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SURVEY PAPER

A survey of Spanish research in mathematics education

Marianna Bosch[1] · Angel Gutierrez[2] · Salvador Llinares[3]

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Abstract

This survey paper presents recent relevant research in mathematics education produced in Spain, which allows the identification of different broad lines of research developed by Spanish groups of scholars. First, we present and describe studies whose research objectives are related to student learning of specific curricular contents and process-oriented competencies, namely arithmetic, algebra, geometry, functions and calculus, probability and statistics, and argumentation or proof in geometric contexts. Next, we present characteristics and foci of investigations dealing with different aspects of mathematics teacher education, encompassing a large part of Spanish research in mathematics education. The descriptions of other transversal lines of research complement the previous two big blocks: research on students with special educational needs and the effects of using technology in different curricular contents and educational levels. Finally, we report on the research activities and advances of Spanish research in mathematics education from two main theoretical frameworks created or developed by Spanish researchers. This plurality of research strands also corresponds to a wide range of international collaborations, especially with Latin American colleagues.

Keywords Spanish education · Teaching and learning of mathematics · Mathematics teacher education · Students with special needs · Technology for teaching and learning · Theories of mathematics education

1 Introduction

Mathematics education is a dynamic area of research in Spain. Spanish researchers work in different national and international research teams and use a variety of approaches to focus on the teaching and learning of mathematics in various contexts and at all education levels, from kindergarten to university. This research community has grown steadily over the past 45 years, especially during the last two decades. The research field is dynamic, and involves different groups using a variety of methodologies to focus on the teaching and learning of mathematics in different contexts. This paper presents an overview of such research in recent years, identifying lines, foci, or trends that are being developed, both locally and with international collaborations.

A milestone in the development of Spanish research in mathematics education was the Spanish law "for the Reform of Universities" published in 1982. It introduced *Didáctica de la Matemática* as one of the "areas of knowledge" that structured university departments and research: it was key to the strengthening of mathematics education as a scientific and academic discipline in Spain, with the consequent institutional support for research in this area and the possibility of developing doctoral programs and creating research groups. The law also consolidated the integration of teacher education in universities.

In the late 1980s, the universities of Valencia, Granada, and the Autonomous University of Barcelona launched the first doctoral programs in mathematics education, followed by other universities a few years later. To run those programs, the universities established fruitful relationships with renowned international researchers and institutions to offer doctoral courses and mentoring. The doctoral students were mostly teacher educators, secondary school teachers, some recent graduates in mathematics, and teachers from

✉ Angel Gutierrez
angel.gutierrez@uv.es

Marianna Bosch
marianna.bosch@ub.edu

Salvador Llinares
sllinares@ua.es

1 University of Barcelona, Barcelona, Spain

2 University of Valencia, Valencia, Spain

3 University of Alicante, Alicante, Spain

Latin American universities, who did not have the opportunity to pursue a doctorate in mathematics education in their countries. The Spanish research community has currently achieved maturity, with more than 300 doctoral dissertations completed in 2010–2019, compared to about 100 in 2000–2009.

The survey presented here, which does not claim to be exhaustive, has been based mainly on literature published between 2015 and 2024 in leading national and international journals and books. The result is a walk through a plurality of research lines developed by teams of different sizes and territorial distributions. They are presented in this survey paper organized according to their research focus: learning school contents (Sect. 2), teacher education (Sect. 3), students with special needs (Sect. 4), the use of technology in the teaching and learning of mathematics (Sect. 5), and contributions to theoretical frameworks (Sect. 6). The papers in this Special Issue are representative examples of the diverse Spanish research activity in mathematics education in recent years and of its international collaborations.

2 Research focused on the teaching and learning of mathematical contents

One of the main research agendas in mathematics education is devoted to analyzing the learning processes of different curricular contents by students of all education levels, from kindergarten to university, including future teachers. This section focuses on future teachers as learners of mathematics, with the professional aspects of future teachers being considered in Sect. 3. Section 2 is organized based on the typical curricular topics (arithmetic, geometry, algebra, etc.).

2.1 Arithmetic thinking

The origins of modern Spanish research on arithmetic thinking at different education levels and in teacher training date back to the early 1980s and it remains active today. The research objectives are diverse and mainly focus on primary and lower secondary students. One of the major topics is the problem of learning the different sets of numbers, from whole to complex numbers. In particular, the understanding of rational numbers and their representations (mainly, fractions) is one of the most important obstacles for primary and secondary school students (González-Forte et al., 2023).

Many researchers have focused on additive operations in verbal problem-solving, while others have paid attention to aspects of the multiplicative structure. In these problems, the choice of an appropriate operation is a permanent difficulty for students, increased when solving problems with fractions (Zorrilla et al., 2024). These results allowed defining hypothetical learning trajectories suitable for different types of numbers and school years.

A line of research with a long tradition in Spain is dedicated to the analysis of historical mathematics textbooks (e.g., Ruiz-Catalán et al., 2024) and to the comparison of textbooks based on different national curricula, to evidence the changes in school-mathematical contents. In a similar direction, the comparison of textbooks from different countries also showed interesting results. For instance, a comparison of the arithmetic problems included in primary school textbooks from Singapore and Spain concluded that the differences found cannot explain the different success of the countries in international assessment (Vicente et al., 2022).

2.2 Algebraic and functional (or variational) thinking

Algebra and functions are the two most important content blocks in secondary school curricula around the world, particularly in Spain. Research on the learning of these areas is very diverse, ranging from algebraic thinking of early primary school students to content learning in high school, university degrees, and teacher training courses. Kieran (2022) presented a synthesis of international mathematics education research in this field, showing a diversity of research objectives that can also be traced in Spanish research.

Functional thinking is considered a component of algebraic thinking that focuses on the relationships between related variables, as a precursor of studying functions and calculus (Ramírez et al., 2022). Students are led to deal with situations involving the covariation of two or more interrelated variable quantities, including the interpretation of the situations in mathematical terms and the identification of relevant covariables (Ramírez et al., 2022). In the Spanish curriculum, the study of functions starts in the first years of secondary school, when students have already learned the basics of algebra. However, research has shown that primary school students, even in early grades, could identify and represent functional relationships: Morales et al. (2018) designed an experiment with grade 1 students to identify their strategies to solve problems involving functional relationships, with the correspondence being the most frequently used relationship.

This Special Issue includes three papers showing the diversity of approaches to the teaching and learning of calculus, focused on the functional thinking of primary school students (Cañadas et al., 2024), on pre-calculus students in high school (Santos-Trigo et al., 2024), and on mathematics graduate students (Trigueros et al., 2024).

Learning to use letters as unknowns and variables, other symbols, and algebraic expressions is an important difficulty for students. Several researchers have therefore focused on the algebraic and functional thinking of primary school students before they begin learning algebra. Their results showed a great diversity of students' behaviors, depending on their grades and mathematical capacities (Callejo et al., 2016; Pinto et al., 2022; Arbona, 2024). Two related, but somehow opposite, strategies have been tried: problem-solving and problem-posing.

On the one hand, solving specific types of problems, in particular geometric pattern problems, has proved to be successful in introducing students to the generalization of numerical relationships, writing verbal and then synthetic expressions of their generalizations, and, finally, learning to write algebraic expressions where letters and other symbols have been endowed with their algebraic meanings (Gutiérrez et al., 2018a). Several longitudinal studies on algebraic (Arbona, 2024) and functional thinking (Callejo et al., 2016) have been conducted in primary schools. Their results showed that students could learn to generalize and verbally describe the variables present in problems and give meaning to the letters used to write algebraic representations, increasing their ability as they progressed through the grades. The studies also reported on the different strategies used by the students and compared the presence of each strategy in the different grades. These studies allowed their authors to draw conclusions about differences in algebraic or functional reasoning among primary school students and suggest a learning trajectory throughout primary school. The same question, description of changes in students' understanding and learning of algebra throughout the years, has been approached from the framework of the ontosemiotic approach (OSA), to define levels and sub-levels of algebrization, allowing a microscopic view of students' reasoning when they generalize, algebraically represent their generalizations, and calculate solutions (Burgos et al., 2024).

Most Spanish textbooks present algebraic expressions from the very beginning of the first lessons on algebra, expecting that students will be able to give meaning to them and to the letters (usually x, y, z) by themselves. However, research shows that providing students with meaningful contexts helps them understand the algebraic meaning and use of letters and other symbols. Some authors used geometric pattern problems as contexts for teaching algebraic or functional thinking. Asking for verbal expressions for the generalizations produced, for shortened verbal-symbolic expressions, and, finally, for meaningful algebraic expressions (e.g., writing *m* for "minute") can guide students, helping both letters and operations to become meaningful to them (Gutiérrez et al., 2018a).

Apart from the interpretation of verbal statements and algebraic expressions, the ways students represent them are also clues to understanding their ways of reasoning, conceptions, and errors. Students in different primary grades can develop different ways of representing general algebraic or functional relationships by themselves. Based on grade 3 students, some researchers concluded that the types of representations of functional relationships naturally used were most often numerical (specific values) and verbal (specific and general relationships). However, some students were able to use letters as undetermined values (Pinto et al., 2022). Older students were able to solve problems involving quadratic functions and a wider diversity of representations, since they most often used numerical and verbal representations, but also tables, manipulatives, pictures, and symbolic representations (Ramírez et al., 2022). Cañadas et al. (2024) presented a study situated within the framework of functional thinking; they explored the abilities of Spanish students in primary school grades 3 and 4 to draw and read Cartesian graphs of linear functions.

On the other hand, problem-posing environments can help secondary school students give meaning to algebraic expressions and give clues to teachers and researchers to identify their difficulties in doing so (Cañadas et al., 2018). By asking students to invent (pose) verbal problems to be solved with given equations, the syntactic and semantic structures of the problems they posed informed about their diverse conceptions regarding the meanings given to unknowns, additive or multiplicative interpretations of relationships, presence of unknowns, relationships between coefficient and unknown, etc. The analysis of the data presented by these authors showed that most of the students had difficulties posing correct problems and that they were more able to give meaning to additive equations than to multiplicative or combined ones. Students' conceptions of algebraic and functional concepts evolve over the years, as can be noted when comparing the studies by Cañadas et al. (2018), who found partially different results.

With regard to learning the usage of variables and other symbols, the translation of verbal problems into algebraic expressions is another relevant teaching step that researchers have used to observe students' understanding of algebraic concepts. A source of difficulties for students is the identification of the algebraic relationships hidden in a verbal statement. Research has shown that additive, multiplicative, and combined relationships had quite different complexities for students to identify them in verbal statements or to convert algebraic expressions into verbal statements (Cañadas et al., 2018). The types of errors found were related to the completeness of students' verbal statements, the arithmetic structure of statements and algebraic expressions, and the use of symbols. A significant conclusion of those studies was that most errors were related to students' lack of knowledge of arithmetic facts and wrong interpretations of arithmetic operations. Researchers have also investigated students' behavior in specific kinds of problems frequently found in Spanish textbooks, such as problems of ages and problems of ratios (Soneira et al., 2023). Research also showed that errors in the correspondence between verbal and algebraic expressions were pervasive in secondary school and were also quite frequent in pre-service primary school teachers.

The processes of secondary students' learning of limits, derivatives, and integrals have also been analyzed. Some Spanish researchers were interested in analyzing the learning of interpolation, limits, and derivatives in upper secondary school. Berciano et al. (2015) presented an empirical study aimed at evaluating high school students' abilities for graphical and algebraic interpolation and extrapolation of functions, and identified possible differences between students' performance and difficulties in graphical and algebraic contexts. Concerning limits, Fernández Plaza et al. (2015) studied high school students' strategies to calculate the finite limit of a function at a point, showing students' limitations when functions were presented graphically and they could not apply the calculation strategies proposed by textbooks. In a similar study, Vargas et al. (2020) analyzed the tasks posed in five grade 11 Spanish textbooks, in which they identified three implicit meanings of derivatives. From a more theoretical perspective, Santos-Trigo et al. (2024) identify strengths of students' systematic use of dynamic geometric systems (DGS) in their learning of the concept of derivative.

The APOS (Action, Process, Object, and Schema) theory, introduced by Ed Dubinsky in the 1980s, has fruitfully been used internationally to analyze students' conceptions of calculus concepts. Based on undergraduate students, (Fuentealba et al., 2019) described the evolution of the students' APOS Schemas of derivative, when they had to use global and punctual data to relate graphical and algebraic representations of functions and their first and second derivatives. Trigueros et al. (2024) present results of a study based on the work of several post-graduate master students tackling complex problems where they had to establish relationships between derivatives. From a different perspective, Orts et al. (2018) identified three learning trajectories in high school students related to their ways of linking graphical and analytical representations and understanding the variation of the slope of the tangent in a neighborhood of a point.

Research carried out in the framework of the Anthropological Theory of the Didactic (ATD) has also explored the teaching and learning of algebra, functions, and elementary calculus in secondary school and the transition to the tertiary level. Researchers first elaborated a model of the school conceptions of the mathematical contents to be taught, based on textbooks and classroom activities. They then compared this model to their own reference model, to identify didactic phenomena that could explain students' (and teachers') difficulties in learning (and teaching) those contents. Finally, they implemented empirical instructional processes to evidence the phenomena under study and make the reference model evolve (Gascón, 2024).

2.3 Geometric thinking, visualization, and argumentation/proof in geometric contexts

Teaching and learning mathematical proof can take place in the context of any area of mathematics. However, the visual and manipulative characteristics of geometry differentiate it from other mathematical areasand make it more suitable when students start learning proof and proving. One characteristic is the help provided by the visual support to identify properties, organize arguments, state conjectures, and express their proofs. This connection between visualization and proof is at the heart of the dialectic between diagrams and figures and between the tangible real and the ideal (Laborde, 2005). Blanco et al. (2024a) report on Spanish mathematics education research focused on exploring the connection between visualization and argumentation and how the former helps students develop their argumentation skills.

The development of kindergarten and primary school students' geometric reasoning has received attention from mathematics education researchers for many decades (e.g., the seminal works by Pierre and Dina Van Hiele in the 1950s). Alsina et al. (2021) analyzed the arguments produced by 5–6 year-old kindergarten children, concluding that they could produce short simple arguments aimed mostly at explaining their actions, but rarely at refuting others' arguments. In primary school, attention has been paid to typical geometric concepts like polygon and types of quadrilaterals and triangles. Researchers have concluded that grade 3 students, when the teaching units offered adequate learning opportunities, could induce definitions of geometric figures (e.g., triangles and quadrilaterals) and develop sophisticated classifications, with different levels of structuration showed by the more and less able students (Bernabeu et al., 2021).

Students are typically first introduced to (informal) mathematical proofs in the intermediate grades of secondary school. Most studies about learning to prove have therefore been based on the outcomes of those students. Based on different theoretical frameworks and diverse types of experiments, several Spanish studies have informed on aspects and characteristics of the arguments and proofs produced by students. There is an important moment when students move from experimental justifications based on specific geometric figures to deductive proofs (Saorín et al., 2019). Students also exhibit different reasoning styles, types of representations, and levels of generality (Ramírez-Uclés & Ruiz-Hidalgo, 2022). Finally, adequate teaching units, based on proof problem-solving, can induce the evolution of students' styles of proving (Fiallo & Gutiérrez, 2017) and evidence the different students' proving abilities (Llinares & Clemente, 2019).

Both the mathematical knowledge of teachers and their ways of reasoning influence their teaching and their pupils' learning. The styles of reasoning of future teachers have therefore been assessed. Manero and Arnal-Bailera (2021) showed that a significant part of the future secondary school teachers observed had a Van Hiele level of proving lower than the expected level of some high school students. It should be known that, in Spain, future secondary school teachers must be graduated in mathematics, in another scientific degree, or prove that they have the equivalent mathematical knowledge.

2.4 Stochastic sense

Stochastic sense refers to the reasoning necessary to deepen the knowledge and daily use of two blocks of school content: probability and statistics. For many decades, there has been live research on teaching and learning probability in Spain and, more recently, on statistics in primary, secondary, and teacher training levels. Batanero and Álvarez-Arroyo (2024) presented surveys of recent international literature, including publications of Spanish authorship.

Experimental research has offered proposals for primary school teachers to organize, and assess their classes, based on several dimensions (Vásquez Ortiz et al., 2020). Probability and proportionality are narrowly linked. Some experiments showed that students in grades 6 and 10 used proportional reasoning differently when they solved arithmetic and probabilistic problems. A reason for that difference seemed to be the different complexity of the problems of each type (Hernández-Solís et al., 2023). The understanding and learning of different statistical concepts by students in primary and secondary school have also been analyzed, like sample proportion (Begué et al., 2018), sample distributions (Batanero et al., 2020), and statistical parameters (Begué et al., 2018). Other researchers proposed levels of reading and semiotic complexity of statistical graphs and diagrams (Arteaga et al., 2021) and analyzed the types of problems requiring statistical tables posed in textbooks of primary (Gea et al., 2022) and secondary education (Pallauta et al., 2023).

3 Research on mathematics teacher education

Two ideas allow us to categorize Spanish research focused on mathematics teacher education. On the one hand, the idea of specific professional practices, emphasizing how student teachers learn to reason about mathematics teaching as a feature of teaching competence (Blömeke et al., 2017; Grossman, 2018). On the other hand, we can consider different reflections on the idea of *specialized knowledge needed to teach mathematics* (Ball et al., 2008; Lin & Rowland, 2016) from the Spanish context (Carrillo et al., 2018; Godino, 2024).

3.1 Learning specific professional practices

One trend in Spanish research is articulated around prospective teachers' learning of different practices articulating the teaching of mathematics. The practice of teaching mathematics is conceived as a conglomerate of specific professional practices. The analysis of learning of these practices provides information about how prospective teachers in initial training or teachers in professional development courses learn to reason about the teaching practice.

One specific professional practice studied is the design of tasks, teaching plans, and modification of mathematical problems. There are different foci represented in this Special Issue. One is about how prospective early childhood teachers learn to design mathematical tasks to promote mathematical processes (Alsina et al., 2024). Another one approaches the way secondary school teachers analyze, design, and implement lessons about modeling while learning on the collective identification of constraints affecting the teaching of modeling (Barquero et al., 2024). Finally, the focus is put on how resources, goals and orientations influence teachers' decisions–making a lesson plan (Lupiáñez et al., 2024).

Other studies that have produced relevant findings are focused on prospective teachers' noticing of students' mathematical thinking (Fernández et al., 2018). These studies are complemented with others about how prospective teachers propose new instructional tasks considering the interpretation of students' mathematical thinking (Ivars et al., 2020; Montero et al., 2020; Moreno et al., 2021). The findings of these studies provide features about how prospective teachers use theoretical information to reason on the teaching (Buforn et al., 2022; Ivars et al., 2018) and about development levels and pathways of noticing (Fernández et al., 2024). Furthermore, these studies indicate some relationships among cognitive skills articulating the noticing (Sánchez-Matamoros, 2019).

Characterizing how prospective teachers interpret the teaching of other teachers and reflecting on one's practice is the focus of other studies which have allowed identifying mechanisms activating specialized knowledge during post-analysis lessons (Garcia et al., 2024; Morales-López et al., 2024). Some of these studies identify development levels of the didactical suitability assessment competence (Font et al., 2024) and how teachers notice the linguistic-mathematical practices in the classroom (Planas et al., 2024).

Learning these specific professional practices is considered central to become a mathematics teacher. The relevance of the role played by the specific professional practices in mathematics teaching shifts the attention towards finding ways to favor their development in training programs

(Grossman, 2018). The idea of "decomposing the practice of teaching mathematics" into specific professional practices allows generating language and categories to discuss and analyze the teaching of mathematics, making visible to prospective teachers the implicit features of such practices. It is a way to make visible relevant aspects of the teaching practice to those who want to become teachers. The training proposals derived from this perspective respond to practice-based teacher education and are the research context for studying prospective teachers' learning of ways of reasoning about mathematics teaching. These formative processes underline a structure of activities that aims to support the prospective teachers' reasoning. The formative interventions introduce theoretical information from mathematics education, such as guides to focus on what to observe and how to define lines of action in the practice (Climent et al., 2024, Fernández et al., 2018).

3.2 Features of specialized knowledge of the mathematics teacher

From the notion of Mathematics Knowledge for Teaching (Ball et al., 2008), some Spanish research groups have made further elaborations (Carrillo et al., 2018, Pino-Fan et al., 2018; Scheiner et al., 2019). Since the seminal works of Shulman (1986), there has been an important development of theoretical models of mathematics teacher specialized knowledge that underline the idea of teaching mathematics as a profession (Schwarz & Kaiser, 2019). The importance of these approaches lies in placing the discipline to be taught at the center, and, more particularly, making the researchers aware of what mathematics knowledge for teaching represents for the understanding of the discipline, in this case mathematics. Characteristics of how prospective teachers know the mathematics necessary to teach have been generated (Alvarez-Arroyo et al., 2024; Begué et al., 2023; Buforn et al., 2022; Climent et al., 2024; Toscano et al., 2024). Later developments showed that teachers' beliefs about content influenced how they taught. In the case of mathematics, some models introduced the notion of beliefs about mathematics and its teaching and learning permeating the teacher's knowledge (Carrillo et al., 2018; Godino, 2024). For example, through the didactical suitability criteria for mediational and affective facets being used to examine the preservice teachers' beliefs (Fernández-López et al., 2024).

We can identify different uses of the models of knowledge (MTSK or the model of teacher knowledge from the Onto-semiotic Approach) to design tasks in training programs that favor the integrated learning of different knowledge domains and develop key competencies for the mathematics teacher's practice (Climent et al., 2024; Font et al., 2024; Pino-Fan et al., 2023). Furthermore, these models open the possibility of being complemented with other theoretical models (such as Mathematical Workspaces) to consider the way student teachers can become aware of the relationships between the affective and epistemological dimensions as an important aspect of the specialized knowledge of the teacher (Gómez-Chacón et al., 2024).

3.3 Features of knowing and doing in mathematics teacher education

An explicit product of Spanish research on mathematics teacher education is that it allows thinking about the research that underpins a particular way of understanding mathematics teacher education. This way of understanding is reflected in identifying specific professional practices that become the focus of attention in teacher education programs, and in the role of theoretical references in the researchers' decisions. A common characteristic, independent of the theoretical approach adopted, the type of participants, or the particular context, is to use "representations of practice" to focus on prospective teachers' ways of reasoning. Another one is to support the processes of reflection/analysis on what has been observed from theoretical knowledge of teaching and learning mathematics. Furthermore, the analysis of how prospective teachers learn to carry out certain specific professional practices suggests that this learning supports the learning of specialized knowledge of teachers, showing the symbiosis between doing and knowing. Furthermore, one additional idea can be inferred from Spanish research on mathematics teacher education is that the contributions of different theoretical perspectives illustrate the symbiosis between knowing and doing (Climent et al., 2024; Fernández et al., 2020; García et al., 2024).

4 Research on the cognitive processes of students with special educational needs

Students with difficulties for learning mathematics have ever been a focus of interest for many researchers in mathematics education. In the past few decades, there has also been a significant amount of research on mathematically gifted students. Recent legislative changes in Spain and more pressure from parents have led to increased recognition by teachers of the need to identify and attend to students with high abilities or giftedness, particularly those with mathematical giftedness. In mathematics education, the main objectives of the international research agenda regarding students with special needs include: actions for the identification of students with mathematics learning difficulties and mathematically gifted students; the design of differentiated teaching in school and out-of-school contexts; and the study of these students' mathematical reasoning and affectivity compared to those of average students. Since the 1980s, mathematics education research on students with special needs has been conducted in Spain (e.g., Soto & Gómez, 1987), but it has been scarce. However, in the past decade, several research groups have been formed to study this issue and have produced interesting results, integrated into international research activity.

The research objectives regarding students with special needs are highly diverse, not only because they include both difficulties and giftedness, but also because the area of learning difficulties comprises a wide variety of causes. This Special Issue includes two papers presenting research results on this topic: one analyzes the learning of arithmetic by a student with autism spectrum disorder (ASD) (Bruno et al., 2024) and the other analyzes the capacity of generalization of mathematically gifted students (Mora et al., 2024b).

4.1 Students with learning difficulties

The efforts of Spanish researchers in the last decades have mainly been directed at students with Down syndrome (DS) and ASD, although other difficulties, like dyscalculia, have occasionally been addressed. The Spanish research on students with ASD has, in some way, paralleled the research on students with DS, since mathematical contents, teaching aims, and research objectives and methodologies are similar. However, the different characteristics of students with DS and ASD make the practice of research very different.

Students with DS have different styles of reasoning depending on how severe the syndrome is, but some common traits have been identified in their memory, capacities for abstraction or deduction, interpretation and processing of information, and impulsiveness when trying to solve tasks. In this context, several studies have informed on students' behavior when solving different types of tasks, like counting, grouping, partitioning, and establishing relationships between whole numbers (Noda & Bruno, 2017) or subitizing during initial learning of numbers (Tuset et al., 2019).

The teaching of students with ASD has often been based on arithmetic topics such as counting, additive or multiplicative calculations, and verbal problem-solving. Analyses of teaching experiments showed that, when students were motivated, they tended to make progress and succeeded in learning the content objectives. Research results also showed the importance of the teaching methodology (Goñi-Cervera et al., 2024). Other teaching experiments carried out by the same team of researchers with older students also produced positive results in verbal additive (Bruno et al., 2021) or multiplicative problem-solving (Bruno et al., 2024; García-Moya et al., 2024).

Research on students with learning difficulties has also considered the case of generalization of geometric patterns (Goñi-Cervera et al., 2022) and the use of manipulatives. In this respect, it is also important to consider students' fine motor difficulties, which may be compensated with technological tools, like touchscreen software (González et al., 2015). Some researchers have designed apps specifically bearing in mind the characteristics of students with SD (González et al., 2015) or ASD (Blanco et al., 2024b).

Dyscalculia is a specific mathematical disorder mainly consisting of difficulties in using numbers when counting and doing simple arithmetic calculations. Research on this topic tried to differentiate the difficulties experienced by students with dyscalculia in different activities, like understanding verbal statements, handling arithmetic symbols, or applying the calculation rules (Castro, & Cañadas, 2016).

4.2 Mathematically gifted students

Analyzing the mathematical reasoning of mathematically gifted students in specific activities such as visualization, creativity, generalization, or proof, among others, is a focus of attention of mathematics education research. Sometimes, mathematically gifted students start using certain kinds of reasoning (e.g., deductive reasoning) much earlier than their grade or age peers. Other times, they follow the same reasoning processes as their peers, but do so in more flexible, complex, or original ways.

Problem-solving is the most frequently used context to identify and encourage mathematically gifted students. Moreover, these students demand challenging problems. An international line of research focuses on the design of teaching units based on problem-solving and sets of problems to train students in problem-solving strategies. The problems are sometimes based on curricular contents, like geometry (Ramírez-Uclés & Ruiz-Hidalgo, 2022), graphs (Blanco & García-Moya, 2021), or generalization of number sequences (Mora et al., 2022).

Despite the predominance of problem-solving, researchers recently showed that problem-posing is just as interesting to identify mathematically gifted students, by comparing the statements invented by potential mathematically gifted and average students (Espinoza et al., 2016).

ESTALMAT (ESTímulo del TALento MATemático) is a Spanish two-year-long out-of-school workshop for potential mathematically gifted students, where mathematics education researchers can observe students' reasoning in problem-solving experiments. Ramírez-Uclés and Ruiz-Hidalgo (2022) posed several geometric proof problems to ESTALMAT students and classified the answers based on Krutetskii's (1976) types of mathematical thinking. The results showed a large number of students' preferred types of thinking, partially dependent on the characteristics of each problem. Mora et al. (2024b) analyze the capacities of students in grades 6, 7, and 8, who applied to the entrance exam of ESTALMAT, to identify and generalize computational and procedural strategies necessary to solve two problems.

The paper also analyzes the validity of the problems to differentiate students with mathematical giftedness from their average peers.

Mathematics competitions are another context useful for mathematics education researchers to obtain information about potential mathematically gifted students' ways of reasoning. Comparing the differences in the solutions of the students with the highest and lowest scores in a mathematical Olympiad can inform about the adequacy of the problems posed to discriminate the potential mathematically gifted participants (Mora et al., 2022). This research also provided interesting and significant results obtained from analyzing and comparing the ways of reasoning and metacognitive strategies of the highest and lowest scorers in the competition (Mora, 2024).

Although Krutetskii (1976) concluded, after his problem-solving experiments, that showing a higher capacity of visualization was not a characteristic of mathematically gifted students, recent international research has proved that visualization can differentiate mathematically gifted students from average students. The results of studies conducted in Spain aligned with previous international research in supporting such conclusion, and provided different tools to assess students' mathematical giftedness in spatial orientation (Gutiérrez et al., 2018b), 3-dimensional geometry (Escrivà et al., 2018), and orthogonal projections (Mora et al., 2024a).

5 Research on the use of technology in mathematics education

Spanish schools incorporated information and communication technologies (ICT) into mathematics teaching many years ago. There has therefore been a great deal of research aimed at exploring various aspects of the presence of ICT in classrooms at the different educational levels. The research has encompassed the emergence and evolution of educational software like Logo (now Scratch) and the variety of apps, intelligent tutorial systems, and DGS; it has also helped create new applications, some of them currently under development. The interest of Spanish mathematics education in ICT-related research is evidenced in this Special Issue with the inclusion of two papers. One is devoted to computational thinking (Del Olmo-Muñoz et al., 2024) and the other to analyzing the possible advantages of the regular use of DGS in calculus subjects of secondary school (Santos-Trigo et al., 2024).

5.1 ICT in early childhood and primary school

The use of ICT with young children has been problematic for a long time, due to their difficulties in handling elements such as commands, the cursor, drop-down menus, the keyboard, the mouse, etc. The popularization of new products such as touch screens and floor robots has helped to partially overcome those difficulties and to create friendlier environments for young children. In addition, the potential of new software and hardware has enabled including computational thinking in early childhood and primary school curricula, like in the current Spanish primary school curriculum of mathematics.

COVID-19 lockdowns revealed numerous deficiencies in most educational systems related to a lack of teachers' training in computer use and to the geographical and economic conditions of schools and students. However, the availability of appropriate software, such as intelligent tutorial systems and video-conferencing platforms, solved some of the difficulties. Some intelligent tutorial systems allowed students to solve problems without the need to interact with their teachers. One of these tutorials posed verbal arithmetic-algebraic problems to primary school students and provided prompts when the user needed them (Del Olmo-Muñoz et al., 2023). These authors conducted several teaching experiments, one of their conclusions being that the students' socio-economic conditions did not significantly influence their learning and success in solving the problems.

Other researchers investigated the development of computational thinking in young children in learning environments mainly based on programmable floor robots or computer simulators of such robots. Students in kindergarten and low primary grades were able to program robots to make them reach given goals, as demonstrated by Del Olmo-Muñoz et al. (2024). These authors present two linked studies with 5 to 7 year-old children programming a floor robot to go from its initial position to another marked location. The paper describes and categorizes the metacognitive control applied by the students and evaluates the possible relationships between metacognitive processes, efficiency, age, and gender.

The teaching and learning of spatial geometry can also benefit from new 3-dimensional DGS. Some research experiments (based on the manipulation of physical and virtual cubes and the drawing of paper nets of cubes) were carried out to teach some properties of cubes to ordinary and mathematically gifted students in grades 5 and 6, and to facilitate the development of their capacity for visualization (Escrivà et al., 2018). The analysis of the students' answers showed that each kind of problem required students to use different kinds of images and abilities.

5.2 ICT in secondary school and university

Researchers in mathematics education have paid attention to some affective aspects of students and their influence on learning. The international literature consistently shows that the use of specialized ITC improves students' interest in mathematics. However, many studies consisted of short experiments and students perceived them as a novelty, in comparison to their ordinary classes, which, after having finished the experiment, led to positive feelings among students in favor of using ITC. A group of Spanish researchers organized a long teaching experiment in grade 9 consisting of 25 class sessions with the students using a DGS. The students were in transition from empirical to abstract approaches to learning geometry (Gómez-Chacón et al., 2016). The researchers looked at the relationship between different affective aspects and learning. They showed that the use of software improved students' attitudes and willingness to study mathematics, with the benefits of gaining intellectual independence and taking advantage of learning opportunities arising in the classroom.

Santos-Trigo et al. (2024) analyzed the possible advantages of the regular use of DGS in calculus subjects in secondary school. Research experiments carried out showed that teaching future teachers to use a DGS helped them develop their mathematical reasoning and understanding of mathematical concepts when solving problems with the DGS (Hernández et al., 2020). Another study showed that the combined use of ICT and online possibilities (e.g., Wikipedia) supported a successful methodology of inquiry-based learning (Santos-Trigo et al., 2021).

6 Contributions to theories of mathematics education

As a young discipline, mathematics education currently integrates a diversity of theoretical approaches of different kinds, which adopt particular ways of conceiving didactic research, its methods, objectives and foundations. In Spain, two research groups operate within a specific theoretical framework, rooting their productions and contributing to its development. In the early 1990s, both groups participated in the so-called Inter-University Seminar for Research in Didactics of Mathematics, which gradually brought together a growing number of researchers from different Spanish universities. The seminar was held twice a year to share the study of the two main approaches then being developed in France: the Theory of Didactic Situations (TDS), created by Brousseau (1986), and the emerging ATD created by Chevallard (1992). Two major research teams remain from this period: one working on the ATD led by Marianna Bosch and Josep Gascón from Barcelona, and another on the OSA that Juan Díaz Godino began to develop in Granada in the late 1990s (Godino, 2024).

This section focuses on the presentation and analysis of these theoretical frameworks, to which this Special Issue devotes two articles, Gascón (2024) on the ATD and Godino et al. (2024) on the OSA. Both approaches bring together a number of Spanish researchers who have presented contributions in different areas of mathematics, from the introduction of number systems and counting to combinatorics, probability and statistics, and from the teaching of algebra to differential calculus. A developed line of research in both frameworks is teacher education and, in the case of the ATD, university education in mathematics and related fields. An overview of the research that clearly falls within these two frameworks will be provided, addressing problems formulated from the theories themselves, and whose results contribute to their development.

Let us begin with the case of the ATD. In the late 1980s and early 1990s, a group of Spanish researchers began to develop doctoral theses under the direction of Brousseau in the TDS or Chevallard in the incipient ATD. Both theories were aligned with the project proposed by Brousseau for the creation of a science he called Didactics of Mathematics (Brousseau, 1997). Gascón (2024) describes precisely this project, its disruptive character regarding other ways of approaching didactics research, and the extensions proposed by the ATD by explicitly including an institutional dimension already in germ in the TDS.

The Inter-University Seminar facilitated the generation of a very active research team whose frame of reference is the ATD, but which readily draws on complementary tools from the TDS. It is a reference group in the international community, and has an active role in collaborations with other teams, including Chevallard's team. The Spanish ATD group has developed research in different areas: number systems and counting; mathematical modelling; elementary algebra; algebraic-functional modelling and differential calculus; geometry in secondary school; university teaching of mathematics and statistics, together with the problems of transition between high school and university; as well as teacher education at all education levels, from preschool to university. During the past 20 years, the group has worked on the development of a main line of research on the study of the conditions necessary for a change in the current educational paradigm, characterized as "visiting works" to foster its evolution into a new paradigm, still incipient, characterized as "questioning the world" (Chevallard, 2015).

On many occasions, these lines of research have entered into "dialogue" with other theoretical approaches, such as the TDS and lesson studies in early childhood and primary school teacher education (Lendínez et al., 2023; García et al., 2024), different conceptions of mathematical modeling (Barquero et al., 2019; Barquero & Ferrando, 2024), or APOS theory and problem-solving in university teaching (Bosch et al., 2017; Florensa et al., 2024). In this dialogue with other approaches, Gascón and Nicolás (2019) have proposed an ongoing study on the nature of didactics as a science and the role played by value judgements. Finally, some researchers in the team use the ATD as an analytical framework for studies on the history of mathematics education (Sánchez et al., 2020).

The main contributions of the Spanish team to the theoretical and methodological development of the ATD must be considered in relation to the epistemological program of research in mathematics education initiated by the TDS (Gascón, 2024). The ATD and the TDS shared its main principles about the consideration of school mathematical activity as a priority object of study and the methodological proposal to develop alternative epistemological models of the knowledge to be taught. The ATD also provides tools to incorporate the institutional relativity of knowledge and the processes of didactic transposition in the modeling of school knowledge. It then substantially broadens the object of study of mathematics education beyond school mathematics. At the same time, problems related to the teaching and learning of mathematics are addressed in terms of the institutional conditions and constraints that facilitate, as well as limit, certain study processes to take place. Using the notion of praxeology (Chevallard, 1992), the ATD proposes new delimitations of the objects of knowledge to identify didactic phenomena that affect the learning of mathematics at different levels. In some cases, alternative study processes are proposed to address these phenomena. For example, research on elementary algebra and differential calculus is approached from the perspective of functional algebraic modeling. It shows both the isolated nature of elementary school algebra, which is limited to equational calculus, and its disconnection from functions and the modeling perspective (Bosch, 2015; Lucas et al., 2019).

Complementarily to the epistemological and didactic analysis of curriculum works, a strong and sustainable effort has been made in the study of the conditions that facilitate—as well as the constraints that hinder—the transition between the paradigm of visiting the works and that of questioning the world. One of these lines addresses tertiary education, in both mathematics and statistics, in the first years of engineering and business degrees. Since the pioneering work of Barquero (2009), a significant number of study processes based on the inquiry of open questions have been designed, implemented, and analyzed in the form of instructional proposals based on the inquiry of open questions called “study and research paths”. These proposals offer a methodological framework broad enough to encompass any possible study process, from the most guided to the most open-ended, but also specific enough to provide the necessary design and management tools to ensure its sustainability in today’s university context (Markulin et al., 2024). They also provide an ideal framework for university teacher education (Florensa et al., 2021).

Research on study and research paths has also been carried out in secondary education, although with more specific interventions, generally associated with the teaching of a specific subject or area. Bosch (2018) presents an overview of research in this line to date, pointing out the relationships between inquiry processes and the epistemological models of the different mathematical fields involved. Since then, progress has been made in the study of the ecology of inquiry processes, i.e. the conditions at different levels (society, school, pedagogy, discipline, etc.) that enable their implementation, as well as the constraints, mainly institutional, that limit it. Currently, research is progressing in the development of this ecology to provide secondary and university teachers with the didactic and mathematical infrastructures necessary for the change of paradigm using different modalities of integrating study and research paths in traditional subjects (Barquero et al., 2021).

Regarding teacher education, two main lines of research are being developed. One extends the proposals of the study and research paths to teacher education Different studies describe these proposals, both for secondary teachers (Barquero et al., 2018, 2024) and preservice primary school teachers in different school areas, like arithmetic, measurement, or probability and statistics (Hakamata et al., 2024). The second line of research focuses on early childhood teacher education, using the ATD as an analytical framework, but adopting the principles of the TDS in the teacher education process (García et al., 2024).

The second theoretical framework in which an important group of Spanish researchers participates is the Onto-Semiotic Approach (OSA) that Juan Díaz Godino began to develop in the 1990s. The aim was to connect the different constructs used in mathematics education to describe cognitive phenomena, as well as the main notions of theoretical frameworks such as the TDS, the ATD, the theory of conceptual fields of Vergnaud (1990), or the theory of registers of semiotic representation of Duval (1995).

At present, we can distinguish five lines of development of the OSA. The first is the continued work of its systematization and the deepening of its foundations and background, an effort that is reflected in Godino (2024). The second line is the application of the theoretical tools of the OSA to different contents of school mathematics, some of which have previously been discussed. In this line, detailed analyses of the mathematical activity are carried out to understand the complexity of the objects and processes involved in problem-solving, providing a basis for the design, planning, and implementation of educational-instructional processes in mathematics using the theory of didactic suitability as a guide (Godino et al., 2023). Finally, we highlight the studies opening a dialogue between the OSA and other approaches, a line of research that has characterized this theoretical framework since its first publications, and that has given rise to recent works (such as Ledezma et al., 2024; Hummes et al., 2023; Manolino et al., 2023).

7 Conclusion

This survey of current research by the Spanish mathematics education community has shown a wide diversity of investigations published mostly during the past ten years. Today, numerous research groups are working on different problems with diverse approaches all over the country, cooperating among them and internationally, following uneven paths, as is normal in the evolution of research. Structuring the survey around strongly connected topics or frameworks would only have betrayed reality. We therefore decided to organize it according to the most important lines of research from an empirical perspective, considering five main blocks. Some lines focus directly on school mathematical contents. Research on teacher education stands out, with contributions from many research groups. These blocks are completed with more cross-curricular lines, such as research on students with special needs and the use of ICTs. The overview ends with an outline of the activities of two groups attached to specific theoretical frameworks, the ATD and the OSA. As in any picture of a dynamic situation, some parts have been more visible than others. Overlaps between the blocks are unavoidable, and there may also be some gaps, especially affecting emerging research lines. However, some general traits can be highlighted in this final section.

The first characteristic of the research carried out in Spain is the strong links maintained with European and, above all, Latin American research communities. This clearly shows that research goes beyond national borders but is still affected by cultural and language closeness. Somehow, the term "Spanish" refers more to a language community than a country delimitation. However, the facility to collaborate with colleagues from other Spanish-speaking countries is a great asset and a constant challenge due to the quite different educational systems in the countries.

A second common characteristic in the diversity of research objectives is the sharing of qualitative research methods and, especially clinical analyses through case studies, to support exploratory research, in line with the tendency of international mathematics education research. The type of problems addressed mostly involve the identification and delimitation of regular phenomena, rather than the effects of widely disseminated instructional proposals or educational policies. Flexibility is required for research methods and conceptualizations to evolve and consider different educational needs and social conditions.

New developments might be necessary for Spanish research in mathematics education to have a greater impact on local, regional, and national educational policies, e.g., large-scale studies at the national or regional levels. This task is still pending for the research community in Spain, which is conscious of its importance. For instance, new curricular requirements in terms of early algebra, computational thinking, and learning through inquiry processes are current challenges to be addressed. They can be used as cornerstones to design new research projects, since they closely correspond to the research of some dynamic Spanish teams.

8 In memoriam

Unfortunately, during the preparation of this Special Issue, the French researcher Guy Brousseau passed away. Brousseau was one of the pioneers in the study of didactic-mathematical problems. The Theory of Didactic Situations (TDS), created by Brousseau, placed mathematical knowledge at the center of research, thus allowing a new way of interpreting the didactic universe, a new research program. His participation in the development of mathematics education research in Spain was also highly significant. From the 1990s onwards, he supervised several doctoral dissertations of researchers from Madrid, Navarra, La Rioja, Comunidad Valenciana, and Aragón. He also collaborated closely with researchers from Andalusia, Catalonia, and Murcia. His participation in the doctoral program in mathematics education at the University of Granada, in 1990–1992, is particularly noteworthy, as well as his frequent participation in seminars, conferences, and congresses organized in Spain, such as the first International Congress of the ATD in 2005. Since 2010, the Jaume I University of Castellón houses the Resource Centre in Didactics of Mathematics that bears his name and contains the archives of the Jules Michelet school and the COREM research center, which Brousseau created and that served as a fundamental empirical support for the development of the TDS.

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