $\eta\beta < 1$, we obtain that

$$\frac{\sigma + \eta\alpha(\sigma - 1)}{\eta} > (\alpha(\sigma - 1) + \sigma\beta)\,\eta\beta B \left(\frac{A}{K}\right)_P^{\eta\beta - 1}. \qquad (53)$$

But this inequality defines a contradiction with respect to (52), so that we have to conclude that $g'_C\,(A/K) > g'_K\,(A/K)$ for $(A/K)_P$.

B The effects of greener preferences

Differentiating the left-hand side of equation (32), we obtain

$$\left[\frac{\sigma + \eta\alpha(\sigma - 1)}{\eta} - (\alpha(\sigma - 1) + \sigma\beta)\,\eta\beta B \left(\frac{A}{K}\right)_P^{\eta\beta - 1}\right] d\left(\frac{A}{K}\right)_P \qquad (54)$$

$$= \left[\rho - (\sigma - 1)\left(\frac{A}{K}\right)_P + (\sigma - 1)B\left(\frac{A}{K}\right)_P^{\eta\beta}\right] d\alpha.$$

The sign of the parenthesis of the left-hand side is *positive* given (53). To know the sign of the parenthesis of the right-hand side, we use again (32) which we rewrite as

$$\rho - (\sigma - 1)\left(\frac{A}{K}\right)_P + (\sigma - 1)B\left(\frac{A}{K}\right)_P^{\eta\beta} = \frac{\sigma}{\alpha}\left(\frac{A}{K}\right)_P\left[\frac{1}{\eta} - \beta B\left(\frac{A}{K}\right)_P^{\eta\beta - 1}\right], \qquad (55)$$

where the sign of the parenthesis of the right-hand side is *positive* as long as condition (27) must be satisfied for the optimal solution. The result is that the partial derivative of $(A/K)_P$ with respect to α is positive.

References

[1] Aghion, P. and Howitt, P. (1998), *Endogenous Growth Theory*, Cambridge, MA: MIT Press.

[2] Bovenberg, A.L. and de Mooij, R.A. (1997), "Environmental tax reform and endogenous growth", *Journal of Public Economics* **63**:207-237.

[3] Bovenberg, A. L. and Smulders, S.A. (1995), "Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model", *Journal of Public Economics* **57**:369-91.

[4] Bovenberg, A.L. and Smulders, S.A. (1996), "Transitional impacts of environmental policy in an endogenous growth model", *International Economic Review* **37**:861-93.

[5] d'Arge, R.C. (1971), "Essay on economic growth and environmental quality", *Swedish Journal of Economics* **73**:25-41.

[6] Elbasha, E.H. and Roe, T.L. (1996), "On endogenous growth: The implications of environmental externalities", *Journal of Environmental Economics and Management* **31**:240-68.

[7] Ewijk, C. van and Wijnbergen, S. van (1995), "Can abatement overcome the conflict between environment and economic growth?", *De Economist* **143**:197-216.

[8] Gradus, R. and Smulders, S.A. (1993), "The trade-off between environmental care and long-term growth: Pollution in three prototype growth models", *Journal of Economics* **58**:25-51.

[9] Grimaud, A. (1999), "Pollution permits and sustainable growth in a Schumpeterian model", *Journal of Environmental Economics and Management* **38**:249-66.

[10] Hettich, F. (1998), "Growth effects of a revenue-neutral environmental tax reform", *Journal of Economics* **67**:287-316.

[11] Huang, C-H. and Cai, D. (1994), "Constant-returns endogenous growth with pollution control", *Environmental and Resource Economics* **4**:383-400.

[12] Hung, T.Y.V., Chang, P. and Blackburn, K. (1994), "Endogenous growth, environment and R & D" in C. Carraro (ed.), *Trade, innovation and environment*, Dordrecht:Kluwer Academic Press.

[13] John, A. and Pecchenino, R. (1994), "An overlapping generations model of growth and the environment", *Economic Journal* **104**:1393-410.

[14] John, A. et al. (1995), "Short-lived agents and the long-lived environment", *Journal of Public Economics* **58**:127-41.

[15] King, R., Plosser, C. and Rebelo, S. (1988), "Production, growth and business cycles I: The basic neoclassical model", *Journal of Monetary Economics* **21**:195-232.

[16] Ligthart, J.E. and van der Ploeg, F. (1994), "Pollution, the cost of public funds and endogenous growth", *Economic Letters* **46**:351-61.

[17] Lucas, R.E. (1988), "On the mechanics of economic development", *Journal of Monetary Economics* **22**:3-42.

[18] Michel, P. and Rotillon, G. (1995), "Disutility of pollution and endogenous growth", *Environmental and Resource Economics* **6**:279-300.

[19] Mohtadi, H. (1996), "Environment, growth, and optimal policy design", *Journal of Public Economics* **63**:119-40.

[20] Rosendahl, K.E. (1996), "Does improved environmental policy enhance economic growth?", *Environmental and Resource Economics* **9**:341-64.

[21] Rubio, S.J. and Fisher, A.C. (1994), "Optimal capital accumulation and stock pollution: The greenhouse effect", *Revista Española de Economía,* Monográfico: Recursos Naturales y Medio Ambiente, pp. 119-40.

[22] Smulders, S. (1999), "Endogenous growth theory and the environment" in Jeroen C.J.M. van der Bergh (ed.), *Handbook of Environmental and Resource Economics*, Cheltenham: Edward Elgar, Chapter 42.

[23] Smulders, S. and Gradus, R. (1996), "Pollution abatement and long-term growth", *European Journal of Political Economy* **12**:505-32.

[24] Stokey, N.L. (1998), "Are there limits to growth?", *International Economic Review* **39**:1-31.