A. Characteristic fields of the RHD equations

Analytical expressions for the spectral decomposition of the three 5×5 (in 3D) Jacobian matrices $\mathcal{B}^i(\mathbf{U})$ associated to the fluxes $\mathbf{F}^i(\mathbf{U})$ of system (2.7),

$$\mathcal{B}^i(\mathbf{U}) = \frac{\partial \mathbf{F}^i(\mathbf{U})}{\partial \mathbf{U}} \tag{A1}$$

have been given by Donat et al. (1998).

In this Appendix, we explicitly show the eigenvalues and the right and left eigenvectors coresponding to matrix $\mathcal{B}^x$, whereas the cases y and z easily follows from symmetry. The eigenvalues are:

$$\lambda_\pm = \frac{1}{1 - \mathrm{v}^2 c_s^2} \left\{ v^x(1 - c_s^2) \pm c_s \sqrt{(1 - \mathrm{v}^2)[1 - v^x v^x - (\mathrm{v}^2 - v^x v^x) c_s^2]} \right\} \tag{A2}$$

$$\lambda_0 = v^x \quad \text{(triple)} \tag{A3}$$

The following expressions define auxiliary quantities:

$$\mathcal{K} \equiv \frac{\tilde{\kappa}}{\tilde{\kappa} - c_s^2}, \quad \tilde{\kappa} = \frac{1}{\rho} \left. \frac{\partial p}{\partial \varepsilon} \right|_\rho, \quad \mathcal{A}_\pm \equiv \frac{1 - v^x v^x}{1 - v^x \lambda_\pm} \tag{A4}$$

A complete set of *right–eigenvectors* is,

$$\mathbf{r}_{0,1} = \left(\frac{\mathcal{K}}{hW}, v^x, v^y, v^z, 1 - \frac{\mathcal{K}}{hW} \right) \tag{A5}$$

$$\mathbf{r}_{0,2} = \left(W v^y, 2hW^2 v^x v^y, h(1 + 2W^2 v^y v^y), 2hW^2 v^y v^z, 2hW^2 v^y - W v^y \right) \tag{A6}$$

$$\mathbf{r}_{0,3} = \left(W v^z, 2hW^2 v^x v^z, 2hW^2 v^y v^z, h(1 + 2W^2 v^z v^z), 2hW^2 v^z - W v^z \right) \tag{A7}$$

$$\mathbf{r}_\pm = (1, hW\mathcal{A}_\pm \lambda_\pm, hWv^y, hWv^z, hW\mathcal{A}_\pm - 1) \tag{A8}$$

The corresponding complete set of *left–eigenvectors* is

$$\mathbf{l}_{0,1} = \frac{W}{\mathcal{K} - 1}(h - W, Wv_x, Wv_y, Wv_z, -W)$$

$$\mathbf{l}_{0,2} = \frac{1}{h(1 - v_x v_x)}(-v_y, v_x v_y, 1 - v_x v_x, 0, -v_y)$$

$$\mathbf{l}_{0,3} = \frac{1}{h(1 - v_x v_x)}(-v_z, v_x v_z, 0, 1 - v_x v_x, -v_z)$$

$$\mathbf{l}_{\mp} = (\pm 1)\frac{h^2}{\Delta}\begin{bmatrix} hW\mathcal{A}_{\pm}(v_x - \lambda_{\pm}) + \aleph_{\pm} \\ 1 + W^2(\mathrm{v}^2 - v_x v_x)(2\mathcal{K} - 1)(1 - \mathcal{A}_{\pm}) - \mathcal{K}\mathcal{A}_{\pm} \\ W^2 v_y(2\mathcal{K} - 1)\mathcal{A}_{\pm}(v_x - \lambda_{\pm}) \\ W^2 v_z(2\mathcal{K} - 1)\mathcal{A}_{\pm}(v_x - \lambda_{\pm}) \\ \aleph_{\pm} \end{bmatrix}$$

where Δ is the determinant of the matrix of right-eigenvectors

$$\Delta = h^3 W(\mathcal{K} - 1)(1 - v_x v_x)(\mathcal{A}_+\lambda_+ - \mathcal{A}_-\lambda_-), \tag{A9}$$

and,

$$\aleph_{\pm} = -\left\{-v_x - W^2(\mathrm{v}^2 - v_x v_x)(2\mathcal{K} - 1)(v_x - \mathcal{A}_{\pm}\lambda_{\pm}) + \mathcal{K}\mathcal{A}_{\pm}\lambda_{\pm}\right\} \tag{A10}$$

For an ideal gas equation of state it can be proven that $\mathcal{K}$ is always greater than one (in fact $\mathcal{K} = h$), and Δ is different from zero ($\mid v^x \mid < 1$).

B. An Efficient Implementation of Flux Formulae in Multidimensional Relativistic Hydrodynamical Codes

This appendix is an adaptation of Aloy, Pons & Ibáñez (1999) to derive and analyze a simplified formulation of the numerical viscosity terms appearing in the expression of the numerical fluxes associated not only to the modified Marquina's flux formula but also, being more general, to several High-Resolution Shock-Capturing schemes. After some algebraic pre-processing, we give explicit expressions for the numerical viscosity terms of two of the most widely used flux formulae, which implementation saves computational time in multidimensional simulations of relativistic flows. Additionally, such treatment explicitely cancells and factorizes a number of terms helping to amortiguate the growing of round-off errors. We have checked the performance of our formulation running GENESIS to solve a standard test-bed problem and found that the improvement in efficiency is of high practical interest in numerical simulations of relativistic flows in Astrophysics.

The numerical study of the evolution of multidimensional relativistic flows turns out to be a topic of crucial interest in, at least, two different scientific fields: Nuclear Physics (studies of the properties of the equation of state for nuclear matter via comparison of simulations and experiments of heavy ion collisions) and Relativistic Astrophysics. The field of Numerical Relativistic Astrophysics is recently undergoing an extraordinary developement after the important efforts of people working in building up robust codes able to describe many different astrophysical scenarios, such that relativistic jets in quasars and microquasars, accretion onto compact objects, collision of compact objects, stellar core collapse and recent models of Gamma-Ray bursts (see, e.g., the recent review in Ibáñez & Martí, 1998, and references therein). Thus, the improvement in the efficiency of multidimensional hydro-codes becomes a necessity.

It is well known the performance of *modern high-resolution shock-capturing* techniques (HRSC) in simulations of complex classical flows. Most of the HRSC methods are based on the solution of local Riemann problems (i.e., initial value problems with discontiuous initial data) and since 1991 (see Martí, Ibáñez & Miralles 1991) several Riemann Solvers or Flux Formulae have been specifically designed in relativistic fluid dynamics (see, e.g., Martí, 1997, Ibáñez & Martí, 1998, for a review on Riemann solvers in Relativistic Astrophysics). In addition, in Pons et al. (1998) it is showed the way for applying special relativistic Riemann solvers in General Relativistic Hydrodynamics, hence any future new Riemann solver, exhaustively analyzed in Special Relativistic Hydrodynamics (SRH), could be applied to get the numerical solution of local Riemann problems in General Relativistic Hydrodynamics. Consequently, the interest of the results we obtain in this note goes beyond the domain of SRH and can be easily extended to General Relativistic Hydrodynamics.

For consistency, we start by summarizing the basics of the HRSC techniques. A system of *conservation laws* (see LeVeque 1991) is a set of partial differential equations of the form (2.7), where $\mathbf{U} \in \Re^d$ is the *vector of unknowns* and $\mathbf{F}^i(\mathbf{U})$ is the *flux* in the i-direction. In the above system (2.7) we can define a $d \times d$-Jacobian matrix $\mathcal{B}^i(\mathbf{U})$ associated to the flux in the i-direction as:

$$\mathcal{B}^i = \frac{\partial \mathbf{F}^i(\mathbf{U})}{\partial \mathbf{U}}. \tag{B1}$$

The system is said to be hyperbolic if the Jacobian matrices have real eigenvalues.

The main ingredients of a HRSC algorithm are:

i) A finite discretization of the equations in conservation form (2.7). Using a *method of lines*, this discretization reads:

$$\frac{d\mathbf{U}_{i,j,k}(t)}{dt} = -\frac{\hat{\mathbf{F}}_{i+\frac{1}{2},j,k} - \hat{\mathbf{F}}_{i-\frac{1}{2},j,k}}{\Delta x} - \frac{\hat{\mathbf{G}}_{i,j+\frac{1}{2},k} - \hat{\mathbf{G}}_{i,j-\frac{1}{2},k}}{\Delta y} - \frac{\hat{\mathbf{H}}_{i,j,k+\frac{1}{2}} - \hat{\mathbf{H}}_{i,j,k-\frac{1}{2}}}{\Delta z} \tag{B2}$$

where subscripts i, j, k are related, respectively, with x, y and z-discretizations, and refer to cell-centered quantities. The cell width, in the three coordinate directions are, respectively, Δx, Δy and Δz.

ii) Quantities $\hat{\mathbf{F}}_{i+\frac{1}{2},j,k}$, $\hat{\mathbf{G}}_{i,j+\frac{1}{2},k}$ and $\hat{\mathbf{H}}_{i,j,k+\frac{1}{2}}$ are called the *numerical fluxes* at the cell interfaces. These numerical fluxes are, in general, functions of the states of the system at each side of the cell interface. Some HRSC methods derive expressions for the numerical fluxes by giving a consistent flux formulae or solving *local Riemann problems*, with an exact (Martí & E. Müller, 1994) or approximate Riemann solver, after a *cell reconstruction procedure* that gives the state at both sides of the interface, denoted by L (left state) and R (right state). Several monotonic cell reconstruction prescriptions have been given in the scientific literature to achieve different orders of spatial accuracy (van Leer 1979, Woodward & Colella 1984, Marquina 1994).

For clarity, from now on we will omit the indexes relative to the grid and restrict our study to the x_1-splitting of the above system (2.7), assuming that the vector of unknowns satisfies $\mathbf{U} = \mathbf{U}(x_1, t)$.

We have focussed our analysis to some of the most popular HRSC algorithms, and analyzed their expressions for the numerical fluxes. Hence, the sample considered is: HLLE (Harten, Lax, & van Leer 1983, Einfeldt, 1998), Roe (Roe 1981), Marquina (M) (Donat et al. 1998), and a modified Marquina's flux formula (MM) (Aloy et al. 1999). The above selection gathers the most fundamental differences among the large sample of HRSC flux formulae. HLLE is the simplest one, it does not need the full spectral decomposition of the Jacobian matrices. Roe's solver linearizes the information contained in the spectral decomposition into an average state. Marquina's (and its modified version) flux formula considers the information coming from each side of a given interface (it is not a Riemann solver) and, in some astrophysical applications, has produced better results in modelling complex flows.

After some algebraic work, all these flux formulae can be cast into the following general form:

$$\hat{\mathbf{F}}(\mathbf{U}^L, \mathbf{U}^R) = \frac{1}{2}\Big((\mathcal{I} + \widetilde{\mathcal{I}}^L)\mathbf{F}^L + (\mathcal{I} - \widetilde{\mathcal{I}}^R)\mathbf{F}^R + (\mathcal{Q}^L\mathbf{U}^L - \mathcal{Q}^R\mathbf{U}^R)\Big) \tag{B3}$$

where $\mathbf{F}^{L,R}$ stands for the fluxes evaluated at the states $\mathbf{U}^{L,R}$ and $\mathcal{I}$ is the unit matrix. Following Harten (1983), the $\mathcal{Q}^{\mathcal{L},\mathcal{R}}$ terms in the above equation will be referred as the *numerical viscosity matrix.*

Matrices $\widetilde{\mathcal{I}}^{L,R}$ and $\mathcal{Q}^{L,R}$ can be expressed as linear combinations of the projectors onto each eigenspace, i.e., the direct product of the corresponding left and right eigenvectors $\mathbf{l}_p, \mathbf{r}_p$ associated to the p-th characteristic field (p=1,...,d),

$$\widetilde{\mathcal{I}}^{L,R} = \sum_{p=1}^{d} b_p \mathbf{l}_p^{L,R} \mathbf{r}_p^{L,R} \tag{B4}$$

$$\mathcal{Q}^{L,R} = \sum_{p=1}^{d} c_p \mathbf{l}_p^{L,R} \mathbf{r}_p^{L,R} \tag{B5}$$

where superscripts L, R indicate that the eigenvectors are evaluated at the state $\mathbf{U}^{L,R}$. The values of the coefficients b_p and c_p appearing in the above definitions of matrices $\widetilde{\mathcal{I}}^{L,R}$ and $\mathcal{Q}^{L,R}$ depend on the eigenvalues λ_p as shown in Table I, for the four flux formulae analyzed.

TABLE I

Parameters in the numerical fluxes.

Flux Formulae	b_p	c_p
HLLE	$\frac{\Psi_+ + \Psi_-}{\Psi_+ - \Psi_-}$	$\frac{2\Psi_+\Psi_-}{\Psi_+ - \Psi_-}$
Roe	0	$\mid \lambda_p(\widetilde{\mathbf{U}}) \mid$
M	β_p	$\alpha_p(1-\beta_p^2)$
MM	0	α_p

Table 4.3: In the above table we have introduced the quantities $\Psi_+ = max(0, \lambda_+^R, \lambda_+^L)$ and $\Psi_- = min(0, \lambda_-^R, \lambda_-^L)$, λ_+ and λ_- are, respectively, the maximum and minimum of λ_p, $\alpha_p = max\left(\mid \lambda_p^L \mid, \mid \lambda_p^R \mid\right)$ and $\beta_p = \frac{1}{2}\left(sgn(\lambda_p^L) + sgn(\lambda_p^R)\right)$. We denote by $\widetilde{\mathbf{U}}$ the state of the system according to Roe's average.

Several comments concerning Table I are in order:

i) If we take into account the orthonormality relations between the right and left eigenvectors

$$\sum_{p=1}^{d} \mathbf{l}_p \mathbf{r}_p = \mathcal{I} \tag{B6}$$

and the fact that the coefficients b_p and c_p are, in the case of HLLE, independents of p, then matrices $\widetilde{\mathcal{I}}^{L,R}$ and $\mathcal{Q}^{L,R}$ are, trivially, the unit matrix multiplied by the corresponding factors.

ii) For HLLE's and Roe's flux formulae their corresponding matrices $\widetilde{\mathcal{I}}^{L,R}$ and $\mathcal{Q}^{L,R}$ satisfy the relations: $\widetilde{\mathcal{I}}^L = \widetilde{\mathcal{I}}^R$, $\mathcal{Q}^L = \mathcal{Q}^R$

iii) As it is well known, the knowledge of the spectral decomposition of the Jacobian matrices is a basic ingredient to build up Riemann solvers or many flux formulae. Nevertheless, while HLLE's flux formula only needs the values of the maximum and minimum speeds of propagation of the signals, Roe's and Marquina's flux formulae need explicitly the full knowledge of the spectral decomposition, including right and left eigenvectors.

The system governing the evolution of multidimensional relativistic perfect fluids can be written in Cartesian coordinates in the form (2.7), with $d = 5$, where, in units such that the speed of light $c = 1$, the vector of unknowns $\mathbf{U}$ is given by (2.8), and the fluxes $\mathbf{F}^i$ are defined by (2.9)

A very worthy simplification on the calculation of matrices $\mathcal{Q}$ arises when some eigenvalue is degenerate, i.e., when the system is not strictly hyperbolic. In SRH, like in multidimensional Newtonian hydrodynamics, there is a *linearly degenerate* field, $p = 0$, such that the corresponding eigenvalue λ_0 is triple (the system in the j-direction splitting is not strictly hyperbolic, although the set of eigenvectors is complete). According to equation (B5), and using the orthonormality relations between the right and left eigenvectors

$$\sum_{k=1}^{3} r^m_{0,k} l^n_{0,k} = \delta^{mn} - r^m_+ l^n_+ - r^m_- l^n_- \tag{B7}$$

where $m, n = 1, \ldots 5$ denote the components of the 5-vector, it is possible to eliminate the three eigenvectors associated to the degenerate field and to write down the following simplified expression (omitting L,R superscripts)

$$\mathcal{Q}^{mn} = c_0 \delta^{mn} + (c_+ - c_0) r^m_+ l^n_+ + (c_- - c_0) r^m_- l^n_- . \tag{B8}$$

Notice that only $r_\pm$ and $l_\pm$ are needed to evaluate the numerical viscosity. The same procedure can be applied to any system of conservation laws where one of the eigenvectors has multiple degeneracy, because orthogonality relations always allow us to skip the explicit dependence on one of the vector subspaces of the spectral decomposition. In particular, it could be of great interest in the case of the equations of relativistic magnetohydrodynamics where, in the ansatz of a directional splitting, similar degeneracy arises in the structure of the characteristic fields associated to each one of the fluxes.

The explicit formulae for the numerical viscosity term corresponding to the system of equations of special relativistic hydrodynamics are:

HLLE's flux formulae. Since the numerical viscosity matrix is proportional to the identity, the application of the above recipes is obvious.

Roe's flux formulae. The numerical flux across some given interface can be written

$$\hat{\mathbf{F}}(\mathbf{U}^L, \mathbf{U}^R) = \frac{1}{2}[\mathbf{F}^L + \mathbf{F}^R + \mathbf{q}] \tag{B9}$$

$\mathbf{q}$ being the five–vector calculated from the corresponding numerical viscosity matrices of Table I:

$$\mathbf{q} = \mathcal{Q}(\mathbf{U}^L - \mathbf{U}^R) \equiv \mathcal{Q}\Delta\mathbf{U} \tag{B10}$$

In Roe's Riemann solver the quantities relative to the spectral decomposition are evaluated using the corresponding Roe-averages of the left and right states, denoted by $\widetilde{\mathbf{U}}$ (see Roe 1981, for the Newtonian case and Eulderink & Mellema, 1995, for the relativistic case). In practice, other particular averaging (e.g., arithmetic means) have also been used. Note that in the following expressions (B11) all quantities are evaluated using Roe's average, except for Δu_m. After some algebra, the viscosity vector associated to the numerical flux in the j-direction is

$$\begin{aligned}
q_1 &= \mid \lambda_0 \mid \Delta u_1 + \chi_a \\
q_2 &= \mid \lambda_0 \mid \Delta u_2 + hW(v_x\chi_a + \chi_b\delta_{jx}) \\
q_3 &= \mid \lambda_0 \mid \Delta u_3 + hW(v_y\chi_a + \chi_b\delta_{jy}) \\
q_4 &= \mid \lambda_0 \mid \Delta u_4 + hW(v_z\chi_a + \chi_b\delta_{jz}) \\
q_5 &= \mid \lambda_0 \mid \Delta u_5 + hW(\chi_a + v_j\chi_b) - \chi_a
\end{aligned} \tag{B11}$$

where

$$\chi_a = \sum_{m=1}^{5} [(\mid \lambda_+ \mid - \mid \lambda_0 \mid) l_+^m + (\mid \lambda_- \mid - \mid \lambda_0 \mid) l_-^m]\, \Delta u_m \tag{B12}$$

$$\chi_b = \sum_{m=1}^{5} \left[(\mid \lambda_+ \mid - \mid \lambda_0 \mid) V_+^j l_+^m + (\mid \lambda_- \mid - \mid \lambda_0 \mid) V_-^j l_-^m\right] \Delta u_m \tag{B13}$$

$$V_\pm^j = \frac{\lambda_\pm - v^j}{1 - v^j\lambda_\pm} \tag{B14}$$

M and MM- flux formulae. The numerical flux across a given interface can be written like equation (B9) with

$$\mathbf{q} = \mathbf{q}^L - \mathbf{q}^R \tag{B15}$$

$$\mathbf{q}^{L,R} = \mathcal{Q}^{L,R}\mathbf{U}^{L,R} \tag{B16}$$

Omitting the superscripts L, R and taken into account the expressions in Table I for MM and the results in Donat el al. (1998), the viscosity vector in the x-splitting is:

$$\begin{aligned}
q_1^{L,R} &= \frac{h^2}{\Delta}\left\{M\left[\mathcal{A}_-\Omega_+ - \mathcal{A}_+\Omega_-\right] + p(c_+\aleph_+ - c_-\aleph_-)\right\} + \\
&\quad c_0 p\frac{W}{h}\left\{\frac{\mathcal{K}}{\mathcal{K}-1} + \frac{v_y^2+v_z^2}{1-v_x^2}\right\} \\
q_2^{L,R} &= \frac{h^3W}{\Delta}\left\{M\mathcal{A}_+\mathcal{A}_-\left[\Omega_+\lambda_+ - \Omega_-\lambda_-\right] + p(c_+\lambda_+\mathcal{A}_+\aleph_+ - c_-\lambda_-\mathcal{A}_-\aleph_-)\right\} + \\
&\quad c_0 pW^2v_x\left\{\frac{1}{\mathcal{K}-1} + 2\frac{v_y^2+v_z^2}{1-v_x^2}\right\} \\
q_3^{L,R} &= \frac{h^2W}{\Delta}v_y\left\{M\left[\Omega_+\mathcal{A}_- - \Omega_-\mathcal{A}_+\right] + p(c_+\aleph_+ - c_-\aleph_-)\right\} + \\
&\quad c_0 p\left\{\frac{W^2}{\mathcal{K}-1} + \frac{1+2W^2(v_y^2+v_z^2)}{1-v_x^2}\right\} \\
q_4^{L,R} &= \frac{h^2W}{\Delta}v_z\left\{M\left[\Omega_+\mathcal{A}_- - \Omega_-\mathcal{A}_+\right] + p(c_+\aleph_+ - c_-\aleph_-)\right\} + \\
&\quad c_0 p\left\{\frac{W^2}{\mathcal{K}-1} + \frac{1+2W^2(v_y^2+v_z^2)}{1-v_x^2}\right\} \\
q_5^{L,R} &= \frac{h^2}{\Delta}\left\{M\left[\mathcal{A}_-\Omega_+\mathcal{D}_+ - \mathcal{A}_+\Omega_-\mathcal{D}_-\right] + p[c_+\aleph_+\mathcal{D}_+ - c_-\aleph_-\mathcal{D}_-]\right\} + \\
&\quad c_0 p\frac{W}{h}\left\{\frac{hW-\mathcal{K}}{\mathcal{K}-1} + \frac{(2hW-1)(v_y^2+v_z^2)}{1-v_x^2}\right\},
\end{aligned} \tag{B17}$$

with the following auxiliary quantities

$$M = \rho h W^2(\mathcal{K}-1), \quad \Omega_\pm = c_\pm(v_x - \lambda_\mp), \quad \mathcal{D}_\pm = hW\mathcal{A}_\pm - 1, \tag{B18}$$

where quantities $c_{\pm,0}$ are given in Table I and Δ is the determinant of the matrix of right-eigenvectors.

The corresponding viscosity vectors in the other directions are trivially obtained by a cyclic permutation of subindices x, y, z.

We have tested the efficiency of our numerical proposal, for Roe's and MM's flux formulae, running GENESIS, without any optimization at compilation level, in a SGI Origin 2000. A standard initial value problem has been chosen: $\rho_L = 10$, $\epsilon_L = 2$, $v_L = 0$, $\gamma_L = 5/3$, $\rho_R = 1$, $\epsilon_R = 10^{-6}$, $v_R = 0$ and $\gamma_R = 5/3$, where the subscript L (R) denotes the state to the left (right) of the initial discontinuity. This test problem has been considered by several authors in the past (see §2.4 for details in 1D, 2D and 3D).

We have compared the performance of two different implementations of the numerical flux subroutine:

i) *Case A*, stands for the results obtained using our analytical prescription. This means to write down, in the numerical flux routine, just the expressions derived here for the viscosity vector $\mathbf{q}$.

ii) *Case B*, stands for the results obtained running the code with a standard high-efficiency subroutine for inverting matrices (we use a LU decomposition plus an implicit pivoting which is, for general matrices, $O(N^3)$). This subroutine is called to get the left eigenvectors from the matrix of right eigenvectors and is adapted to the particular dimensions of the matrices (3×3, in 1D, 4×4, in 2D and 5×5, in 3D). Hence, unlike case A, now we have to calculate numerically the following quantities: the matrix of left eigenvectors, the characteristic variables and, finally, the components of the viscosity vector $\mathbf{q}$.

TABLE II

CPU time (in microseconds).

		TCI (μs)			
		Roe		MM	
Case	# Zones	Case *A*	Case *B*	Case *A*	Case *B*
1D	$100 \times 1 \times 1$	12.2	53.8	23.8	118.9
2D	$20 \times 20 \times 1$	25.5	181.8	49.0	373.5
3D	$14 \times 14 \times 14$	39.4	431.9	75.7	879.0

Table 4.4: Time per numerical cell and iteration (TCI) in microseconds employed by the numerical flux routine in our test-bed problem, for three different grids.

Table II summarizes the results: the direct implementation of our numerical viscosity formulae leads to an improvement of the efficiency (in terms of CPU time) of the numerical fluxes subroutine in a factor which, in 3D calculations, ranges between about *eleven* and *twelve* depending on the particular flux formula used. When comparing Roe's and MM's cases a factor two –in favour of Roe– arises due to the fact that MM's flux formulae needs to compute two viscosity vectors (one per each side of a given interface), unlike Roe's flux formula which needs only one viscosity vector evaluated at the average state. As it must be, the efficiency increases with the number of spatial dimensions involved in the problem due to the computationally expensive matrix inverting operations performed at each interface to get the numerical fluxes. Since the numerical flux routine is, typically, one of the most time-consuming, it translates into a speed up factor between *two* and *four* in the total execution time, depending on the specific weight of the flux formulae routine in each particular application.

Our formulation also gives a unified description of the numerical fluxes (B3), permitting a unique implementation with the possibility of switching in cases when the utilisation of a specific flux formula is more appropriate. In addition, due to the fact that we have eliminated, in the generalized MM's flux formula, all the *conditional clauses*, the efficiency is ensured either for scalar or vectorial processors.

Another worthy by-products of our algebraic pre-processing concerns with the significant reduction of round-off errors, as a consequence of the number of operations suppressed and factorization. One of the important issues in designing a multidimensional hydro-code is the accurate preservation of any symmetries present in a physical problem. A numerical violation of these symmetries could arise as a consequence of accumulation of round-off errors in the calculation of the numerical fluxes, as we have explained in §2.3.10. The algebraic simplifications, shown in the present appendix, reduce the number of operations and cure such problem.

Two last additional consequences arise from our work. First is that similar expressions can be worked out for any non-linear hyperbolic system of conservation laws for which the full spectral decomposition is known. In particular, when some of the vectorial subspaces has multiple degeneracy, a similar algebraic preprocessing is very convenient. The other important consequence is that an appropriate combination of a simplified formulation of the numerical viscosity together with the use of special relativistic Riemann solvers in General Relativistic Hydrodynamics (Pons et al. 1998), should allow a very easy and efficient extension to General Relativistic Hydrodynamics.

C. Explicit algorithm to recover primitive variables

In any RHD code evolving the conserved quantities Eq. (2.8) in time, the variables $\{p, v^1, v^2, v^3, \rho, \varepsilon\}$ have to be computed from the conserved quantities at least once per time step. In GENESIS this is achieved using Eqs. (2.1)–(2.3) and the equation of state. For an ideal gas equation of state with constant γ, this implies to find the root of the function

$$f(p) = (\gamma - 1)\rho_* \varepsilon_* - p \tag{C1}$$

with ρ_* and ε_* given by

$$\rho_* = \frac{D}{W_*} \tag{C2}$$

and

$$\varepsilon_* = \frac{\tau + D\,(1 - W_*) + p\,(1 - W_*^2)}{D\,W_*}, \tag{C3}$$

where

$$W_* = \frac{1}{\sqrt{1 - \mathbf{v}_* \cdot \mathbf{v}_*}}, \tag{C4}$$

and

$$\mathbf{v}_* = \frac{\mathbf{S}}{\tau + D + p}. \tag{C5}$$

The zero of $f(p)$ in the physically allowed domain $p \in]p_{\mathrm{min}}, \infty[$ determines the pressure. The monotonicity of $f(p)$ in that domain ensures the uniqueness of the solution. The lower bound of the physically allowed domain, p_{min}, defined by

$$p_{\mathrm{min}} = |\mathbf{S}| - \tau - D, \tag{C6}$$

is obtained from Eq. (C5) taking into account that (in our units) $|\mathbf{v}| \leq 1$. Knowing p, Eq. (C5) then directly gives $\mathbf{v}$, while the remaining state quantities are straightforwardly computed from Eqs. (2.1)–(2.3) and the definition of the Lorentz factor.

In GENESIS, the solution of $f(p) = 0$ is obtained by means of a Newton–Raphson iteration in which the derivative of f, f', is approximated by

$$f' = |\mathbf{v}_*|^2 c_{s*}^2 - 1, \tag{C7}$$

where c_{s*} is the sound speed given by

$$c_{s*} = \sqrt{\frac{(\gamma - 1)\gamma\varepsilon_*}{1 + \gamma\varepsilon_*}}. \tag{C8}$$

This approximation tends to the exact derivative when the solution is approached. On the other hand, it easily allows one to extend the present algorithm to general equations of state.

D. Transfer equations for the synchrotron radiation

The presence of shock waves into the a jet fluid enhance the emission because a shock is usually a very efficient converter of bulk kinetic energy into internal energy of the electrons and energy associated to the magnetic field, and we know that the total emitted power of a charged particle in the form of synchrotron emission is (see *e.g.*, Pacholczyk 1970)

$$\frac{dE}{dt} = \frac{2e^4}{3m^4c^7} B^2 \sin^2 \alpha E^2 \tag{D1}$$

where e and m are the charge and the mass of the particle (electrons or positrons in our case), B is the magnetic field strength, E is the energy of the particle and α is the angle between the velocity and the magnetic field direction.[18]

In order to calculate the synchrotron emission that reaches us coming from a high number of electrons in a relativistic jet, it is necessary (as mentioned before) to solve the transfer equations along the line of sight. To do it we need to know first the emission and absorption coefficients for a set of electrons (expressed in the reference frame of the magnetic field (1,2), see below), which are respectively (see Gómez 1993)

$$\epsilon^{(i)} = \frac{\sqrt{3}e^3}{8\pi mc^2} B \sin\vartheta \int_{E_{min}}^{E_{max}} N(E)[F(x) \pm G(x)]dE \tag{D2}$$

$$\kappa^{(i)} = -\frac{\sqrt{3}e^3}{8\pi m\nu^2} B \sin\vartheta \int_{E_{min}}^{E_{max}} E^2 \frac{d}{dE}\left(\frac{N(E)}{E^2}\right)[F(x) \pm G(x)]dE \tag{D3}$$

where ν is the frequency, $N(E)$ is the electron density with an energy between E and $E + dE$, ϑ being the angle between the magnetic field and the line of sight, the functions F and G are related with the second order Bessel functions as follows

$$F(x) = x \int_x^\infty K_{5/3}(z)dz$$
$$G(x) = xK_{2/3}(x)$$

with $x = \nu/\nu_c$, ν_c being the critical frequency, at which an electron of energy E emits approximately the largest quantity of synchrotron emission, and corresponds

[18]Note that the synchrotron emission grows as the second power of the magnetic field and the energy of the electrons.

to

$$\nu_c = \frac{3e}{4\pi m^3 c^5} B \sin\vartheta E^2.$$

In the equations (D2) and (D3) superindex (i) denotes polarization. The '+' sign corresponds to the polarization $i = 1$ (*i.e.,* in the direction of the axis 1 –normal to the projection of the magnetic field over the plane of the sky–), and the sign '–' to $i = 2$ (*i.e.,* in the direction of the axis 2 –which is the direction of the projection of the magnetic field over the plane of the sky). In general, as E_{min} and E_{max} may have any value in the range $[0, \infty]$, there is no analytical expression of the emission and absorption coefficients and they must be calculated numerically.

Another element that we need to establish is how the internal energy is distributed among the relativistic electrons. Given that observed radio spectra from emission regions which are transparent to radiation are of the power law form $dE/dt \propto \nu^{-\alpha}$, where α is a constant (Kembhavi & Narlikar 1999), and since the electrons receive their random motions through shock-heating, usually it is assumed (following the treatment of non-relativistic shocks) that they develop a power law distribution of energies (see *e.g.,* Blandford 1990):

$$N(E)dE = N_0 E^{-p} dE, \quad E_{min} \le E \le E_{max} \tag{D4}$$

where p is the spectral index of the electrons, which usually takes values between one and three. Neglecting radiative energy losses, the ratio C_E between the maximum and minimum energy can be considered constant along all the jet, which allows us to treat this as a parameter of the model.

The power law is fully determined by the equations

$$N_0 = \left[\frac{\mathcal{U}(p-2)}{1 - C_E^{2-p}}\right]^{p-1} \left[\frac{1 - C_E^{1-p}}{\mathcal{N}(p-1)}\right]^{p-1} \tag{D5}$$

and

$$E_{min} = \frac{\mathcal{U}}{\mathcal{N}} \frac{p-2}{p-1} \frac{1 - C_E^{1-p}}{1 - C_E^{2-p}}, \tag{D6}$$

$\mathcal{U}$ and $\mathcal{N}$ being the electron energy density and the number density, respectively; the values of these two parameters as a function of the position in the jet are calculated by the hydrodynamical code. The largest number of electrons is around E_{min} and hence this is also the characteristic electron energy.

The next element that we have to consider in order to evaluate the synchrotron emission is the strength and orientation of the magnetic field. As the hydro-code is not able to calculate the magnetic field evolution we have to establish it in a somewhat arbitrary way. Following Wilson & Scheuer (1983) and Gómez *et al.* (1995, 1997), we assume that the magnetic energy density remains a fixed fraction of the particle energy density, which leads to a field with magnitude proportional to $\mathcal{U}^{1/2}$.

The minimum value E_{min} plays an important role in the *Faraday rotation* and *depolarization* of the polarized flux. The Faraday rotation is the rotation of the polarization plane of a linearly polarized wave which is the result of the superposition of two circularly polarized waves with the same amplitude but rotating in opposite senses and with different initial phases.

The differential change in the polarization angle, $d\chi_F$, (in rad) per unit of length ds (in cm) due to Faraday rotation is determined by

$$\frac{d\chi_F}{ds} = 2.36 \times 10^{-17} N_e B_{||} \lambda^2 \tag{D7}$$

where λ is the wavelength (in cm), $B_{||}$ is the projection of the magnetic field (in Gauss) along the line of sight, and N_e is the electronic density (cm^{-3}) (which is the result of the integration of (D4) in the allowed range of energies).

Of course, different parts of the jet may present different Faraday rotations (due *e.g.*, to differences in the path length followed by the waves), being the global effect of such variations a depolarization (Garrington *et al.* 1988; Laing 1988) or decrease in the polarization degree of the synchrotron emission (hence, the inclusion of this mechanism is necessary to calculate properly the polarization properties of the radiation). The Faraday rotation or depolarization can take place everywhere between the source and the observer, which makes very difficult to determinate exactly the intrinsic polarization of the source.

The last elements that we need to know in order to write the transfer equations are the Stokes parameters. The Stokes parameters are four quantities that express in a convenient form the radiation field. Their definition is (see *e.g.,* Gómez 1993):

$$
\begin{aligned}
I &= \frac{c}{4\pi}[<(\mathcal{E}_0^a)^2> + <(\mathcal{E}_0^b)^2>] \\
Q &= \frac{c}{4\pi}[<(\mathcal{E}_0^a)^2> - <(\mathcal{E}_0^b)^2>] \\
U &= \frac{c}{4\pi}2<\mathcal{E}_0^a\mathcal{E}_0^b\cos\delta> \\
V &= \frac{c}{4\pi}2<\mathcal{E}_0^a\mathcal{E}_0^b\sin\delta>
\end{aligned}
$$

where $\delta = \phi_a - \phi_b$ is the phase shift, $\mathcal{E}^{a,b}$ being the electric vector of a monochromatic electromagnetic wave, of frequency ν, which may be written as

$$
\begin{aligned}
\mathcal{E}^a &= \mathcal{E}_0^a \sin(2\pi\nu - \phi_a) \\
\mathcal{E}^b &= \mathcal{E}_0^b \sin(2\pi\nu - \phi_b)
\end{aligned}
$$

$\mathcal{E}_0^a$ and $\mathcal{E}_0^b$ being the amplitudes in two perpendicular directions, namely (a, b), which form, together with the wave propagation direction, the *observer frame.*

The physical meaning of the Stokes parameters is the following: I is the total specific intensity, Q and U tell us the type of polarization, and V indicates if the polarization is *left* or *right*, and is zero if the polarization is linear. For the synchrotron radiation, the polarization is always linear, so $V_{syn} = 0$.

Two additional quantities related with the Stokes parameters are the *degree of polarization*, $\Pi = (Q^2 + U^2)^{1/2}$, and the *polarization angle*, $\chi = \frac{1}{2}\arctan(U/Q)$. Finally, the *transfer equations for the synchrotron radiation in the observer frame* are (Gómez 1993)

$$
\begin{aligned}
\frac{dI^{(a)}}{ds} &= I^{(a)}\left[-\kappa^{(1)}\sin^4\chi_B - \kappa^{(2)}\cos^4\chi_B - \frac{1}{2}\kappa\sin^2 2\chi_B\right] \\
&\quad + U\left[\frac{1}{4}(\kappa^{(1)} - \kappa^{(2)})\sin 2\chi_B + \frac{d\chi_F}{ds}\right] \qquad \text{(D8)} \\
&\quad + \epsilon^{(1)}\sin^2\chi_B + \epsilon^{(2)}\cos^2\chi_B \\
\frac{dI^{(b)}}{ds} &= I^{(b)}\left[-\kappa^{(1)}\cos^4\chi_B - \kappa^{(2)}\sin^4\chi_B - \frac{1}{2}\kappa\sin^2 2\chi_B\right] \\
&\quad + U\left[\frac{1}{4}(\kappa^{(1)} - \kappa^{(2)})\sin 2\chi_B - \frac{d\chi_F}{ds}\right] \qquad \text{(D9)} \\
&\quad + \epsilon^{(1)}\cos^2\chi_B + \epsilon^{(2)}\sin^2\chi_B
\end{aligned}
$$

$$
\begin{aligned}
\frac{dU}{ds} &= I^{(a)}\left[\frac{1}{2}(\kappa^{(1)}-\kappa^{(2)})\sin 2\chi_B - 2\frac{d\chi_F}{ds}\right] \\
& \quad I^{(b)}\left[\frac{1}{2}(\kappa^{(1)}-\kappa^{(2)})\sin 2\chi_B + 2\frac{d\chi_F}{ds}\right] \\
& \quad -\kappa U - (\epsilon^{(1)}-\epsilon^{(2)})\sin 2\chi_B
\end{aligned} \tag{D10}
$$

where $\kappa = 1/2(\kappa^{(1)} - \kappa^{(2)})$ is the mean value of the absorption coefficient; the superscripts $(a),(b)$ and $(1),(2)$ refer respectively to the axes (a) and (b) of the observer frame and the frame in which the axis (2) is parallel to the projection of the magnetic vector in the plane perpendicular to the direction of propagation of the electromagnetic field (axis (1) is perpendicular to this direction). χ_B is the angle that the direction (2) (*i.e.*, the projection of the magnetic field over the plane of the sky) forms with the (a) axis. $I^{(a)}$ and $I^{(b)}$ are the specific intensities over the axis (a,b) respectively, and are related with I by $I = I^{(a)} + I^{(b)}$ ($I^{(a),(b)} = (c/4\pi) < (\mathcal{E}_0^{a,b})^2 >$).

The emission is then calculated by integrating equations (D8) to (D10) while accounting for relativistic effects such as Doppler boosting and light aberration. The Doppler boosting affects the transfer equations by changing the emission and absorption coefficients as follows

$$\epsilon(\nu) = \mathcal{D}^2\epsilon^{'}(\nu^{'}) \tag{D11}$$

$$\kappa(\nu) = \mathcal{D}^{-1}\kappa^{'}(\nu^{'}) \tag{D12}$$

where variables without primes are measured in the source rest frame and the primed ones are measured in the comoving frame. $\mathcal{D}$ is the Doppler factor

$$\mathcal{D} = W^{-1}(1 - v\cos\Theta)^{-1}, \tag{D13}$$

Θ being the angle between the fluid direction and the observer.

The light aberration changes the angle of the line of sight when a relative movement between two inertial frames exists. This aberration has two effects in the emission model (Gómez 1993): (1) in the calculus of the emission and absorption coefficients (eqs. D2 and D3) by changing ϑ (the angle between the line of sight and the magnetic field), because both coefficients where expressed in the fluid frame; and (2) changing the polarization properties through the angle χ_B.

Let us remark, that if the magnetic field were completely random in direction, both, ϑ and χ_B would be totally random too, and the light aberration would not cause any effect on the emission. Actually, the randomness degree of the magnetic field determinates, with the Lorentz factor, the corrections due to the effect of light aberration.

Another important effect in the emission time evolution is the light-travel time delays, especially for small viewing angles (*e.g.,* Gómez *et al.* 1997). Such time delays whithin the jet have been ignored assuming that the calculated emission corresponds to that of a stationary jet. This is a plausible approximation for the case of the slow evolving large scale structure in jets. We have not included such effect because it introduces an enormous amount of computational resources and/or complexity. The reason for this is that it involves computation of the emission and absorption coefficients at different (source) times in order to compute a single image in the observer frame. Considering that each 3D RHD model needs $\sim$ 225 Mb and we need to save about 1000 models, about 225 Gb are requested in order to produce a radio image. Although this represents a significant computational effort, it allows the calculation of different emission models (*i.e.,* different viewing angles, observation time and frequency, etc.) with a single hydrodynamical run. A different approach that avoids the storage problems is to calculate the emission at run time. In this alternative it is necessary to pre-select one or several viewing angles before executing the hydrodynamical run. Then, only those data in the appropriate time intervals and predefined directions are saved. The main advantage of the first approach is that after the run it is possible to calculate the radio maps for any desired angle, while in the second case, one needs to infer in advance which are going to be the more interesting viewing angles (and experience shows us that this is usually a *fine tuning* task).

REFERENCES

Abell, G.O. & Margon, B., 1999, *Nature*, 279, 701.

Akerlof, C.W., *et al.* 1999, Nature, 398, 400.

Akujor, C.E. 1992, A&A, 259, L61.

Aloy, M.A., Ibáñez, J.Mª., Martí, J.Mª., & Müller, E. 1999a, ApJS, 122, 151.

Aloy, M.A., Ibáñez, J.Mª., Martí, J.Mª., Gómez, J.L., & Müller, E. 1999b, ApJ, 523, L100.

Aloy, M.A., Ibáñez, J.Mª., Martí, J.Mª., Gómez, J.L., & Müller, E. 1999c, ApJ, submitted.

Aloy, M.A., Ibáñez, J.Mª., Martí, J.Mª., Gómez, J.L., & Müller, E. 2000, in preparation.

Aloy, M.A., Pons, J.A., & Ibáñez, J.Mª. 1999, Comp. Phys. Communications, accepted.

Aloy, M.A., Müller, E., Ibáñez, J.Mª., Martí, J.Mª., MacFadyen, A. 1999c, ApJ, submited.

Andersen, M.I. (1999), *Science* **283**, 2075.

Anile, A.M. 1989, Relativistic Fluids and Magneto-Fluids (Cambridge University Press).

Apple, S.,& Camenzind, M. 1992, A&A, 256, 354.

Attridge, J.M., Roberts, D.H., & Wardle, J.F.C. 1999, ApJ, 518, L87.

Balsara, D.S., & Norman, M.L. 1992, ApJ, 393, 631.

Baring, M.G. 1995, Ap&SS, 231, 169.

Band, D.L., *et al.* 1993, ApJ, 413, 281.

Blandford, R.D., & McKee, C.F. 1976, *Phys. Fluids*, 19, 1130.

Blandford, R.D., & Rees, M.J. 1974, MNRAS, 169, 395.

Blandford, R.D., & Königl, A. 1979, ApJ, 232, 34.

Blandford, R.D., & Znajek, R.L. 1977, MNRAS, 179, 433.

Blandford, R.D. 1990, in *Active Galactic Nuclei*, SSAS Saas-Fee Advanced Course 20, Lectures Notes, eds. T.J.-L. Courvoisier & M. Mayor. (Springer-Verlag), p. 161.

Blaes, O.M. 1994, ApJS, 92, 643.

Begelman, M.C., & Cioffi, D.C. 1989, ApJ, 345, L21.

Begelman, M.C., Blandford, R.D., & Rees, M.J. 1984, Rev. Mod. Phys., 56, 255.

Begelman, M.C., Rees, M.J., & Sikora, M. 1994, ApJ, 429, L57.

Begelman, M.C., Rees, M.J., & Blandford, R.D. 1979, Nature, 279, 770.

Benson, J.M., Walker, R.C., Unwin, S.C., Muxlow, T.W.B., Wilkinson, P.N., Booth, R.S., Pilbratt, G., & Simon, R.S. 1988, ApJ, 334, 560.

Bicknell, G.V. 1984, ApJ, 239, 433.

Bicknell, G.V. 1996, in *Energy Transport in Radio Galaxies and Quasars*, eds. P.E. Hardee, A.H. Bridle, J.A. Zensus. ASP Conference Series, Vol. 100, p. 253.

Biretta, J.A., & Owen, F.N. 1990, in *Parsec-scale Radio Jets*, ed. J.A. Zensus & T.J. Pearson (Cambridge: Cambridge University Press), p. 125.

Biretta, J.A., Zhou, F., & Owen, F.N. 1995, ApJ, 447, 582.

Blinnikov, S.I., Dunina-Barkovskaya N.V., & Nadyozhin D.K. 1996, ApJS, 106, 171.

Bodo, G., Massaglia, S., Rossi, P., Rosner, R., Malagoli, A., & Ferrari, A. 1995, A&A, 303, 281.

Bodo, G., Rossi, P., Massaglia, Ferrari, A., Malagoli, A., & Rosner, R. 1998, A&A, 333, 1117.

Bodo, G., 1998, in *Astrophysical jets: Open problems*, eds. S. Massaglia & G. Bodo, Gordon and Breach, p.161.

Bowman, M., 1994, MNRAS, 269, 137.

Bridle, A.H., Hough, D.H., Lonsdale, C.J., Burns, J.O., & Laing, R.A. 1994, AJ, 108, 766.

Briggs, M.S., Band, D.L., Kippen, R.M., Preece, R.D., Kouveliotou, C., *et al.* (1999), ApJ, submitted; and astro-ph/9903274.

Browne, I.W.A. 1987, in *Superluminal Radio Sources*, eds. J.A. Zensus & T.J. Pearson (Cambridge: Cambridge University Press), p. 111.

Camenzind, M. 1993, in *Jets in Extragalactic Radio Sources*, eds., H.J. Röser & K. Meisenheimer (Springer-Verlag, Berlin), p. 109.

Castro-Tirado, A.J. 1999, A&SS, in press; and astro-ph/9903187.

Cavallo, G. & Rees, M.J., 1978, MNRAS, 183, 359.

Centrella, J. & Wilson, J.R. 1984, ApJS, 54, 229.

Chirstiansen, W.A., Scott, J.S., & Vestrand, W.T., 1978, ApJ, 223, 13.

Clarke, D.A., Norman, M.L., & Burns, J.O. 1989, ApJ, 342, 700.

Cohen, M.H., Cannon, W., Purcell, G.H., Shaffer, D.B., Broderick, J.J., Kellermann, K.I., & Jauncey, D.L. 1971, ApJ, 170, 207.

Colella, P., & Woodward, P.R. 1984, JCP, 54, 174.

Colgate, S.A., & Johnson, M.H. 1960, Phys. Rev. Lett., 5, 235.

Costa, E., *et al.* 1987, Nature, 387, 783.

Cox, J.P., & Giulli, R.T. 1968, in *Principles of Stellar Structure*, ed. Gordon & Breach, New York.

Curtis, H.D. 1918, *Publ. Lick Obs.*, 13, 9.

Daigne, F. 1999, Ph. D. Thesis, Universite de Paris Sud, U.F.R. Scientifique d'Orsay.

Daigne, F., & Moschkovitch, R. 1999, in Procedings of the Rome Conference *Gamma-Ray Bursts in the Afterglow Era*, astro-ph/9901343.

Daly, R.A. 1995, ApJ, 454, 580.

Daly, R.A., & Marscher, A.P., 1988, ApJ, 334, 539.

Davis, S.F. 1984, NASA Contractor Rep. 172373, ICASE Rep., No.84-20.

Dermer, C.D., & Mitman, K.E. 1999, ApJ, 513, 656.

De Young, D. 1993, ApJ, 405, L13.

Dezalay, J., Lestrade, J., Barat, C., Talon, R., Terenkhov, O., Sunyaev, R., & Kuznetsov, A. 1996, ApJ, 471, L27.

Djorgovski, S.G., Kulkarni, S.R., Bloom, J.S., Goddrich, R., Frail, D.A., Piro, L. and Palazzi, E. 1998, ApJ508, L17.

Donat, R., Font, J.A., Ibáñez, J.M$^{\underline{\rm a}}$., & Marquina, A. 1998, *J. Comp. Phys.*, 146, 58.

Donat, R., & Marquina, A. 1996, JCP, 125, 42.

Duncan, G.C., & Hughes, P.A. 1994, ApJ, 436, L119.

Duncan, G.C., & Hughes, P.A., & Opperman, J., 1996, in *ASP Conf. 100, Energy Transport in Radio Galaxies and Quasars*, ed. P.E. Hardee, A.H. Bridle, & J.A. Zensus (San Francisco: ASP), p. 143.

Dubal, M.R., & Pantano, O. 1993, MNRAS, 261, 203.

Eichler, D., Livio, M., Piran, T. & Schramm, D.N., 1989, Nature, 340, 126.

Einfeldt, B. 1998,*SIAM J. Numer. Anal.*, 25, 294.

Elgroth, P.G. 1971, *Phys. Fluids*, 14, 2631.

Elgroth, P.G. 1972, *Phys. Fluids*, 15, 2140.

Eulderink, F., & G. Mellema, G. 1994, A&A, 284, 652.

Eulderink, F., & G. Mellema, G. 1995, A&AS, 110, 587.

Fanti, C., Fanti, R., Parma, P., Schilizzi, R.T., & van Breugel, W.J.M., 1985, *Astr. Ap.*, 143, 292.

Fanti, C., Fanti, R. 1986, in *Superluminal Radio Sources*, eds. J.A. Zensus & T.J. Pearson (Cambridge: Cambridge University Press), p. 174.

Fanti, C., Fanti, R., Schilizzi, R.T.,Spencer, R.E., Nanrendog, Parma, P., van Breugel, W.J.M., & Venturi, T. 1990, A&A, 231, 333.

Falle, S.A.E.G., & Komissarov, S.S. 1996, MNRAS, 278, 586.

Faranoff, B.L.,& Riley, J.M. 1974, MNRAS, 167, 31.

Fenimore, E., Epstein, R.I., & Ho, C.H. 1993, A&AS, 97, 59.

Fenimore, E., Madras, C., & Nayakshin, S. 1996, ApJ, 473, 998.

Fenimore, E., Ramirez, E., & Summer, M.C. 1998, in *Gamma-Ray Bursts, 4th Huntsville Symposium*, C. Meegan, R. Preece & T. Koshut, Eds. AIP Conf. Proc. 428 (New York: AIP).

Fishman, G. & Meegan, C., 1995, ARA&A, 33, 415.

Font, J.A., Ibáñez, J.M$^{\underline{a}}$., Marquina, A., & Martí, & J.M$^{\underline{a}}$. 1994, A&A, 282, 304.

Frail, D.A., *et al.* 1997, Nature, 389, 261.

Fryer, D.A., $ Wooslwey, S.E. 1998, ApJ, 502, L9.

Fryer, D.A., $ Wooslwey, S.E., Herant, M., & Colgate, S.A. 1999, ApJ, 520, 650.

Fruchter, A.S., *et al.* 1999, ApJ, 519, L13.

Galama, T.J., Vreeswijk, P.M., van Paradijs, J. Kouveliotou, C., Augusteijn, T., *et al.*. 1998, Nature395, 670.

Garrington, S.T., Leahy, J.P., Conway, R.G., & Laing, R.A. 1988, Nature, 331, 147.

Ghisellini, G., Padovani, P., Celotti, A., & Maraschi, L. 1993, ApJ, 407, 65.

Gómez, J.L. 1993, Ph. D. Thesis, Instituto de Astrofísica de Andalucía.

Gómez, J.L., Alberdi, A., & Marcaide, J.M. 1993, A&A, 274, 55.

Gómez, J.L., Alberdi, A., & Marcaide, J.M. 1994, A&A, 284, 51.

Gómez, J.L., Alberdi, A., Marcaide, J.M., Travis, J.P. 1994, A&A, 292, 33.

Gómez, J.L., Martí, J.M$^{\underline{a}}$, Marscher, A.P., Ibáñez, J.M$^{\underline{a}}$ & Marcaide, J.M. 1995, ApJ, 449, L19.

Gómez, J.L., Martí, J.M$^{\underline{a}}$, Marscher, A.P., Ibáñez, J.M$^{\underline{a}}$., & Alberdi, A. 1997, ApJ, 482, L33.

Gómez, J.L., Marscher, A.P., Alberdi, A., Martí, J.M$^{\underline{a}}$, & Ibáñez, J.M$^{\underline{a}}$. 1998, ApJ, 499, 221.

Gómez, J.L., Marscher, A.P., Alberdi, A., & Gabuzda, D.C. 1999, ApJ, accepted, or astro-ph/9906199.

Gómez, J.L., Marscher, A.P., & Alberdi, A. 1999, ApJ, in press.

Goodman, J., 1986, ApJ, 308, L47.

Goodman, J., 1997, *New Astron.*, 5, 449.

Granot, J., Piran, T., & Sari, R. 1999, ApJ, 513, 679.

Grot, P.J., Piran, *et al.* 1998, ApJ, 502, L123.

Halpern, J.P, Kemp, J., Piran, T. and Bershady, M.A. 1999, ApJ, 517, 105 ; and astro-ph/9903418.

Hardee, P.E., & Norman, M.L. 1988, ApJ, 334, 70.

Hardee, P.E., Cooper, M.A., Norman, M.L., & Stone, J.M. 1992, ApJ, 399, 478.

Hardee, P.E., Clarke, D.A., Howell, D.A. 1995, ApJ, 441, 644.

Hardee, P.E. 1996, in *Energy transport in radio galaxies and quasars*, ASP Conference Series, Vol. 100, eds. P.E. Hardee, A.H. Bridle, J.A. Zensus, p. 273.

Hardee, P.E., & Rosen, A. 1999, ApJ*accepted*; and astro-ph/9906490.

Harten, A., Lax P.D., & van Leer, B. 1983,*SIAM Rev.*, 25, 35.

Harten, A. 1983, *J. Comp. Phys.*, 49, 357.

Harrison, F.A., *et al.* 1999, ApJ, submitted, or astro-ph/9905306.

Hartmann, D., 1996, A&AS, 120, 31.

Hawley, J.F., Smarr, L.L., & Wilson, J.R. 1984, ApJS, 55, 221.

Hooimeyer, J.R.A., Barthel, P.D., Schilizzi, R.T., & Miley, G.K. 1992, A&A, 261, 25.

Hughes, P.A., Aller, H.D., & Aller, M.F. 1985, ApJ, 298, 301.

Hughes, P.A., Aller, H.D., & Aller, M.F. 1989a, ApJ, 341, 54.

Hughes, P.A., Aller, H.D., & Aller, M.F. 1989b, ApJ, 341, 68.

Hummel, C.A., *et al.* 1992, A&A, 257, 489.

Ibáñez, J.Mª., & Martí, J.Mª. 1998, *J. Comp. Applied Math.*, in press.

Iwamoto, T.J., Mazzali, P.A., Nomoto, K., Umeda, H., Nakamura, T., *et al.* (1998), Nature, 395, 672.

Jenkins, E.B. 1997, IAU Symposia, vol. 188, E97.

Jenninson, R.C., & Das Gupta, M.K. 1953, Nature, 172, 996.

Johnson, M.H., & McKee, C.F. 1971, Phys. Rev. D, 3, 858.

Jones, T.W. 1988, in *Supermassive black holes*, Proc. of the Third George Mason Astrophysics Workshop, Fairfax, VA, (Cambridge: Cambridge University Press), p. 59.

Jones, T.W., & O'Dell, S.L. 1977a, ApJ, 214, 522.

Jones, T.W., & O'Dell, S.L. 1977b, ApJ, 215, 236.

Jones, T.W., Tregillis, I.L., & Dongsu Ryu, 1999, astro-ph/9908315.

Junor, W., & Bireta, J.A. 1995, ApJ, 109, 500.

Katz, J.I., 1994, ApJ, 432(2), L107.

Katz, J.I., 1997, ApJ, 490, 633.

Katz, J.I., & Canel, L.M. 1996, ApJ, 471, 915.

Katz, J.I., & Piran, T. 1997, ApJ, 490, 772.

Kembhavi, A.K., & Narlikar, J.V. 1999, *Quasars and Active Galactic Nuclei. An introduction* (Cambridge: Cambridge University Press).

Klebesadal, R.W., Strong, I.B., & Olsen, R.A. 1973, ApJ, 182, L85.

Krichbaum, T.P., Quirrenbach, A., & Witzel 1992, in Variability of Blazars, ed. E. Vataoja & M. Valtonen (Cambrige: Cambridge Univ. Press), 331.

Kobayashi, S., Piran, T. and Sari, R. 1999, ApJ, 513, 669.

Kössl, D., & Müller, E. 1988, A&A, 206, 204

Koide, S. 1997, ApJ, 478, 66

Koide, S., Nishikawa, K.-I., & Muttel, R.L. 1996, ApJ, 463, L71

Koide, S., David, L.M., Shibata, K. & Kudoh, T. 1999a, to appear in *Proceedings of the 19th Texas Symposium on Relativistic Astrophisics*, or astro-ph/9907434.

Koide, S., David, L.M., Shibata, K. & Kudoh, T. 1999b, ApJ, *submitted*, or astro-ph/9907435.

Komissarov, S.S. 1990, Sov. Astron. Lett. 16(4), 284

Komissarov, S.S., & Falle, S.A.E.G. 1996, in Energy Transport in Radio Galaxies and Quasars, eds. P.E. Hardee, A.H. Bridle, J.A. Zensus. ASP Conference Series, Vol. 100, p. 327

Komissarov, S.S., & Falle, S.A.E.G. 1997, MNRAS, 288, 833.

Komissarov, S.S., & Falle, S.A.E.G. 1998, MNRAS, 297, 1087.

Königl, A., 1980, *Phys. Fluids*, 23, 1083.

Königl, A., 1981, ApJ, 243, 700.

Kössl, D., Müller, E., & Hillebrandt, W. 1990, A&A, 229, 378.

Kouveliotou, C., Meegan, C., Fishmann, G., Bhat, N., Briggs, M., Koshut, T., Paciesas, W., & Pendleton, G. 1993, ApJ, 413, L101.

Krolik J.H., & Pier, E.A. 1991, ApJ, 373, 277.

Krolik J.H. 1999, in *Active Galactic Nuclei*, ed. Ostriker J.P., & Spergel D.N., Princenton University Press.

Kulkarni, S.R., Djorgovski, S.G., Ramaprakash, A.N., Goodrich, R., Bloom, J.S., *et al.* 1998a, Nature, 393, 35.

Kulkarni, S.R., Frail, D.A., Wieringa, M.H., Ekers, R.D., Sadler, E.M., Wark, R.M., Higdon, J.L., Phinney, E.S. and Bloom, J.S. 1998b, Nature, 395, 663.

Kulkarni, S.R., Djorgovski, S.G., Odewahn, S.C., Bloom, J.S., Gal, R.R., *et al.* 1999a, Nature, 398, 389.

Kulkarni, S.R., Frail, D.A., Sari, R., Moriarty-Schieven, G.H., Shepherd, D.S., Udomprasert, P., Readhead, A.C.S., Bloom, J.S., Feroci, M. and Costa, E. 1999b, preprint astro-ph/9903441.

Laing, R.A. 1988, Nature, 331, 149.

Laing, R.A. 1996, in Energy Transport in *Radio Galaxies and Quasars*, eds. P.E. Hardee, A.H. Bridle, J.A. Zensus. ASP Conference Series, Vol. 100, p. 241.

Lind, K.R., Payne, D.G., Meier, D.L., & Blandford, R.D. 1989, ApJ, 344, 89.

Lobanov, A.P., Krichbaum, T.P., Witzel, A., Kraus, A., Zensus, J.A., Britzen, S., Otterbein, K., Hummel, C.A., & Johnston, K. 1998, A&A, 340, L60.

Loken, C., Burns, J.O., Clarke, D.A. & Norman, M.L. 1992, ApJ, 392, 54.

Loken, C., Roettiger, K., Burns, J.O., & Norman, M.L. 1995, ApJ, 445, 80.

LeVeque, R.J. 1991, Numerical Methods for Conservation Laws (Birkhäuser).

MacFadyen, A. & Woosley, S.E. 1998, ApJ, submitted; and astro-ph/980102748.

Mao, S., Narayan, R., & Piran, T. 1994, ApJ, 420, 171.

Maraschi, L. 1992, in *Variability of Blazars*, ed. E. Valtaoja & M. Valtonen (Cambridge: Cambridge University Press), p. 205.

Marquina, A., Martí, J.Mª., Ibáñez, J.Mª., Miralles, J.A., & Donat, R. 1992, A&A, 258, 566.

Marquina, A. 1994, SIAM J. Scient. Comp., 15, 892.

Marscher, A.P. 1980, ApJ, 235, 386.

Marscher, A.P., & Gear, W.K. 1985, ApJ, 298, 114.

Marscher, A.P., Gear, W.K., & Travis, J.P. 1992, in *Variability of Blazars*, ed. E. Vataoja & M. Valtonen (Cambridge: Cambridge University Press).

Martí, J.Mª., Ibáñez, J.Mª., & Miralles, J.A. 1991, Phys. Rev. D, 43, 3794.

Martí, J.Mª., & Müller, E. 1994, J. Fluid Mech., 258, 317.

Martí, J.Mª., Müller, E., & Ibáñez, J.Mª. 1994, A&A, 281, L9.

Martí, J.Mª, Müller, E., Font, J.A., & Ibáñez, J.Mª 1995, ApJ, 448, L105.

Martí, J.Mª., & Müller, E. 1996, JCP, 123, 1.

Martí, J.Mª., Font, J.A., Ibáñez, J.Mª.& Müller, E. 1996, in Energy Transport in Radio Galaxies and Quasars, eds. P.E. Hardee, A.H. Bridle, J.A. Zensus. ASP Conference Series, Vol. 100, p. 149.

Martí, J.Mª., Müller, E., Font, J.A., Ibáñez, J.Mª. & Marquina, A. 1997, ApJ, 479, 151.

Martí, J.Mª. 1997a, in Relativistic Jets in AGNs, ed. by M. Ostrowski, M. Sikora, G. Madegski, M. Begelman (Astronomical Observatory of the Jagiellonian University, Cracovia), p. 90.

Martí, J. Mª. 1997b, in *Relativistic Gravitation and Gravitational Radiation*, ed. J-A. Marck and J-P. Lasota (Cambridge University Press, Cambridge, UK).

Martí, J.Mª., Müller, E. & Ibáñez, J.Mª. 1998, in *Astrophysical jets: Open problems*, eds. S. Massaglia & G. Bodo, Gordon and Breach, p.149.

Massaglia, S., Bodo, G., & Ferrari, A. 1996, A&A, 307, 997.

Metzger, M.R., Djorgovski, S.G., Kulkarni, S.R., Steidel, C.C., Adelberger, K.L., Frail, D.A., Costa, E. and Frontera, F. (1997), Nature, 387, 878.

Mészáros, P. 1995, in *Proc. of the 17th Texas Symp. on Relativistic Astrophysics and Cosmology*, H. Böhringer, G.E. Morfill, & J.E. Trümper (eds.), N. Y. Acad. Sci., p. 440.

Mészáros, P. 1999, in *Proc. of the 19th Texas Symp. on Relativistic Astrophysics & Cosmology*, Paris, Dec. 1998; and astro-ph/9904038.

Mézáros, P., & Rees, M.J. 1992, MNRAS, 257, 29.

Mézáros, P., Laguna, P. & Rees, M.J. 1993, ApJ, 415, 181.

Mézáros, P., & Rees, M.J. 1997, ApJ, 482, L29.

Mioduszewski, A.J., Hughes, P.A., & Duncan, G.C. 1997, ApJ, 476, 649.

Mirabel, I.F., & Rodriguez, L.F. 1994, Nature, 371, 46.

Mochkovitch, R., Hernanz, M., Isern, J. & Martin, X. 1993, Nature, 361, 236.

Mochkovitch, R., Hernanz, M., Isern, J. & Loiseau, S. 1995, A&A, 293, 803.

Muxlow, T.W.B., & Garrington, S.T. 1991, in *Beams and Jets in Astrophysics*, ed. P.A. Hughes (Cambridge: Cambridge Univ. Press), 52.

Muxlow, T.W.B., & Wilkinson, P.N. 1991, MNRAS, 251, 54.

Narayan, R., Paczyński, B., & Piran, T. 1992, ApJ, 395, L83.

Narayan, R., & Piran, T. 1999, in preparation.

Nishikawa, K.–I., Koide, S., Sakai, J., Christodoulou, D.M., Sol, H., & Mutel, R.L. 1997, ApJ, 483, L45.

Nishikawa, K.–I., Koide, S., Sakai, J., Christodoulou, D.M., Sol, H., & Mutel, R.L. 1998, ApJ, 498, 166.

Norman, M.L., Smarr, L., Winkler, K.-H.A., & Smith, M.D. 1982, A&A, 113, 285.

Norman, M.L., & Winkler, K.-H.A. 1986, in *Astrophysical Radiation Hydrodynamics*, ed. M.L. Norman, K.-H.A. Winkler. Reidel, Dordrecht, p. 227.

Norman, M.L., Burns, J.O., & Sulkanen, M.E. 1988, Nature, 335, 146.

Norman, M.L., & Hardee, P.E. 1988, ApJ, 334, 80.

Norman, M.L. 1996, in *Energy Transport in Radio Galaxies and Quasars*, eds. P.E. Hardee, A.H. Bridle, J.A. Zensus. ASP Conference Series, Vol. 100, p. 319.

Norris, J., Nemiroff, R., Bonell, J., Scargle, J., Kouveliotou, C., Paciesas, W., Meegan, C., & Fishmann, G. 1996, ApJ, 459, 393.

Noh, W.F. 1987, JCP, 72, 1.

Owen, F.N., & Rudnick, L. 1976, ApJ, 205, L1.

Pacholczyk, Q.G. 1970, *Radio Astrophysics*, Freeman, San Francisco.

Pacyński, B. 1986, ApJ, 308, L43.

Pacyński, B., & Rhoads, J.E. 1993, ApJ, 418, L5.

Pacyński, B., & Xu, G. 1994, ApJ, 427, 708.

Pacyński, B. 1998, ApJ, 494, L45.

Padman R., Lasenby A.N., & Green 1991, in *Beams and Jets in Astrophysics*, ed. P.A. Hughes (Cambridge: Cambridge Univ. Press), 484.

Panaitescu, A., Wen, L., Laguna, P. and Mészáros, P. 1997, ApJ, 482, 942.

Panaitescu, A., & Mészáros, P., 1998, ApJ, 492, 683.

Panaitescu, A. & Mészáros, P. 1999, ApJ, in press; and astro-ph/9810258.

Pen, U.-L. 1998, ApJS, 115, 19.

Pearson, T.J., Unwin, S.C., Cohen, M.H., Linfield, R.P., Readhead, A.C.S., Seielstad, G.A., Simon, R.S., & Walker, R.C. 1981, Nature, 290, 365.

Philips, R.B., & Mutel, R.L. 1980, ApJ, 236, 89.

Phinney E.S. 1987, in *Superluminal Radio Sources*, eds. J.A. Zensus & T.J. Pearson (Cambridge: Cambridge University Press), p. 301.

Pian, E., Fruchter, A.S., Bergeroni, L.E., Thorsett, S.E., Frontera, F., *et al.* 1998, ApJ492, L103.

Pian, E., Amati, L., Antonelli, L.A., Butler, R.C., Costa, E., *et al.* (1999), A&A, submitted; and astro-ph/9903113.

Piran, T., Shemi, A. & Narayan, R. 1993, MNRAS, 263, 861.

Piran, T. 1997, in *Unsolved Problems in Astrophysics*, J.N. Bahcall & J.P. Ostriker (eds.), Princeton Univ. Press, p. 343.

Piran, T. 1999, Physics Reports, in press; and astro-ph/9810256.

Pons J.A., Font J.A., Ibáñez, J.Mª., Martí, J.Mª., & Miralles, J.A. 1998, A&A, 339, 638.

Popham, R., Woosley, S.E. & Fryer, C. 1998, ApJ, submitted; and astro-ph/9807028.

Porter, D.H. & Woodward, P.R. 1994, ApJS, 93, 309.

Prilutskii, O.F. & Usov, V.V. 1975, Ap&SS, 34, 395.

Rawlings, S., & Saunders, R. 1991, Nature, 349, 138.

Rayburn, D.R., 1977, MNRAS, 179, 603.

Rees, M.J., & Mészáros, P. 1994, ApJ, 430, L93.

Rees, M.J. 1997, in *Relativistic Astrophysics*, ed. Jones, B.J.T., & Marković, Cambridge Univ. Press, 251.

Rhoads, J.E. (1999), ApJ, submitted; and astro-ph/9903399.

Roe, P.L. 1981, *J. Comput. Phys.*, 43, 357.

Rosen, A., Hughes, P. A., Duncan, G. C., & Hardee, P. E. 1999, ApJ, in press.

Ruderman, M. 1975, *Ann. N.Y. Acad. Sci.*, 262, 164.

Ruffert, M. & Janka, H.-T., Shäfer G. 1996, A&A, 319, 122.

Ruffert, M., Janka, H.-T., Takahashi, K. & Schäfer, G. 1997, A&A, 319, 122.

Ruffert, M. & Janka, H.-T. 1998, A&A, 338, 535.

Ruffert, M. & Janka, H.-T. 1999a, A&A, 344, 573.

Ruffert, M. & Janka, H.-T. 1999b, ApJ, *submitted*, or astro-ph/9908290.

Rybicki, G.B., Lightman, A.P. 1979, *Radiative Processes in Astrophysics.* Wiley, New York, p. 110.

Shakura, N.I., & Sunyaev, R.A. 1973, A&A, 24, 337.

Sari, R., & Piran, T. 1995, ApJ, 455, 143.

Sari, R., & Piran, T. 1997, ApJ, 485, 270.

Sari, R., Piran, T., & Halpern, J.P. 1999, ApJsubmitted; and astro-ph/9903339.

Sari, R., Piran, T., & Narayan, R. 1998, ApJ, 497, 17.

Scheuer, P.A.G. 1984, MNRAS, 166, 513.

Schneider, V., Katscher, V., Rischke, D.H., Waldhauser, B., Marhun, J.A., & Munz, C.-D. 1993, JCP, 105, 92.

Sedov, L.I. 1959, in *Similarity and Dimensional Methods in Mechanics*, London, Chapt. 4.

Seielstad, G.A., Cohen, M.H., Linfield, R.P., Moffet, A.T., Rommey, J.D., Schilizzi, R.T., & Shaffer, D.B. 1979, ApJ, 229, 53.

Shapiro, P.R. 1979, ApJ, 233, 831.

Shapiro, P.R. 1980, ApJ, 236, 958.

Shone, D.L., Porcas, R.W., & Zensus, J.A. 1985, Nature, 314, 603.

Shu, C.W., & Osher, S.J. 1989, JCP, 83, 32.

Soffitta, P., Feroci, M., Pior, L., In'tzand, J., Heise, J., DiCiolo, L., Muller, J.M., Palazzi, E. and Frontera, F. 1998, *IAU Circ.*, 6884.

Strang, G. 1968, SIAM J. Num. Anal., 5, 506.

Swain, M.R., Bridle, A.H., & Baum, S.A. 1998, ApJ, 507, L29.

Tabani, M. 1998, ApJ, 497, 21.

Tingay, S.J., *et al.* 1995, Nature, 374, 141.

Tinney, C., Stathakis, R. & Cannon, R. 1998, *IAU Circ.*, 6896.

Tutukov, A.V., & Yungelson, L.R. 1994, MNRAS, 268, 871.

Usov, V.V. 1994, MNRAS, 267, 1035.

van Leer, B. 1979, JCP, 32, 101.

van Paradijs, J., *et al.* 1997, Nature, 386, 686.

van Putten, M.H.P.M. 1993, ApJ, 408, L21.

van Putten, M.H.P.M. 1996, ApJ, 467, L57.

Vietri, M. 1997, ApJ, 488, L105.

Vitello, P., & Salvati, M. 1976, *Phys. Fluids*, 19, 1523.

Vreeswijk, P.M., *et al.* 1999, ApJ, *in press*, or astro-ph/9904286.

Witti, J., Janka, H.-T., & Takahashi, K. 1994, A&A, 286, 841.

Walker, R.C., Benson, J.M., & Unwin, S.C. 1987, ApJ, 316, 546.

Walker, R.C. 1997, ApJ, 488, 675.

Weynmann, R.J., Morris, S.L., Foltz, C.B. & Hewett, P.C. 1991, ApJ, 373, 23.

Wen, L., Panaitescu, A., & Laguna, P. 1997, ApJ, 486, 919.

Whitney, A.R., Shapiro, I.I., Rogers, A.E.E., Robertson, D.S., Knight, C.A., Clark, T.A., Goldstein, R.M., Marandino, G.E., & Vandenberg, N.R. 1971, *Science*, 173, 225.

Wijers, R.A.M., & Galama, T.J. 1988, ApJ, *in press*, or astro-ph/9805341.

Wilson, J.R. 1979, in *Sources of Gravitational Radiation*, ed. L.L. Smarr (Cambridge University Press).

Wilson, M.J. 1987, MNRAS, 216, 971.

Wilson, M.J., & Scheuer, P.A.G. 1983, MNRAS, 205, 449.

Woodward, P.R., & Colella, P. 1984, *J. Comput. Phys.*, 54, 115.

Woosley, S.E. 1993, ApJ, 405, 273.

Woosley, S.E., Eastman, R.G. and Schmidt, B.P. 1998, ApJ, submitted; and astro-ph/9806299.

Zel'dovich, Y.B., & Raizer, Y.P. 1966, in *Physics of Shock Waves and High-Temperature Phenomena* (New York: Academic Press), p. 101.

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