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On the Assessment of the Van Hiele Levels of Reasoning

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Introduction

There is a scarcity of research related to the van Hiele model of reasoning in which the authors have not needed to assess the students' van Hiele level. From the beginning of application of the van Hiele model, researchers have felt the necessity of creating instruments of assessment, to be used to identify the level of reasoning of students. Two questions have to be answered: 1) What type of test should be used? 2) How should the students' responses be evaluated? The aim of this paper is to present our answers to these questions.

Different answers to the questions have been given through the time. An early attempt was made by Usiskin (1982), who designed a paper and pencil multiple-choice test in which each item was intended to evaluate a specific level of reasoning. The answers were marked as right or wrong, and students were assigned to a van Hiele level depending on the number of correct answers at each particular level. Some years later, Burger and Shaughnessy (1986) worked at the other end of the spectrum of possibilities by creating a clinical interview test based on a set of problems and semi-structured dialogues. For each problem, each student's answer was analyzed and a van Hiele level was assigned on the basis of the dominant level evidenced in the answer. Finally, from the levels assigned to a student's set of answers, an overall van Hiele level was assigned to the student.

Other researchers have adopted positions between these two. Most people agree that a clinical interview is the most accurate way of assessment of the van Hiele levels, since it provides more information about the student's way of reasoning than other procedures. However this procedure is not feasible when many students have to be tested since it is so time consuming. Multiple-choice items, on the other hand, are efficient, as they can be administered in a short time to many people and the scoring can be done by a computer, but they are far from being valid and reliable (see Crowley, 1989, 1990; Wilson, 1990).

Aware of the need for a van Hiele test without the limitations mentioned above, we have been working for several years in the design of such a test, with the following objectives in mind:

1. To define a procedure to design valid reliable tests with a minimum number of items.
2. To obtain a pool of paper and pencil items from which one could construct tests meeting minimum requirements. Each item should require the students to give detailed answers, in order to obtain a clear picture of their reasoning.

In this paper we describe the background and main characteristics of a procedure to select items for a test and we show its application to a specific case - a longitudinal study were we assessed the van Hiele level of reasoning of Spanish students from grades 6th of Primary (11-12 year old students) to the last grade of Secondary (17-18 year old students). In this study, three linked tests were designed, each adapted to the difference in knowledge and reasoning among students in Primary, lower Secondary, and upper Secondary. The content of the items were polygons and other related concepts.

We use here the standard description of the van Hiele levels that can be found in many references (Burger & Shaughnessy, 1986; Crowley, 1987; Hoffer, 1983; Jaime & Gutiérrez, 1990). We have numbered the van Hiele levels from 1 to 5. Level 5 is not considered in our study, since its existence has not been clearly established. This does not affect the results of the study, as Spanish primary or secondary students are far from performing the kind of reasoning associated with level 5.

Analysis of the van Hiele Levels: Key Components of Each Level

Many descriptions of the van Hiele levels can be found in the literature, from the early papers of the van Hieles to the most recent publications that identify several general aspects of the levels, their particularities in different contexts, or the kind of answers to a specific test anticipated from students reasoning at different levels. Particularly useful are lists of descriptors (Burger & Shaughnessy 1986; Crowley, 1987; Fuys, Geddes & Tischler, 1988; Jaime & Gutiérrez 1990; Usiskin, 1982). These descriptions implicitly lead to the conclusion that one cannot consider a level of reasoning as a singular process that is atained (or not) by students, but must be considered as a set of processes. Students can rather be considered as reasoning in a level *n* only when they show mastery of the processes integrating such level. Therefore, to be valid, any test intended to assess the van Hiele levels must evaluate the different main processes integrating each level.

Very few authors have explicitly considered the van Hiele levels as sets of processes of mathematical reasoning. De Villiers (1987) identifies six “geometric thought categories”:

1) Recognition and represent of figure-types (level 1)
2) Use and understanding of terminology (level 2)
3) Verbal description of properties of figure-types (level 2)
4) Hierarchical classification (level 3)
5) One step deduction (level 3)
6) Longer deduction (level 4)

Hoffer (1981) also considers the van Hiele levels as integrated by several components, identifying five “skills in geometry” to be taken into account in the assessment of the students’ level of reasoning: 1) Visual; 2) Verbal; 3) Drawing; 4) Logical; 5) Applied. Hoffer then describes the characteristics of each skill in each van Hiele level.

While the categories identified by de Villiers (1987) belongs to a specific van Hiele level, all the skills described by Hoffer (1981) are part of each level. From an analysis of these and other publications mentioned, we have adopted an intermediate position, by identifying different processes of reasoning as characteristic of several (but not all) van Hiele levels:

1. **Recognition** of types and families of geometric figures, identification of components and properties of the figures.
2. **Definition** of a geometrical concept. This process can be viewed in two ways: As the students *formulate* definitions of the concept they are learning, and as the students *use* a given definition read in a textbook, or heard from the teacher or another student.
3. **Classification** of geometric figures or concepts into different families or classes.
4. **Proof** of properties or statements, that is, to explain in some convincing way why such property or statement is true.

It is possible to make a more detailed list of processes of reasoning characterizing the van Hiele levels, since some of the processes listed above can be decomposed into more specific parts. For instance, the process of proof, as characteristic of level 4, corresponds to the students’ ability to write formal proofs. Thus, a student reasoning in level 4 has to show the ability to differentiate between the several related statements (direct, converse, etc.) and to write the different usual types of proofs (direct, converse, ad absurdum, etc.). Therefore, level 4 could be considered the set of processes corresponding to the different types of proofs instead of the general process of proving. However, a much more detailed list of processes would lead to the impossibility of putting into practice the theoretical framework because of the practical constraints in the administration of assessment instruments (primarily time constraints).

Each process (recognition, definition, classification, proof) is a component of two or more levels of reasoning. How a student considers and uses the processes is an indicator of the student’s level of reasoning:

1. Recognition by students at level 1 is limited to physical, global attributes of figures. They sometimes use geometric vocabulary (more often students in up-

per primary or secondary), but such terms have a visual meaning more than a mathematical one. For instance, when describing a rectangle some students use the term "perpendicular" for a side when they mean "vertical." In other cases, they are able to notice correctly some mathematical properties of figures, but these are simple properties, such as the number of sides.

Students at level 2 or higher, however, are able to use and recognize mathematical properties of geometric concepts. It is important to notice that the ability of recognition does not discriminate among students in the van Hiele levels 2, 3 or 4.

2. Students at level 1 are not able to use given mathematical definitions. The only definitions they can formulate consist of descriptions of physical attributes of the figure they are looking at, such as "round" or "longer than wider" and perhaps some basic mathematical property.

When students on level 2 are provided with a mathematical definition, and they know every property contained in the definition, they can use it. These students, however, may experience difficulties when using some logical expressions, such as "and," "or" or "at least." Students on level 2 do not understand the logical structure of definitions (i.e., sets of necessary and sufficient properties of the defined concept), so when they are asked for a definition that has not been learned by rote, they often provide a long list of properties of the concept, unaware of redundancies. Or the definitions may not include some necessary property that the students use implicitly.

As the students in level 3 can establish logical relationships between mathematical properties, they are able to use and formulate mathematical definitions. Therefore, when formulating a definition, students try to be non redundant, although redundancies may appear when the relationships among the properties do not consists on one-step implications.

The progress of students in level 4 with respect to those in level 3 consists of a better understanding of the logical structure of mathematics, so the former admit the existence of several definitions of the same concept and can prove their equivalence.

3. Students in level 1 can only understand only exclusive classifications, since they do not accept nor recognize any kind of logical relationship between classes nor, many times, among two elements of the same class having quite different physical appearance.

The students in level 2 experience difficulty in establishing logical connections between properties. Therefore, the classifications produced by students in level 2 are usually exclusive. In the same way, when students in this level of reasoning are provided with a definition different from the one they have learned previously, the students usually do not accept the new definition, and they continue using their "own" definition. This behavior is quite prevalent in Spanish students, since textbooks for different grades or from different publishers may use different definitions of geometrical concepts, for instance the types of quadrilaterals.

Usually, researchers identify students in level 3 as those having the ability to make inclusive classifications of families. Thus, those students who say, for instance, that squares are not rectangles, are assigned to level 2. This criterion undervalues students who have been taught only exclusive definitions. A more accurate discrimination between students in levels 2 or 3 is based on the ability to accept and identify non-equivalent definitions of the same concept, and to change one's mind about the kind of classification, exclusive or inclusive, when the definitions are changed. Students in level 3 have achieved the maximum degree of ability of classification, so this process cannot discriminate among students in the van Hiele levels 3 or 4.

4. Students in level 1 cannot understand the concept of proof. For students in level 2 a typical proof consists of some experimental verification of the truth of the property in one or a few cases. Depending on the students' degree of acquisition of this ability, they may be convinced with just a special example, or they may need a more elaborated set of examples. Here is one student's answer when she was asked to prove that the angles of a quadrilateral add up to 360°:

> Let's suppose that we have a square (Figure 1-a). Each angle is 90°. They add 90° times 4 (sides) = 360°.
>
> Now, let's suppose any quadrilateral (Figure 1-b). Each angle is 80°, 92°, 66°, 122°. They add 80°+92°+66°+122° = 360°.
>
> If the angles add more than 360° the figure is no longer a quadrilateral.

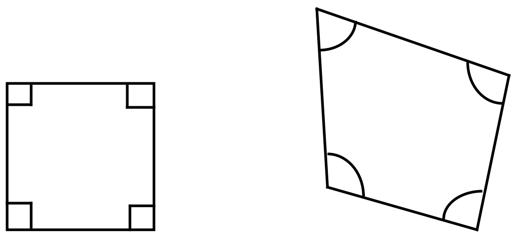

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Figure 1.

Students on level 3 are able to make deductions and logical proofs. The students at this level are able to give informal reasons for the truth of properties, the specific examples being only a help and no longer the proof itself.

Finally, students in the van Hiele level 4 may understand and write standard formal proofs. Specific figures are used only sometimes, to help select the adequate properties for the proof, but the students are aware that a figure is only a case and that to prove a statement it is necessary to develop a sequence of implications based on already established properties.

Table 1 summarizes the main characteristics of each process used to distinguish among students at the different van Hiele levels.

TABLE 1
Distinctive Attributes of the Processes of Reasoning in each van Hiele Level

	Level 1	Level 2	Level 3	Level 4
Recognition	Physical attributes	Mathematical properties	---------	---------
Use of definitions	---------	Only definitions with simple structure	Any definition	Accept several equivalent definitions
Formulation of definitions	List of physical properties	List of mathematical properties	Set of necessary and sufficient properties	Can prove the equivalence of definitions
Classification	Exclusive, based on physical attributes	Exclusive, based on mathematical attributes	Can move among inclusive and exclusive	---------
Proof	---------	Verification with examples	Informal logical proofs	Formal mathematical proofs

Assessment of the van Hiele Levels: Design of Paper and Pencil Tests

As mentioned in the introduction, one aim of our research was to find appropriate paper and pencil items. A test able to give a valid assessment of a student's van Hiele level of reasoning should meet certain requirements:

A) It must evaluate the five processes listed above (recognition, formulation and use of definitions, classification, proof).

B) It must evaluate the four levels of reasoning, that is, every student should have the possibility of answering questions according to his/her maximum capability of reasoning.

C) It must provide the students with an opportunity to explain the reasons for their answers so the researcher can determine which level of resoning was behind every answer.

Therefore, any test should have a number of items sufficient to fit both requirements A and B. A key problem in the design of a test is to find an optimal number of items, i.e. containing at least one item evaluating every process in every van Hiele level, that can be administered in a reasonable amount of time.

The items in a test answered by a student do not define his/her level of reasoning, but the kind of answers to such items do. An item cannot be pre-assigned to a certain van Hiele level, but may be pre-assigned to a range of levels, depending on the process evaluated by the item, according to Table 1. For instance, an item of recognition discriminates between students in levels 1 and 2, but cannot provide the information necessary to discriminate among students in levels 2, 3 and 4. In the same way, an item of proof is pre-assigned to levels 2, 3 and 4, although we cannot conclude from a blank answer that the student is functioning at level 1.

With respect to requirement C (allowing students to explain their answers), the items need to be open-ended since the choice of the correct answer to a multiple choice item may be based on different reasons, corresponding to different van Hiele levels. Until now, no attempt to solve this problem of multiple choice items has been successful (Crowley, 1989).

In our search for items fitting the three above mentioned requirements, we have developed a kind of paper and pencil item composed of several questions related to a problem, often divided into several parts. We want these written items to be as close as possible to an item for a semi-structured clinical interview, where it is possible for the interviewer to modify the questions, to give some hint, etc. depending on the previous student's answers and the reflected thinking level. When necessary, the items provide the students with extra information in every new part of the item, in order to help them if they have not been able to answer correctly before.

This technique has proved to be particularly useful in items of proof, where it frequently happens that students in levels 3 or 4 cannot answer because they do not find a suitable way to the result, and students in lower levels need more help than just the statement of the problem. When the first part of an item asks the students to make some deduction or proof, they have to turn the page to go to the second part of the item, providing them with some extra information and asking them, again, to solve the problem. This may happen several times in complex items. Students are not allowed to go backwards after they have turned a page. In this way, it is possible to know how much information was used for each answer. See, for instance, items 5 and 6 in the next section.

A Longitudinal Assessment Study

Using the framework presented in the previous section, we have organized a longitudinal study to assess the van Hiele level of reasoning of students in grades 6 of Primary to 4 of Secondary. To evaluate the students' answers and to assign students to van Hiele levels, we have used the model introduced in Gutiérrez, Jaime and Fortuny (1991), according to which, a) every student uses different levels of reasoning depending on the kind of problem to be solved and its difficulty, so it is necessary to assess the use a student makes of each van Hiele level, and b) the progress through the van Hiele levels is continuous, so the students are not assigned a level, but a “degree of acquisition” of each level of reasoning. The result of the assessment of a student is a row of 4 percentages corresponding to the student’s degrees of acquisition of levels 1 to 4.

To assign the degrees of acquisition, both researchers evaluated the tests independently, based on a list of descriptors obtained from the piloting of the items (see Jaime, 1993 for a complete list of the descriptors). The researchers then compared their evaluations and discussed the discrepancies, looking for an agreement.

The Sample

The study was carried out with 309 administrations of the test to students in 10 classroom groups from grades 6th to 8th of Primary (aged 11 to 14) and first to fourth grades of Secondary (aged 14 to 18). Table 2 shows the number of students in each grade. The test was administered in May or June to the whole classroom groups by their teachers of mathematics during a class slot (50-60 minutes).

TABLE 2

Distribution of Students in the Longitudinal Study

Grades	Number of Students
6th Primary	34
7th Primary	62
8th Primary	83
1st Secondary	35
2nd Secondary	36
3rd Secondary	28
4th Secondary	31

The mathematics curriculum in Spanish primary school includes geometry in every grade. In secondary school the mathematics does not include any Euclidean (synthetic) geometry. The mathematics concepts used in the test (types of polygons and related concepts) are taught in grades 1 to 7 of Primary school. Thus, all the students in the sample from the eighth grade of primary school to the fourth grade of secondary school had essentially the same background in relation to the content of the test.

The Test

A pool of 8 items was used in this study, the result of several previous pilot studies where we administered items (either in written or interview version) to students at very different knowledge and reasoning levels. After each administration, a new improved version of the items was obtained. Now we present the items and some examples of students' answers.

Item 1. - Write [in Figure 2] a **P** on the polygons, write an **N** on the nonpolygons, write a **T** on the triangles, and write a **Q** on the quadrilaterals. If necessary, you may write several letters on each shape.

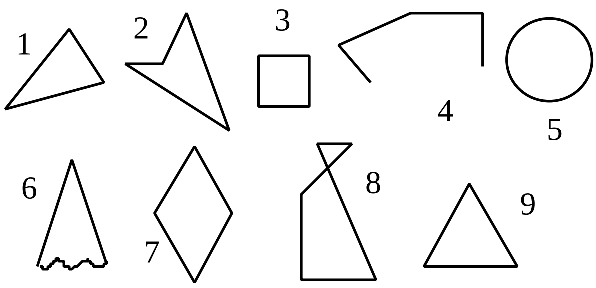

Figure 2.

- Write the numbers of the shapes which *are not polygons* and explain, for each of them, why it is not a polygon.
- The same questions for shapes which *are triangles,* and shapes which *are quadrilaterals.*
- Is shape 8 a polygon? Why? Is shape 2 a triangle? Why?

This item can be answered with a level 1 response. For instance, a student wrote that shape 2 *is not* a polygon because *it does not follow any rule,* and shape 7 *is* a quadrilateral because *it is a rhombus with its sides diagonal and parallel* (obviously, diagonal stands for slanted). Item 1 can also be answered with a level 2 response, when the reasons for classification are bon the ased number of sides and, in shapes 4, 5, and 6, on their openness or curvature.

Item 2. - Write [in Figure 3] an **R** on the regular polygons, an **I** on those that are irregular, a **V** on those that are concave, and an **X** on those that are convex. If necessary, you may write several letters on each shape.

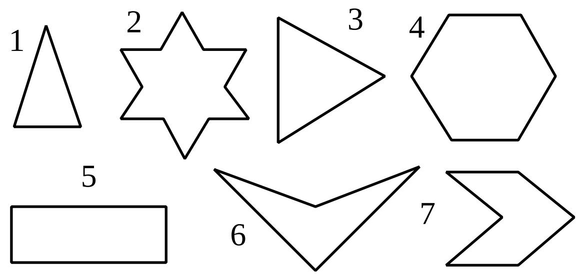

Figure 3.

- For polygons 2, 4, 5, and 7, explain your choice of letters or why you did not write any letter.

This item can be answered in level 1 or 2. Some level 1 students classified as regular polygons those that were "familiar" to them (1, 3, 4, and 5), and as irregular the "estrange" shapes. The reasons of students in level 2 for their classification were based on the (in)equality of angles and/or sides.

Item 3. A) - Write all the important properties which are *shared* by *squares and rhombi.*
- Write all the important properties which are *true* for *squares* but *not* for *rhombi.*
- Write all the important properties which are *true* for *rhombi* but *not* for *squares.*

B) - The same questions as in A) for *equilateral triangles* and *acute triangles.*

To answer this item, the students have to apply their own definitions. Some level 1 students found the number of sides as the only property common to squares and rhombuses, and mentioned as difference among these shapes that *rhombuses are pointy, and squares are not pointy but they end in a side*, or the like. Most of the students in the sample beginning the acquisition of level 2 were not able to differentiate properties that are shared by two polygons from those that belong to only one. For instance the property of squares and rhombuses of having 4 equal sides was written as shared by them and as differentiating each one. The students who had completely acquired the level 2 of reasoning made exclusive classifications, usually justified by lists of properties of angles, sides, and diagonals. The students in level 3 were able to justify either inclusive or exclusive classifications. For instance, a 2nd secondary grader wrote as properties differentiating squares from rhombuses that squares have *right angles* and rhombuses have *2 acute angles and 2 obtuse angles.* The same student made an inclusive classifications of triangles, explained in terms of sets of properties, although he made the wrong inclusion: *All the properties of equilateral triangles are also in the properties of the acute triangles.*

Item 4. A) - You can see a shape in Figure 4-a (a rhombus). Make a list of all the properties that you find for this shape (you can draw to explain the properties).

B) - The same question for shape in Figure 4-b.

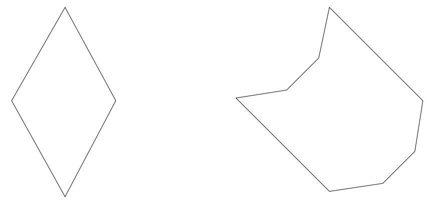

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Figure 4.

Item 4 is an item of recognition so it can be answered in levels 1 and 2. We had very different kinds of answers:

a) Some students just marked the vertices or sides in the shape, without any other comment. We considered these answers as “non-codifiable”, since they did not provide us with enough information to discriminate among reasoning of level 1 or 2.

b) A 7th grader, referring to shape -b-, wrote: *This shape is abstract. It looks like the face of a cat, with a bit of imagination. It has 8 sides.* This answer corresponds to a student beginning the acquisition of level 1.

c) Other level 1 answers were lists of basic mathematical properties, like the number of sides and angles, and the (in)equality of sides or angles. This answer corresponds to students ready to begin the acquisition of level 2.

d) The usual answers by students in level 2 were detailed lists of properties like those mentioned in c) and also parallelism, diagonals, symmetries, etc.

Item 5.1. - Recall that a *diagonal* of a polygon is a segment that joins two non adjacent vertices of the polygon. How many diagonals does an n-sided polygon have? Give a proof for your answer.

Item 5.2. - Complete the three following statements (you can draw if you want): In a 5-sided polygon, the number of diagonals which can be drawn from each vertex is ..... and the total number of diagonals is .....

> In a 6-sided polygon, the number of diagonals which can be drawn from each vertex is ..... and the total number of diagonals is .....
> In an n-sided polygon, the number of diagonals which can be drawn from each vertex is ..... Justify your answer.
> Using your answers above, tell how many diagonals an n-sided polygon has. Prove your answer.

The students in level 2 usually drew some polygons with their diagonals, counted the *number of diagonals,* and tried to deduce a formula, although most of them were not successful since they either drew too few polygons or did not draw all the diagonals of each polygon. The students in level 3 usually followed the same strategy for solving the problem as those in level 2, but they were more careful, and in most cases they arrived at a (almost) correct result, although they could not prove it accurately.

Some students in level 4 obtained in 5.1 the formula suggested in 5.2, but other students obtained and proved in 5.1 a different formula, and afterwards they wrote a different proof after the hints contained in 5.2. For instance, a 1st year secondary student obtained the formula $\frac{n(n-1)}{2} - n$ *because I connect every vertex to each other:*
n(n - 1), I take the half to avoid repetitions: $\frac{n(n-1)}{2}$*, and I subtract the sides which are not diagonals:* $\frac{n(n-1)}{2} - n$. This student obtained in 5.2 the formula $\frac{n(n-3)}{2}$ because *n - 3 are the diagonals from each vertex. I multiply it for the number of vertices: n(n - 3) and I divide it by 2 to avoid repetitions:* $\frac{n(n-3)}{2}$.

Item 6.1. - Prove that the sum of the angles of any acute triangle is 180°.

Item 6.2. - Recall that, if you have two parallel straight lines cut by another straight line (Figure 5-a): all the acute angles in the figure (A, G, C, E) are equal. All the obtuse angles in the figure (B, H, D, F) are equal.

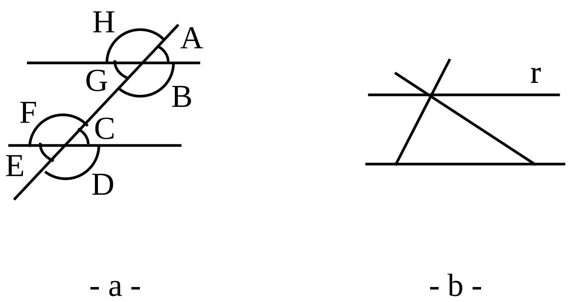

Figure 5.

Taking into account Figure 5-b (line r is parallel to the base of the triangle) and the properties mentioned above, prove that the sum of the angles of any acute triangle is 180°.

Item 6.3. - Here is a complete proof that the sum of the angles of any acute triangle is 180°. Read it and try to understand it.

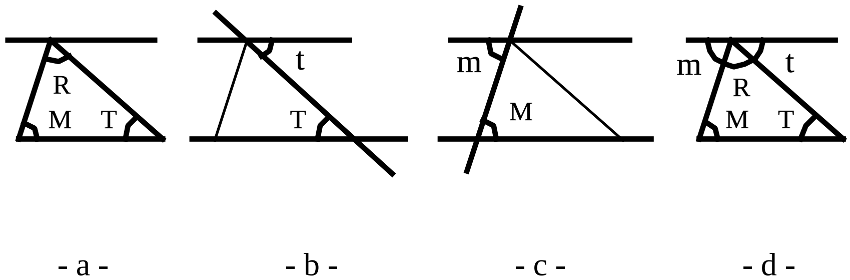

Figure 6.

- The sum that we are supposed to calculate is M + R + T (Figure 6-a).
- Construct a parallel to the base of the triangle through the opposite vertex R (Figure 6-a). By extending a side, we have two parallel lines cut by a transversal, so T = t (Figure 6-b).
- By extending the other side we have two parallel lines cut by a transversal, so M = m (Figure 6-c).
- Therefore, M + R + T = m + R + t = 180°, as the latter three angles form a straight angle (Figure 6-d).

- You have seen above a proof that the sum of the angles of an acute triangle is 180°. Is it true that the sum of the angles of a right triangle is 180°? Prove your answer.

- Tell how much is the sum of the angles of an obtuse triangle: Exactly 180°, more than 180°, or less than 180°. Prove your answer.

This item can discriminate among answers reflecting levels 2, 3, and 4. For instance:

a) Most level 2 students drew a triangle after each question, measured the angles and verified that they did (or did not) add up to 180° (Figure 7). They were assigned a low degree of acquisition of level 2. The very few students who showed the necessity of checking several triangles after the same question were assigned a high degree of acquisition of level 2.

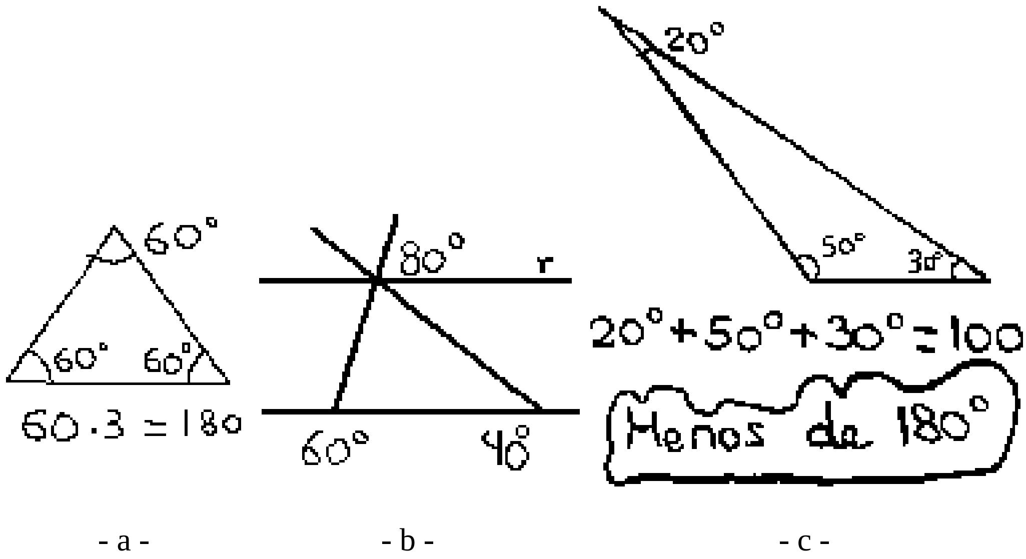

Figure 7. Answers to 6.1, 6.2, and 6.3.

b) Those students who were not able to write a proof in 6.1 but did in 6.2 were assigned to level 3, since they needed the hint included in this part of the item. The correct answers to 6.3 were also assigned to level 3, since those students were able to understand the proof included in the test, and to write it down with some adjustments to the particular cases of right and obtuse triangles.

c) The students who wrote a proof in 6.1 were assigned to level 4. In some cases, the students were able to write a different proof in page 6.2, such as a second year secondary student, whose correct proofs on items 6.1 and 6.2 were based on Figure 8-a and -b respectively.

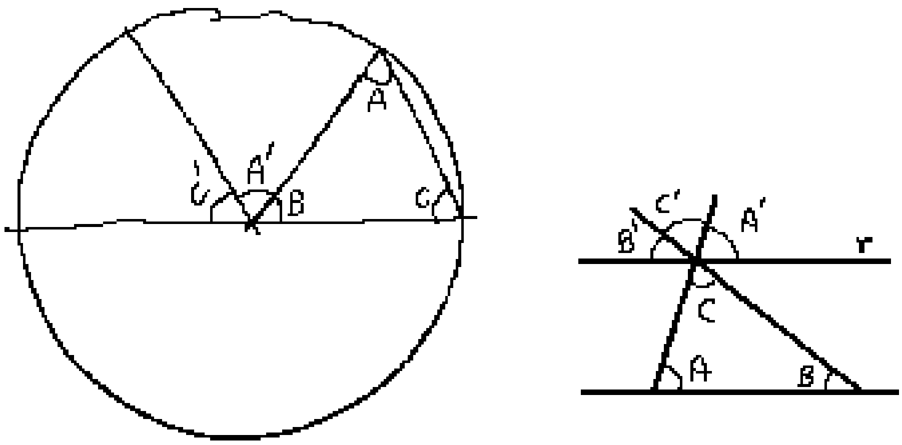

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Figure 8. Drawings for the proofs in 6.1 and 6.2.

Item 7. A) - Prove that the two diagonals of any rectangle have the same length.

B) - Recall that the *perpendicular bisector* of a segment is the line perpendicular to that segment that cuts it through its midpoint (line *r* is the perpendicular bisector of segment AB in Figure 9).

Prove that any point of the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

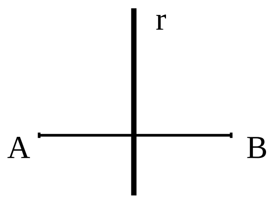

Figure 9.

Items 7 and 8 were intended mainly to identify students with high level of reasoning and good knowledge of geometry. For this reason, unlike items 5 and 6, they did not include hints to help with the proofs. The most usual answer to item 7 from students in level 2 was to draw a figure and to measure the lengths of the diagonals or the segments, to check that they are equal.

Item 8. - Usually a *parallelogram* is defined as a quadrilateral having two pairs of parallel sides.
Could a *parallelogram* also be defined as a quadrilateral in which the sum of any two consecutive angles is 180°? Justify your answer: If your answer is affirmative, prove that both definitions are equivalent. If your answer is negative, draw some example.

This item asks for proof of the equivalence of two definitions of parallelogram, so it can only discriminate students in levels 3 and 4. In our sample we did not find any student solving correctly this item. A few students in level 4 were able to prove only one of the implications.

Table 3 summarizes the processes evaluated by each item and the possible van Hiele levels of students' answers. It supports the validity and reliability of the test, since *every process* is evaluated at least by an item, and there are several items evaluating each level of reasoning. Furthermore, levels 2 and 3 are evaluated by a higher number of items since it is likely that these are the predominant levels of reasoning for most students in upper Primary and Secondary.

TABLE 3

Levels and Processes Evaluated by the Items in the Test

	Levels				Processes				
						Definition			
Items	**1**	**2**	**3**	**4**	**Recognit.**	**Use**	**Formulat.**	**Classific.**	**Proof**
1	•	•			•				
2	•	•			•				
3	•	•	•			•		•	
4	•	•			•	•			
5		•	•	•		•			•
6.1		•	•	•					•
6.2/3		•	•						•
7		•	•	•		•			•
8			•	•		•	•		•

Administration of the Test

Since students from a wide range of grades had to be assessed, it was necessary to optimize the administration of the test, trying to avoid too many difficult problems for the students in the lower grades, and too many easy problems for the students in the upper grades. Thus only a part of the eight items were administered to each student. Our previous experience, and that of other researchers mentioned above, with students in those grades allowed us to guess that most primary school students would reason in levels 1 or 2, while most upper secondary school students would reason in levels 3 or 4. Then, we reduced the number of items evaluating the van Hiele levels 3 and 4 administered to students in Primary and Lower Secondary and reduced the number of items evaluating the levels 1 and 2 administered to students in Upper Secondary. The result was a set of three different sub-tests with five items each one, three of the items being the same in all the tests to guarantee the validity of the comparison of results:

Test A, for grades 6, 7, and 8 of Primary contained items: 1, 3, 4, 6, 7.
Test B, for grades 1 and 2 of Secondary contained items: 1, 2, 3, 5, 6.
Test C, for grades 3 and 4 of Secondary contained items: 1, 3, 5, 6, 8.

Some Results of the Study

The main sources of validation of these tests are the several pilot studies we have accomplished, and the analysis made by several researchers with long expertise in the field of the van Hiele levels. Furthermore, we calculated the Guttman Coefficient of Scalability, measuring the hierarchy of the items in a test (see Mayberry, 1981, for a description of this parameter). We applied this coefficient to the degrees of acquisition of the levels assigned to the 309 students in the sample, divided by school grades. The values of the Guttman Coefficient obtained ranged from 0.98 to 1.00, confirming the reliability of the tests.

The result of the evaluation of each student was a row with four percentages indicating the student's degrees of acquisition of the levels 1 to 4. More meaningful results are obtained by changing the numeric values into a qualitative scale, as follows: Values in the interval [0%, 15%] mean “No Acquisition” of the level. Values in the interval (15%, 40%) mean “Low Acquisition” of the level. Values in the interval [40%, 60%] mean “Intermediate Acquisition” of the level. Values in the interval (60%, 85%) mean “High Acquisition” of the level. Finally, values in the interval [85%, 100%] mean “Complete Acquisition” of the level.

The chart in Figure 10 shows the distribution (percentage) of students according to their van Hiele level. It is evident that most students in the sample, except 6th graders, had a high or complete acquisition of the first level, and they were progressing in the acquisition of the level 2. Another consequence apparent from the chart is that the higher the course, the better the acquisition of the levels, although there is a notorious exception in the acquisition of level 1 by the students in first grade of secondary. The reason is that some students in this group had not enough time to answer all the items in the test, and the last items were items 2 and 3, which influence the results for levels 1 and 2; 23% of the students did not answer the last item in the test, and half of them did not answer the previous item.

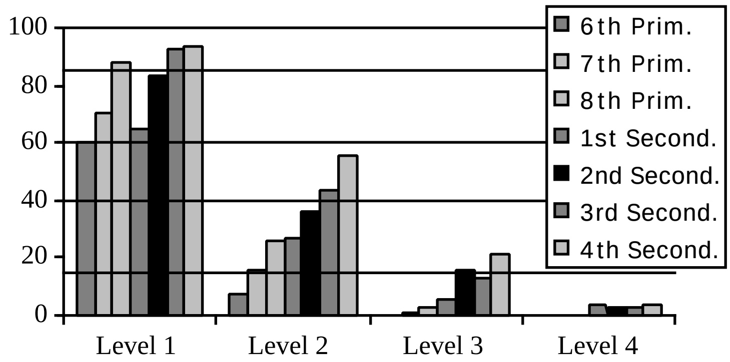

Figure 10.

Another interesting observation is that many students in the sixth and second grades had not completed the acquisition of level 1, but they were progressing toward acquisition of level 2. This same behavior was observed with students in the third and fourth year of secondary school between levels 2 and 3. This behavior seems to contradict the hierarchical structure of the van Hiele levels, as defined by van Hiele (1986):

> The ways of thinking of the base level, the second level, and the third level have a hierarchic arrangement. Thinking at the second level is not possible without that of the base level; thinking at the third level is not possible without that of the second level. (p. 51)

In the past, this statement has been used either to question the validity of the van Hiele model, or to measure the quality of tests, in that if there were students who solved correctly the items in level 2, for instance, but not the items in level 1, that was because of a defect in the test. However, in our framework, another interpretation is possible.

The van Hiele levels of reasoning are integrated by several abilities which must be mastered by students. Thus, it is possible that a student may progress in the mastering of some abilities but not of others, so the student cannot demonstate complete acquisition of a level of reasoning but he/she can show progress in some ability on the higher level. Furthermore, the reality of teaching mathematics is that students are often taught at a level of reasoning higher that the student's level, and the teachers force students to answer according to the rules of that level. Thus, students are not allowed to have enough experience to complete the acquisition of the lower level but, sometimes, some of them acquire practice in the higher level of reasoning (or they appear to reason in that level; van Hiele, 1986, discounts the phenomenon of “reduction of level”). This is often the case in secondary schools.

Conclusions and Final Remarks

We have shown in this paper the basic characteristics of a revised interpretation of the van Hiele levels of reasoning in mathematics: For many years, the levels were considered as having some global properties differentiating each level from the others. Only recently researchers have shown that the van Hiele levels of reasoning should be considered as the addition of some simpler reasoning processes sharing basic characteristics. For instance, students reasoning in level 2 recognize shapes, use and state definition of concepts, classify families, and deduct and prove properties, the four abilities having in common that student's activity is based on mathematical properties and that students are unable to establish logical relationships among such properties.

This new vision of the van Hiele levels allows us to consider the achievement of the levels by the students in a way different from what has been usual in the classical paradigm, were students are assigned to a level of reasoning, like a label, without any other nuance. Students may have a higher or lower acquisition of the different abilities characterizing a given van Hiele level, so it is necessary to establish a scale to measure the quality of a student's reasoning.

As application of the above mentioned theoretical viewpoint, we have designed a test where the items are not intended to assess a certain van Hiele level, but the use of some of the processes integrating the levels. The test was administered to a sample of students in high primary and secondary schools and the results show small differences in the levels of reasoning of different students that would not have been noticed by the classical ways of evaluation of students' level of reasoning. This technique of assessment is clearly an advance in the use of the van Hiele levels.

Further research should be carried out in some directions. Appropriate tools for the assessment of the students' level of reasoning need to be designed, piloted, and validated. Teaching units taking advantage of the recent advances in the understanding of the van Hiele model need to be developed. The application of the van Hiele model in geometry needs to be extended to other fields of mathematics.

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