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A MODEL OF TEST DESIGN TO ASSESS THE VAN HIELE LEVELS

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We propose a framework for designing tests to assess the Van Hiele level of reasoning. The framework is based on the consideration of the different key processes involved in each thinking level and the use of open-ended questions. We present a proposal of paper and pencil super-items with a structure approaching that of the clinical interviews, in order to obtain as much information as possible from the students' written answers.

Introduction.

The Van Hiele model describes the evolution of the kind of reasoning of a student in geometry. It establishes a sequence of 5 levels of reasoning, labelled 1 to 5 in this paper. In this paper we do not try to summarize the general characteristics of the Van Hiele levels. Such description can be found, for instance, in Crowley (1987), Hoffer (1983) and Jaime, Gutiérrez (1990).

We can hardly meet any researcher on the Van Hiele model who has not needed to assess the students' Van Hiele level; this implies the use of a test (written or oral). The Usiskin's test (Usiskin, 1982) and the Burger and Shaughnessy's test (Burger, Shaughnessy, 1986) are the most frequently used, but both tests have some objections:

- The Usiskin's test is based on paper and pencil multi-choice items, and there are some doubts about the possibility of measuring reasoning by means of this kind of items (Crowley, 1990 and Wilson, 1990). Nevertheless, this test has as its main advantage that it can be administered to many individuals and it is easy and quick to asses a level of reasoning to the students.

- The Burger and Shaughnessy's test has to be administered by an interview, and it is very time consuming; this makes the test unsuitable for assessing many people. However, the great advantage of this test is that the information obtained from interviews results in a deeper knowledge of the way students reason and, therefore, in a more reliable assessment of the Van Hiele level than that obtained by paper and pencil tests.

Aware of the necessity stated by many researchers of having a Van Hiele test without the inconveniences mentioned above, we have been working for several years in the design of such a test. Some previous results can be found in Shaughnessy et al (1991), where different ways of assessing the Van Hiele levels were analyzed. Now, the objective of our ongoing research is twofold:

1- To offer a procedure enabling the design of reliable and valid tests to measure the Van Hiele levels of reasoning.

2- To implement a pool of items from which one should be able to make several of such tests.

We present here a theoretical model of design of items and tests (first objective) and also some examples of such items (second objective).

A model for the design of Van Hiele tests.

The core of our proposal is the consideration of each Van Hiele level of reasoning as integrated by several *key thinking processes*. Then, to evaluate a student's thinking level means to evaluate the way this student uses each key thinking process characteristic of that level. This interpretation of the assessment of the Van Hiele levels is implemented by means of paper and pencil open-ended items, designed in a way that they provide an amount of information that approaches the obtained by means of clinical interviews.

1. Key processes of the Van Hiele levels.

As a thinking level is integrated by some thinking processes, quite different one from the others, the items in a test should not be intended to assess a whole level, but one or more of the key processes involved in this level. Then an ideal test should contain at least an item able to assess each key process of each Van Hiele level. For instance, as we shall see below, for the assessment of level 3, the items should allow to assess the way students' use the processes of definition, proof, and classification.

The view of considering the kind of reasoning of a Van Hiele level divided into several components is not new. De Villiers (1987) makes this distinction. Also Hoffer (1981) shows a characterization of 5 geometric skills to be considered for the assessment of each Van Hiele level.

In the following paragraphs, we describe the main key processes we have identified for the Van Hiele levels 1 to 4. We does not consider level 5, as our research is directed to primary and secondary school students. It would be possible to make a more detailed list of key process, since some of the processes stated below can be decomposed in sub-processes. For instance, in level 4, the key process of formal proof could be divided into the processes corresponding to the different ways of proving that students should know. However, this would conduct us to a position impossible to be put into practice because of the length of the tests. These key processes characterizing the Van Hiele levels 1 to 4 are:

Identification of the family a geometric object belongs to.

Definition of a concept, understood from two different points of view: To read definitions, that is to use given definitions, and to state definitions, that is to formulate a definition for a class of geometric objects.

Classification of geometrical objects into different families.

Proof of properties or statements, that is ways of convincing someone else of the truth of a statement.

The table below summarizes the key processes characteristics of each Van Hiele level. An X or the name of a process in a cell means that this process is a part of the reasoning of the level, so it has to be assessed in this level. The "---" means that this process is not a part of the reasoning of the level, so it has not to be assessed in this level.

	Identification	**Definition**	**Classification**	**Proof**
Level 1	X	State	X	---
Level 2	X	Read & State	X	X
Level 3	---	Read & State	X	X
Level 4	---	Read & State	---	X

When assessing the Van Hiele level of reasoning of a student, it is important to notice that, in some levels, some of the processes just mentioned do not have their usual mathematical meaning. Then, when analyzing a student's answer, we have to consider the processes from the perspective of the level exhibited by the student. For this purpose, we specify in the following paragraphs the meaning of each process integrating the levels 1 to 4.

Level 1:

Identification: The students recognize figures on the basis of physical global characteristics, like aspect, size of elements, position, etc.

Definition: Students take into consideration only attributes which refer to physical objects in a global way, or non-mathematical properties like "round" for circle, so they are not able to read a mathematical definition. When stating a definition, the students refer to this same kind of attributes. Sometimes the name of the concept is the definition itself; for example, children quite often say that "a square is a square".

Classification: Students use the same kind of properties of the figures as in the previous processes. They do not accept any relationship among two different families nor, many times, among two elements of the same family having quite different physical aspect (for instance, two isosceles triangles having angles of 50°, 50° and 80°, and 82°, 82°, and 16°, respectively).

Level 2:

Identification: Students recognize geometrical figures on the basis of their mathematical properties.

Definition: The students pay attention to mathematical properties but, when reading or stating definitions, they may have problems with some logical particles, such as "at least".

When stating a definition, sometimes the students omit a necessary property, which they are using implicitly. Other times, they provide a list with more properties than needed, even when the dependence among them is easy to realize. For example, some students define a rectangle as "a parallelogram having two pairs of equal sides, being two sides longer than the other two" (they omit the reference to the right angles). Other students define a rectangle as "a parallelogram having two pairs of equal parallel sides, being two sides longer than the other two, four right angles and two equal diagonals (they include an extra property).

Classification: It is exclusive, that is, the students do not relate families based on the attributes provided in the definitions. When they are given a new definition of a concept, different from the one they already knew, the students do not admit the new definition. This happens very often with quadrilaterals, when the students are habituated to use the exclusive definitions and they are given the inclusive definitions.

Proof: A typical proof in this level consists on verifying the truth of the property to be proved in one or a few examples.

Level 3:

Definition: The students are able to interpret and state mathematical definitions, being conscious that a necessary and sufficient set of properties is needed and that adding more properties to the definition does not result in a better one. Therefore, when providing a definition, the students try not to be redundant, although some redundancies may appear when the relationships among the properties do not consists on one-step implications.

Classification: The students may do inclusive classifications, based on the properties stated in the given definitions of the concepts. The students are able to change their mind when a new definitions of a concept is given, even when there is a change from exclusive to inclusive, or vice versa.

Proof: The students may check the property to be proved in some examples, but they look also for some informal explanation based on mathematical properties, or the examples are selected.

Level 4:

Definition: The progress from the level 3 reasoning consists on a better understanding of definitions and the ability to prove the equivalence of different definitions of the same concept.

Proof: Students in this level are able to do standard formal mathematical proofs. Specific figures are used only sometimes to help to choose the adequate properties for the proof, but the students are aware that a figure is only a case and that to prove a statement it is necessary to make a sequence of implications based on already proved properties.

2. Open-ended items for assessing the Van Hiele levels.

Paper and pencil open-ended items, where the students can freely explain the reason for their answer, are more reliable than multi-choice items for assessing the Van Hiele levels of reasoning. On the other side, what defines a student's level of thinking are not the items administered but the student's answer to such items, since most of the questions can be answered according to several levels of thinking. Therefore, we defend the administration of tests based on open-ended items that are not pre-assigned to a specific level, but to a range of the levels in which answers can be given. In this way, an item contributes to the assessment of each level in this range.

A useful characteristic of clinical interviews is the possibility for the interviewer to modify the questions, to give some hint, etc., depending on the previous student's answers and the reflected thinking level. This is what makes interviews so useful for the assessment of the Van Hiele levels and the reason why they provide more information than any paper and pencil test. In order to approach the amount of information obtained by written tests to that obtained by interviews, we have designed super-items divided in several parts. Students are provided with extra information in every new part of the item, in order to help them if they have not been able to answer correctly before (Jaime, 1993).

This technique has proved to be useful, for instance, in items about proof, where it happens quite often that level 4 students cannot answer because they do not find a suitable way to the result. In ordinary items, with only a statement and a question, often these students do not write anything or they erase what they have written, since they believe they are wrong. This behaviour results in the assignation of the student to a Van Hiele level lower than their real one.

The structure of the super-items we have designed is the following:

- The first part of the item just state the problem and the question.
- When this part has been answered, students have to turn the page and they answer the next part of the item, that provides them with some more information and states again the same question. This may happen several times in complex problems.
- When answering these super-items, students are not allowed to go backwards after they have turned a page. That is, when they have answered a part of the item, they are not allowed to answer again a previous part.

In the annex we show, as examples of the notions introduced above, some items taken from the pool of items we have designed for this research. The table below, that refers to those items, summarizes the characteristics of each item, that is what levels and what key processes of each level are evaluated.

V.H. levels / Key proces.	Level 1		Level 2				Level 3				Level 4
Identification	1	2	1	2							
Definition **Read**				2			2	3			3
Definition **State**	1	2	1	2	4		2		4		
Classification		2		2			2				
Proof						5		3		5	3

Annex: Examples of super-items.

Item 1

(The students are given several figures). For the following figures, write a T inside of the triangles and a Q inside of the quadrilaterals.

Explain how do you know which shapes are triangles and which are not. (The same question for quadrilaterals).

Write the numbers of the figures which are not triangles and explain, for each of them, why it is not a triangle. (The same question for quadrilaterals).

Item 2

2.1. (The students are given several figures). For each figure, write all the names in the following list that are appropriate for the figure: Square, rectangle, rhombus, parallelogram and rhomboid.

Explain the assignations that you have done for ... (the numbers of several figures are given, among which there should be at least a rectangle or a rhombus, and a square).

2.2. The students are given the inclusive definitions for rhombus and rectangle, and they are requested to use these definitions for answering to the same questions as in 2.1.

Item 3

Here are two definitions of certain polygons:

Definition A: It is a quadrilateral having two pairs of parallel sides.

Definition B: It is a quadrilateral in which the sum of any two consecutive angles is 180°.

Do A and B define the same quadrilaterals? Why, or why not? Give a proof for your answer.

Item 4

(The students are given a list of true properties for rhombi). Write a definition of rhombus down taking properties from the list. Remember that you are asked to write a definition down, so you have not to use more properties than needed.

Is it possible to solve again the same task but using a different set of properties from the list?

Item 5

5.1. Recall that a diagonal of a polygon is a segment joining two non adjacent vertices of the polygon. How many diagonals does an n-sided polygon have? Give a proof for your answer.

5.2. Complete the following statements (you can draw if you want):

In a 5-sided polygon, the number of diagonals which can be drawn from each vertex is . . . . . and the total number of diagonals of the polygon is . . . . .

In a 6-sided polygon, the number of diagonals which can be drawn from each vertex is . . . . . and the total number of diagonals of the polygon is . . . . .

In an n-sided polygon, the number of diagonals which can be drawn from each vertex is . . . . . Justify your answer to the last statement.

Using your answers above, tell how many diagonals an n-sided polygon has. Prove your answer.

References.

Burger, W.F.; Shaughnessy, J.M. (1986): Characterizing the van Hiele levels of development in geometry, *Journal for Research in Mathematics Education* vol. 17, pp. 31-48.

Crowley, M.L. (1987): The Van Hiele Model of the development of geometric thought, in *N.C.T.M. (1987): Learning and teaching geometry, K-12.* (N.C.T.M.: Reston, USA), pp. 1-16.

Crowley, M.L. (1990): Criterion-referenced reliability indices associated with the van Hiele Geometry Test, *Journal for Research in Mathematics Education* vol. 21 n° 3, pp. 238-241.

De Villiers, M.D. (1987): *Research evidence on hierarchical thinking, teaching strategies and the van Hiele theory: Some critical comments.* (Research Unit for Mathematics Education, Univ. of Stellenbosch: Stellenbosch, R. of South Africa).

Hoffer, A. (1981): Geometry is more than proof, *The Mathematics Teacher* vol. 74 n° 1, pp. 11-18.

Hoffer, A. (1983): Van Hiele-based research, in *Lesh, R.; Landau, M. (1983): Acquisition of mathematics concepts and processes.* (Academic Press: N. York, USA), pp. 205-227.

Jaime, A. (1993): *Aportaciones a la interpretación y aplicación del modelo de Van Hiele: La enseñanza de las isometrías del plano. La evaluación del nivel de razonamiento* (Doctoral dissertation). (Univ. de Valencia: Valencia, Spain).

Jaime, A.; Gutiérrez, A. (1990): Una propuesta de fundamentación para la enseñanza de la geometría: El Modelo de Van Hiele, in *Llinares, S.; Sánchez, M.V. (1990): Teoría y práctica en educación matemática.* (Alfar: Sevilla, Spain), pp. 295-384.

Shaughnessy, J.M.; Burger, W.F.; Gutiérrez, A.; Jaime, A.; Fuys, D. (1991): Analyzing and describing students' thinking in geometry: Continuity in the Van Hiele levels, in *Proceedings of the 13th Annual Meeting of the P.M.E.-N.A.* vol. 1, pp. 183-188.

Usiskin, Z. (1982): *Van Hiele levels and achievement in secondary school geometry.* (ERIC: Columbus, USA).

Wilson, M. (1990): Measuring a van Hiele geometry sequence: A reanalysis, *Journal for Research in Mathematics Education* vol. 21 n° 3, pp. 230-237.