On the number of zeros in the columns of the character table of a group

by

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Both authors research is partially supported by the Feder, the Spanish Ministerio de Ciencia y Tecnología, grant BFM2001-0180, and the University of the Basque Country, grant 1/UPV00127.310-E-13817/2001. The first author is also supported by the Basque Government.

1 Introduction

Let G be a finite group. In [10] we obtained group theoretical properties of G in terms of $m(G)$, which was defined to be the maximum number of zeros in a row of the character table, and $n(G)$, the minimum number of zeros in a row corresponding to a non-linear character of the character table. Motivated by this work, we study here the dual question. Given $x \in G$, we define $m^*(x) = |\{\chi \in \mathrm{Irr}(G) \mid \chi(x) = 0\}|$ and we put $m^*(G) = \max\{m^*(x) \mid x \in G\}$ and $n^*(G) = \min\{m^*(x) \mid m^*(x) > 0, x \in G\}$. With this notation, we pose the following question: What group theoretical properties can we deduce if we know $m^*(G)$ or $n^*(G)$?

It was proved in [12] that the Fitting height $h(G)$ of a solvable group G is bounded in terms of $m(G)$. Qian's bound was improved in Theorem A of [10] as a consecuence of results of [11]. In [10] we also conjectured that the derived length of a solvable group is bounded in terms of $m(G)$. Our main result shows that, using the classification of finite simple groups, much more can be said if we consider $m^*(G)$. We wirte $k^*(G)$ to denote the number of non-linear irreducible characters of G.

Theorem A. Let G be an arbitrary finite group. Then $k^*(G)$ is bounded in terms of $m^*(G)$.

Thus, if G is solvable, then the derived length of G is bounded in terms of $m^*(G)$. In fact, it follows easily from Theorem A that for any group G the order of G' is bounded in terms of $m^*(G)$. Note that since there are groups of large order with fixed number of non-linear characters (like abelian groups or extraspecial p-groups) it is not possible to obtain any bound for the order of G.

It seems reasonable to hope that the "right" order of magnitude for this bound should be at least logarithmic, as happens with the bound in terms of $m(G)$, but we have been unable to prove this.

We also discuss some more analogs to results of [10]. For p-groups, there is a lower bound for the number of zeros in a row of the character table in terms of the degree of the corresponding character (see Theorem C of [10]). Similarly, we can bound the number of zeros in a column of the character table in terms of the size of the corresponding conjugacy class.

Theorem B. Let x be an element in a p-group P and suppose that the size of its conjugacy class is p^b. Then $m^*(x) \geq b(p-1)$.

Finally, in [10] we obtained some properties of a p-group P in terms of $n(P)$ showing for instance that the derived length of P is bounded by a function of $n(P)$. Our next result shows that we cannot expect to find any analog if we replace $n(P)$ by $n^*(P)$.

Theorem C. For any p-group H there exists a p-group P with $n^*(P) = p-1$ such that H can be embedded in $P/Z(P)$.

2 Nilpotent groups

In this section we prove Theorems B and C and Theorem A for nilpotent groups. We begin with the proof of Theorem C, which is a consecuence of a construction of S. Gagola.

Proof of Theorem D. By Theorem 1.2 of [5], there exists a p-group P and $\chi \in \mathrm{Irr}(P)$ with $|Z(P)| = p$ and $\chi(1) = |P : Z(P)|^{1/2}$ such that H is isomorphic to a subgroup of $P/Z(P)$. Now, pick $x \in Z_2(P) - Z(P)$. It is clear that $\varphi(x) \neq 0$ for all $\varphi \in \mathrm{Irr}(P/Z(P))$. On the other hand, since $\chi(1) = |P : Z(P)|^{1/2}$ and $|Z(P)| = p$, there exist exactly $p-1$ irreducible characters whose kernel does not contain $Z(P)$, namely, the Galois conjugates of χ. Thus $m^*(x) = p - 1$ and the proof is complete. □

It is a well-known theorem of W. Burnside that any non-linear character of a finite group vanishes on some element. The dual question for solvable groups was studied in [7], where it was conjectured that if $\chi(x) \neq 0$ for all $\chi \in \mathrm{Irr}(G)$ and some $x \in G$, then $x \in F(G)$. This was proved, for instance, for odd order groups. It can be deduced form the results in [7] that if P is a p-group and $\chi(x) \neq 0$ for all $\chi \in \mathrm{Irr}(P)$ and some $x \in P$ then $x \in Z(P)$. For completeness, we include a short proof of a slightly more precise version of this result.

Lemma 2.1. *Let P be a p-group of order p^n. Then there exist $\chi_1, \ldots, \chi_n \in \mathrm{Irr}(P)$ such that $\chi_1 \ldots \chi_n(x) = 0$. for any $x \in P - Z(P)$.*

Proof. □

It would be interesting to decide whether or not it suffices to take a constant number of characters for any p-group.

Next, we prove Theorem B.

Proof of Theorem B. We argue by induction on the order of P, noting that the result is trivial if $b = 0$. Suppose then that $b \geq 1$ and take a minimal normal subgroup N of P. If the size of the conjugacy class of $\overline{x} \in P/N$ is p^b, the result follows directly by induction. Otherwise the size is p^{b-1} and all the elements in the coset xN are conjugate in P, so there exists $g \in P$ such that $1 \neq z = [x, g] \in N$ and we can use the same trick as in the proof of the last lemma to conclude that $\chi(x) = 0$ for any irreducible character χ whose kernel does not contain N. In particular

$$m^*(x) \geq k(P) - k(P/N) + m^*(\overline{x}),$$

where $k(P)$ and $k(P/N)$ are the number of conjugacy classes of P and P/N, respectively. It is clear that $k(P) - k(P/N) \geq p - 1$, so the result follows directly from the inductive hypothesis. □

This theorem implies that the largest size of a conjugacy class of a nilpotent group P is bounded in terms of $m^*(P)$. Note that by a theorem of Vaughan-Lee (see [13]) this yields that if P is nilpotent then P' is bounded in terms of $m^*(P)$.

Now, we want to prove Theorem A for nilpotent groups. We need the following folklore lemma.

Lemma 2.2. *Let P be a nilpotent group and $\chi \in \mathrm{Irr}(P)$ non-linear. Then $\chi_{P'}$ is not irreducible.*

Proof. Since nilpotent groups are monomial, there exists a maximal subgroup M of P and $\varphi \in \mathrm{Irr}(M)$ such that $\varphi^P = \chi$. Hence, χ_M is not irreducible and, in particular, $\chi_{P'}$ is not irreducible. □

Theorem 2.3. *Let P be a nilpotent group. Then $k^*(P)$ is bounded in terms of $m^*(P)$.*

Proof. As we have observed before, $|P'|$ is bounded in terms of $m^*(P)$. Therefore it suffices to pick $1_{P'} \neq \varphi \in \mathrm{Irr}(P')$ and bound $|\mathrm{Irr}(P|\varphi)|$ in terms of $m^*(P)$. Let $T = I_P(\varphi)$. Applying Lemma 2.2 of [14] to the character triple (T, P', φ), we deduce that there exists a subgroup U with $P' \leq U \leq T$ such that φ extends to U and all the extensions of φ to U are fully ramified with respect to T.

If $U = T = P$ this means that φ extends to P and hence (using Lemma 2.2) $\varphi = 1_{P'}$, a contradiction. It follows that U is proper in P, and that every character of P lying over φ vanishes on $P - U$. Therefore, $|\mathrm{Irr}(P|\varphi)|$ is bounded in terms of $m^*(P)$, as we wanted to prove. □

3 Solvable groups

Our goal in this section is to prove Theorem A for nilpotent groups. First, we bound the Fitting height $h(G)$ of a solvable group G in terms of $m^*(G)$

and then we argue by induction on the Fitting height. In order to obtain the bound for the Fitting height, we need the following result.

Lemma 3.1. *Let V be a faithful completely reducible G-module (possibly of mixed characteristic) for a finite solvable group G. Then either the Fitting height of G is less than or equal to 5 or there exist at least 5 orbits of elements $x \in V$ with $C_G(x) \subseteq F_9(G)$, the ninth term in the Fitting series of G.*

Proof. The case when V is irreducible is a simplified form of Theorem 4.6 of [11]. The completely reducible case can be obtained using the same argument as in the proof of Theorem E of [11]. □

Theorem 3.2. *Let G be a solvable group. Then $h(G) \leq m^*(G)/5 + 10$.*

Proof. We may assume that $h = h(G) > 10$ and fix $1 \leq i \leq h - 10$. Since $G/F_i(G)$ has Fitting height $h-i \geq 10$, the last theorem applied to the action of the group $G/F_i(G)$ on $V = \mathrm{Irr}(\frac{F_i(G)/F_{i-1}(G)}{\Phi(G/F_{i-1}(G))})$ yields the existence of at least 5 G-orbits of linear characters of $F_i(G)$ containing $F_{i-1}(G)$ in their kernel and whose inertia group is contained in $F_{i+9}(G)$. For each or these orbits we take a representative and then an irreducible character of G lying over it. Clifford's theory ensures that all these characters of G are different and vanish outside $F_{i+9}(G)$, so they vanish at x for any $x \in G - F_{h-1}(G)$. Notice also that when we take $i' \neq i$ between 1 and $h - 1$, this procedure gives rise to different characters of G. Hence $m^*(x) \geq 5(h - 10)$ and the result follows. □

It seems reasonable to hope that the "right" order of magnitude for this bound should be at least logarithmic, as happens with the bound in terms of $m(G)$, but we have been unable to prove this.

In order to complete the proof of the solvable case of Theorem A we need two more lemmas that will also be useful in the proof of the general case. As usual, we say that something is $(a_1, \ldots, a_t)$-bounded if it is bounded by some function of $a_1, \ldots, a_t$.

Lemma 3.3. *Let G be a finite group and $N \trianglelefteq G$. Then $m^(N)$ is $(|G : N|, m^*(G))$-bounded.*

Proof. Let $x \in N$ such that $m^*(x) = m^*(N) = n$ and $\varphi_1, \ldots, \varphi_n \in \mathrm{Irr}(N)$ such that $\varphi_i(x) = 0$ for all i. By considering the irreducible characters of G that lie over the G-invariant characters of N, one can see that at most $m^*(G)$ of these characters are G-invariant. Since the G-orbits of characters of N have size at most $|G : N|$, it suffices to prove that the number of G-orbits of characters of N of size bigger than one that intersect $\{\varphi_1, \ldots, \varphi_n\}$ non-trivially is $(m^*(G), |G : N|)$-bounded.

Let $\mathcal{O}_1, \ldots, \mathcal{O}_k$ be these orbits. Without loss of generality, we may assume that $\varphi_j \in \mathcal{O}_j$ for $j = 1, \ldots, k$. Write $T_1, \ldots, T_k$ to denote the inertia subgroups of $\varphi_1, \ldots, \varphi_k$, respectively. The number of subgroups of G/N, which will be denoted by $s(G/N)$, is $|G : N|$-bounded. We will prove that $k \leq s(G/N)m^*(G)$, and the result will follow. Assume that this inequality is false. Then, we would have that $m^*(G)+1$ of the inertia subgroups $T_1, \ldots, T_k$ coincide. So we may assume, for instance, that $T_1 = \cdots = T_{m^*(G)+1}$. Now, pick $\chi_i \in \mathrm{Irr}(G|\varphi_i)$ for all i. By Clifford's theorem, χ_i vanishes on $G - \cup_{g\in G} T_i^g$ and since $\varphi_1 \ldots, \varphi_{m^*(G)+1}$ lie in different G-orbits, we have found $m^*(G)+1$ different characters $\chi_1, \ldots, \chi_{m^(G)+1}$ of G that vanish on $G - \cup_{g\in G} T_1^g$, a contradiction. □

Lemma 3.4. *Let G be a finite group. Then either $k*(G) \leq m^*(G)^2$ or $|G : G'| \leq m^*(G)(m^*(G)+1)$.*

Proof. Let φ be a non-principal irreducible character of G'. If φ is linear then it does not extend and if it is non-linear and extends to $\hat{\varphi} \in \mathrm{Irr}(G)$ then $\{\lambda\hat{\varphi} \mid \lambda \in \mathrm{Irr}(G/G')\}$ are $|G : G'|$ irreducible characters of G (by Corollary 6.17 of [6]) that vanish at the elements where φ vanishes. In particular, we deduce that $|G : G'| \leq m^*(G)$. Therefore, we may assume that the only irreducible character of G' that extends to G is the principal character.

For every non-principal $\varphi \in \mathrm{Irr}(G')$ let G_φ be the inertia group of φ and U_φ the subgroup of G_φ whose existence is guaranteed by Lemma 2.2 of [14], i.e., the subgroup of G_φ such that φ extends to U_φ and any of these extensions are fully ramified with respect to G_φ. Note that by this fact and Clifford theory we can associate to every extension of φ to U_φ a unique irreducible character of G that lies over it and vanishes on $G - U_\varphi$. Also, by the result proved in the first paragraph this set is always non-empty. This implies that $|U_\varphi : G'| \leq m^*(G)$ for all φ.

Assume now that G' has at most $m^*(G)$ non-principal characters. Since we have just proved that there are at most $m^*(G)$ characters lying over each of the on-principal characters of G', it follows that $k^*(G) \leq m^*(G)^2$. Thus, we may assume that G' has at least $m^*(G)+1$ irreducible characters $\varphi_1, \ldots, \varphi_{m^*(G)+1}$. By the previous paragraph, we can deduce that

$$G = \bigcup_{i=1}^{m^*(G)+1} U_{\varphi_i}$$

and it follows that $|G : G'| \leq m^*(G)(m^*(G)+1)$, as desired. □

Now, we can complete the proof of the main result of this section.

Theorem 3.5. **??** *Let G be a solvable group. Then $k^*(G)$ is bounded in terms of $m^*(G)$.*

Proof. Let $h = h(G)$ be the fitting height of G. We will prove by induction on h that there exist a function $f = f(h, m^*(G))$ such that $k^*(G) \leq f(h, m^*(G))$. Since we have proved in theorem 3.2 that h is $m^*(G)$-bounded, the result will follow. The case $h = 1$ has been proved in Theorem 2.3, so we assume that $h > 1$.

Write G^∞ to denote the largest normal subgroup of G whose quotient is nilpotent. By Theorem 2.3, for instance, we have that $|G' : G^\infty|$ is $m^*(G)$-bounded and by Lemma 3.4 we may assume that $|G : G'|$ is also $m^*(G)$-bounded, so we have that $|G : G^\infty|$ is $m^*(G)$-bounded.

Since $h(G^\infty) = h - 1$, we can apply the inductive hypothesis to deduce that $k^*(G^\infty) \leq f(h-1, m^(G^\infty))$. Using Lemma 3.3, we deduce that $m^*(G^\infty$ is $m^*(G)$-bounded and since $|G : G^\infty|$ is $m^*(G)$-bounded we deduce that the number of irreducible characters of G that lie over non-linear characters of G^∞ is $m^*(G)$ bounded.

Now, it sufficed to prove that the number of characters of G that lie over linear characters of G^∞ is $m^*(G)$ bounded. In order to do this, it is no loss to assume that G^∞ is abelian. It suffices to prove that $|G^\infty|$ is $m^*(G)$-bounded. By Theorem 6.23 of [6] G is an M-group, and by Problem 6.11 of [6] we deduce that for every $\lambda \in \mathrm{Irr}(G^\infty)$ there exists $H_\lambda \leq G$ and $\chi_\lambda \in \mathrm{Irr}(G)$ such that λ extends to $\hat{\lambda} \in \mathrm{Irr}(H_\lambda)$ and $\chi_\lambda = \hat{\lambda}^G$.

Now, an argument similar to that of the proof of Lemma 3.3 allows us to complete the proof, so we will just sketch it. It suffices to bound the number of G-orbits of characters of G^∞. The number of subgroups of G/G^∞ is $m^*(G)$-bounded, so the cardinality of $\{H_\lambda \mid \lambda \in \mathrm{Irr}(G^\infty)\}$ is $m^*(G)$-bounded. Since χ_λ vanishes on $G - \cup_{g \in G} H_\lambda^g$, the result follows. □

4 Arbitrary groups

In this section we complete the proof of Theorem A. It is partially inspired by the proof of Theorem A of [9]. We begin by solving the problem for simple groups.

Lemma 4.1. *The order of an alternating group A_n is bounded in terms of $m^*(A_n)$.*

Proof. As is well-known, the characters of the symmetric group S_n are labelled by the partitions of n. A character of S_n restricts irreducibly to

A_n if the corresponding partition is not self-asssociate (see Theorem 2.5.7 of [8]). Also, any character of S_n such that the corresponding partition is $\alpha = (\alpha_1, \dots, \alpha_t)$ with $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_t$ and $\alpha_2 \geq 2$ vanishes on the n-cycles (by 2.3.17 of [8]). Therefore, the result follows easily when the n-cycles belong to A_n, i.e., when n is odd.

Thus, we may assume that n is even. Consider all the partitions α that are not self-associate and such that $\alpha_2 \geq 3$. As in the first paragraph, it suffices to show that the characters of S_n associated to these partitions vanish on the (n-1)-cycles. This follows using the branching theorem (2.4.3 of [8]) and 2.3.17 of [8]. $\square$

The proof of the next lemma is essentially the same as the proof of Lemma 3.1 of [9].

Lemma 4.2. *Let $G = G_l(q)$ be a simple group of Lie type of rank l over the field with q elements. Then $|G|$ is $m^*(G)$-bounded.*

Proof. Let T be a maximally split torus of the group G of Lie type, and let R_T^G denote the Deligne-Lusztig operator (see [1], p. 205). If θ is a character of T in general position (see [1], p. 219) then $R_T^G(\theta)$ is, up to sign, an irreducible character of G. By Proposition 7.2.4 of [1], these characters are induced from a Borel subgroup B, so they vanish on $G - \cup_{g \in G} B^g$. The number of such characters is the number of regular semisimple classes in the dual group G^* which contain elements in a torus T^* of G^* dual to T (see [1], p.291 or [3], Proposition 13.13). Now the number of such regular semisimple classes is a polynomial in q of degree equal to the semisimple rank l of G (see [2] or [4], p. 483). This proves the result. $\square$

We need two more results. Given a group G, we write $O_\infty(G)$ to denote the largest normal solvable subgroup of G and $F^*(G)$ to denote the generalized Fitting subgroup of G.

Theorem 4.3. *Assume that $O_\infty(G) = 1$. Then $|G : F^*(G)|$ is $m^*(G)$-bounded.*

Proof. By our hypothesis, $F^*(G)$ is the direct product of the minimal normal subgroups of G, say $F*(G) = N_1 \times \cdots \times N_k$, where N_i is a direct product of non-abelian isomorphic simple groups, say $N_i = S_{i1} \times \cdots \times S_{ij_i}$ where $S_ij \cong S_i$ for all j and some simple non-abelian S_i. For every i and j let $\varphi_i \in \mathrm{Irr}(S_i)$ that extends to $\mathrm{Aut}(S_{ij})$ (such character exists by Lemma ?? of [9]) and let $\psi_i \in \mathrm{Irr}(N_i)$ be the tensor product of j_i copies of φ_i. Fix $x_i \in N_i$ such that $\psi_i(x_i) = 0$. . Also, for every i let $C_i = C_G(N_i)$. We have that $G/(C_i \times N_i)$ is isomorphic to a subgroup of $\mathrm{Out}(S_i) \wr S_{j_i}$.

By our choice of the characters ψ_i, we have that they are G-invariant. It follows that the characters $\gamma_1 \times \cdots \times \gamma_t \in \mathrm{Irr}(F^*(G))$ where $\gamma_i \in \{1_{N_i}, \psi_i\}$ are G-invariant. For any of the $2^k - 1$ non-linear characters γ that can be built this way, we pick an irreducible character of G lying above γ. Thus, we have found $2^t - 1$ different characters of G that vanish at $(x_1, \ldots, x_k) \in F^*(G)$. This means that k is $m^*(G)$-bounded.

Hence, since $\cap C_i \leq F^*(G)$, it suffices to bound $|G : C_i \times N_i|$. First, we want to see that j_i is $m^*(G)$-bounded. This can be done using the same argument of the previous paragraph, using the characters $1_{C_i} \times \delta_{i1} \times \cdots \times \delta_{ij_i} \in \mathrm{Irr}(C_i \times N_i)$, where $\delta_{ij} \in \{1_{S_{ij}}, \varphi_i\}$.

This way we have bounded the order of the permutation part of $G/(C_i \times N_i)$ and it suffices to bound the order of the part corresponding to $\mathrm{Out}(S_{i1}) \times \cdots \times \mathrm{Out}(S_{ij_i})$. Let H be this part of the group. Since the characters of $C_i \times N_i$ we are looking at extend to this part of the group, using Gallagher's theorem, we can find $|\mathrm{Irr}(H/(C_i \times N_i))|$ irreducible characters of H lying over any of the characters $1_{C_i} \times \delta_{i1} \times \cdots \times \delta_{ij_i} \in \mathrm{Irr}(C_i \times N_i)$. Also, these $|\mathrm{Irr}(H/(C_i \times N_i))|$ characters vanish at $(x_{i1}, \ldots, x_{ij_i})$. They lie in G-orbits whose size is $m^*(G)$-bounded, so either their number is $m^*(G)$-bounded, which would yield that $|G : C_i \times N_i|$ is $m^*(G)$-bounded, as desired, or else the number of G-orbits cannot be bounded by a function of $m^*(G)$ and this would give more than $m^*(G)$ characters of G that vanish at the same element, a contradiction. □

Now, we are ready to conclude the proof of Theorem A.

Proof of Theorem A. Let $n = |G/O_\infty(G) : F^*(G/O_\infty(G))|$. We will prove by induction on n that there exists a function $f = f(n, m^*(G))$ such that $k^*(G) \leq f(n, m^*(G))$. Since we have just proved that n is $m^*(G)$-bounded, the result will follow.

Assume that $n = 1$. In this case G has a normal solvable subgroup N such that $G/N \cong S_1 \times \cdots \times S_t$ for some simple non-abelian groups S_i. By Lemmas 4.1, 4.2 and the classification of finite simple groups, we know that $|G/N|$ is $m^*(G)$-bounded. Now, it follows from Lemma 3.3 that $m^*(N)$ is $m^*(G)$-bounded and using Theorem **??** we deduce that $k^*(N)$ is $m^*(G)$-bounded and we conclude that the number of characters of G lying over non-linear characters of N is $m^*(G)$-bounded.

Now, we may assume that N is abelian and we want to prove that $|N|$ is $m^*(G)$-bounded. It is clear that G does not have non-abelian solvable quotients, so we have that G' is perfect. By Lemma 3.4, we may assume that $|G : G'|$ is $m^*(G)$-bounded and it is not difficult to see that we may assume that G is perfect.

By the argument that has appeared already in the proof of Lemma 3.3 or of the solvable case, we have that the number of characters of N that are not G-invariant is $m^*(G)$-bounded. This implies that $|N : C_N(G)|$ is $m^*(G)$-bounded. Observe that $C_N(G) = Z(G)$. We know therefore that $|G : Z(G)|$ is $m^*(G)$-bounded, and we want to bound $|Z(G)|$. Let $H = G/Z(G)$. Our perfect group G is a central extension of H, so by Corollary 11.20 of [6], we have that $Z(G)$ is isomorphic to a subgroup of the Schur multiplier $M(H)$, whose order is bounded in terms of $|H|$(by the proof of Theorem 11.15 of [6]), and therefore $m^*(G)$-bounded.

Now, we may assume that $n > 1$. Let $N = O_\infty(G)$, $F/N = F^*(G/N)$ and M a maximal subgroup of G containing F. If G/M is abelian, then its order is $m^*(G)$-bounded (by Lemma 3.4) and it is easy to conclude the proof using arguments that have already appeared.

Thus, we may assume that G/M is simple non-abelian. We have that $t = |G/M|$ is $m^*(G)$-bounded and by the inductive hypothesis we have that

$$k^*(M) \leq f(n/t, m^*(M)).$$

We also know using Lemma 3.3 that $m^*(M)$ is $m^*(G)$-bounded and therefore we can bound the number of characters of G lying over non-linear characters of M. Now, we can assume that M is abelian and the argument used in the case $n = 1$ allows us to complete the proof. □

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