ICME-11 ST3: Survey on the Impact of Research Findings in Mathematics Education on Students' Learning of Mathematics

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1. Introduction: Global objective and definition of the key term "impact"

This Survey Team was asked to prepare a report about the impact of findings from research in mathematics education on students' learning of mathematics. For simplicity, and to avoid repetitions, we'll use the term *research* meaning *research in mathematics education*, and the terms *teaching* and *learning* meaning *teaching* and *learning of mathematics*.

A first naive look at this assignment may induce to believe that this is an easy topic, since research journals are plenty of papers reporting on research projects that have succeeded in making teachers teach very efficiently or making students learn more meaningfully, bypass an obstacle, etc. But a deeper analysis shows that the key term "impact" has a more subtle meaning, so it is convenient first to fix the meaning we have given to the term "impact" (or "influence", we'll use them as equivalent).

When looking at research and mathematics classrooms, we must differentiate among two contexts.

On the one side, we find experimental classrooms, where some external elements have been introduced to influence teachers and/or students' behaviour, as part of a research experiment being conducted. External elements introduced may be mathematical contents, teaching methodologies, ways to organize students' activity, new tools, etc. Either teachers or students, or both, participating in research experiments are not acting as they used to do before beginning the experiment.

On the other side, we find ordinary classrooms, those where no research experiment is being, or has recently been, conducted, so teachers and students are working without any external atypical influence. These classrooms are our objective.

Depending on the circumstances, after the experiment was completed and the researchers left the schools, teachers and students in experimental classrooms may go back to their traditional teaching and learning habits, or they may adopt some results of the experiment and integrate them into their ordinary teaching and learning since then. But we cannot say that some research results have impact on students' learning just because the researchers have demonstrated a clear improvement of students' learning of mathematics. In the same way, we cannot consider that the research results have impact just because a few students have benefited from them.

Rather, some research results do have impact on students' learning when such results have influenced on a significant number of students in a regional, national, multinational, or worldwide base. In other words, research results do have impact when they are merged into the ordinary activity of a significant number of teachers or students, so the research results are used by them as an integrated element of their ordinary classes. Therefore, we reformulate the initial question in this way:

Do research findings in Mathematics Education impact on students' learning of mathematics in ordinary classrooms?

The initial objective of our survey was to know about the worldwide impact of research findings on students' learning. However, nowadays, mathematics education research embraces so many educational levels, objectives, questions, and focuses that made impossible for us to cope with such task. Furthermore, other obstacles, the main being language, impeded us to access information from some parts of the world. Then, we have focused on research results most directly related to the learning of mathematics in classrooms, and we have taken our own countries as cases to study the impact of research results. We don't claim that this report is worldwide valid, nor the cases presented are directly generalizable to other contexts, but we are convinced that the cases we are showing you are similar to many other cases in other countries, educational levels, etc., so they are a projection of the worldwide situation.

2. Background

Although we don't have time for a detailed review of literature, let me mention briefly other approaches and opinions. The relationship among research and ordinary teaching is a permanent topic of discussion.

This is not the first time that the question of the impact of research results on teaching and learning has been raised in ICME conferences. 20 years ago, a Topic Group in ICME6 was devoted to the relationship among practices of teaching mathematics and didactical research. Rouchier and Steinbring presented a final synthesis of the work of the group, published the same year in Recherches en Didactique des Mathematiques. They reflected on the relationships among didactical theories, the experimental research based on them, and the practice of teaching in what they call "real classrooms". One of their conclusions was that research and practice are away one from the other, so it was necessary to explore ways to connect research results and practice. In particular, they asserted that:

- It is necessary a kind of transposition of the didactical theories to the teaching practice, by means of a negotiation between researchers and teachers.

- Teacher training programs and courses should serve as a main way to put teachers in contact with research, that is to connect action with reflection, and to build a professional culture of integration of research into teachers' practice.

However, the interest of researchers to produce useful results and to make them available for teachers is not sufficient to guarantee the link among research and practice. The educational authorities have a decisive role. As an example, one of the keys for the success of Finland in the PISA tests is that their authorities make teachers to be involved in a continuous training.

Some French researchers provide us with other examples of such a role. Brousseau has reported that he couldn't convince French decision makers to introduce a curricular change to adopt the utilization of the Russian lattice algorithm in the learning of multiplication with natural numbers, even when he had demonstrated, with very consistent data, the advantages of this algorithm over the one used in French schools. Brousseau emphasizes the strength of cultural traditions over research results.

In the same vein, a few years ago, Guin and Trouche reported that, in France, the ministry of education had funded for about 10 years, from 1991, research experiments to promote the integration of CAS and symbolic calculators into the secondary schools, but, until the time of their report in 2002, the results of that research only had marginally been implemented into ordinary classes, most times in teacher training courses by the IREMs.

The same problem is reported in USA. In a paper by Bracey entitled "Of educational research and other glacial activities", it is said that, in a recent conference about this theme, speakers repeatedly lamented the relative impotence of educational research to move policy. The speakers also observed that universities, when giving tenure to their researchers, privilege those studies placed in flagship journals, instead of those having impacted more on educational policy or practice. We are sure that this is true in most, if not all, universities in the world.

On the other side, teachers argue that researchers seem to live in their ivory tower, far from real life. Just as an example, in 1997 there was a seminar in Madrid (Spain) devoted to analyze the relationship among research and practice where researchers and teachers presented their views. Related to Rouchier and Steinbring claim for a transposition, a question to the participants in the seminar was: "Are there easy ways for teachers to access texts of research in mathematics education?" The answer by a teacher was:

"This is another of the serious problems for most teachers to access to [research] results: If they want to access a 'universal bibliography', it is very broad even that referred to a single topic, so the task is unapproachable. Nowadays there are not fundamental texts about mathematics education where the theory is approachable, presented in a systematic way, and with enough amount of examples of different levels."

Another teacher participating in the seminar pointed at three reasons for the distance among researchers and ordinary teachers:

- The language used is characteristic of the researchers' culture, and it is estrange to teachers.

- The area of application of most research results is very limited. Usually, researchers focus on a very specific question, making their results look artificial and useless to teachers.

- Most research reports are mainly descriptive. They may be very useful to illuminate the problems of students' learning, but they are not practical enough for the everyday work of teachers in their classrooms.

So, from both sides, teachers and researchers, we can perceive a pessimistic feeling about the impact of research results on students' learning of mathematics. However, there are also reasons to foresee a more optimistic future, since, as we shall show you now, there are cases of clear impact of research results on students' learning thanks, in some of them, to the involvement of educational authorities. These cases, if carefully analyzed, can give us clues about ways for researchers to act to be more successful in influencing teachers' teaching and students' learning.

3. Routes of research results to impact on students' learning

Most frequently, the impact of research results on students' learning doesn't come from a direct exposition of students to the results, but from the influence of the research findings on some mediators. Those mediators may be a new official or school curriculum, the choice of different textbooks, the decision of a school to use technology, the participation of teachers in a training course or a congress, and others less relevant. A mediator may influence on other mediators, or may directly influence on students.

By official curriculum we mean one ruled by a government. Different countries are different cases. Some countries, like Mexico, Malaysia and many others have a national curriculum that has to be implemented by all schools of the country. The opposite case are USA and other countries where neither the country nor the states, regions or departments have any official curricula but, at most, a set of curricular guidelines that in some cases are just prescriptions of contents to be learned to pass an exam at the end of a school cycle. In those countries, schools are more or less free to organize their own school curriculum. A third case are countries, like Spain, with several state, regional or departmental governments ruling their own curriculum.

By school curriculum we mean the curriculum designed by the teachers of a school and directly applied to organize their classes of mathematics. Usually school curricula are mainly based on textbooks and on teachers' beliefs and their pedagogical and mathematical content knowledges.

3.1. Routes of research results to impact on students' learning: Curricula

Problem solving was the star mathematics education research topic in the 1980s. Years later, many curricula all over the world implemented results from that research. Let's see two examples, from Spain and Malaysia. In Spain regional governments are competent to decide their own official curriculum.

These excerpts are taken from the official curricula implemented in 1992 in the region of Valencia for primary and compulsory secondary schools. The contents are organized in "blocks", and you can see that problem solving is one of them with the same status as arithmetic, geometry, algebra, etc.

This is the detailed content of the block of problem solving for Primary School. The structure of the problem solving block for Secondary School is similar to this one, but including some more sophisticated heuristics related to deductive reasoning and proof. When reading the contents of the block, you can immediately recognize, first, the four Polya's phases and, second, a list of heuristics to solve problems that was originated by the research on problem solving made in the 1980s.

You can find similar contents in the curricula of many countries all over the world. For instance, the Malaysian official curricula for Primary and Secondary Schools provide these guidelines, making emphasis on Polya's phases and problem solving heuristics.

These guidelines are explicitly implemented in the contents of the curricula for upper grades at primary school and lower grades at secondary school. This is an example from the Malaysian curriculum for primary school year 5. In this lesson (look at the green box) teachers are suggested to teach the four Polya's phases, and they are also provided (look at the blue box) with an example where each phase is contextualized.

A case different from the previous ones is the NCTM Principles and Standards for School Mathematics. It is different because the Principles and Standards are not an official curriculum designed by a government, but a private initiative aimed to articulate a coherent set of learning objectives, mathematical contents and teaching methods at a national scale in a country where differences between the guidelines published by different states are quite big.

To write the Principles and Standards, NCTM created a team of experts, most of them teachers from different educational levels. And also NCTM asked a group of mathematics education researchers to produce a set of papers summarizing the most relevant research in eight areas of mathematics, intended to advice the writers of the Principles and Standards to integrate research results into the document they were to produce. Furthermore, the Research Companion to the Principles and Standards was published in an effort to explicitly articulate the research foundation that guided the content and recommendations of the Principles and Standards document.

It is significant, and a proof of the influence of research on the Principles and Standards, that this document includes many explicit references to research publications. Just as an example of such influence, the suggestion included in a geometry standard that children "learn to recognize a shape by its appearance as a whole or through qualities such as ‘pointness’” is a clear consequence of the use of van Hiele levels as a theoretical background for the geometry standards.

4. Routes of research results to impact on students’ learning: Textbooks

It is widely recognized that, more than official curricula, textbooks are the main element influencing teachers and students. If the authors of a textbooks series know and use results from research when writing the books, they are in a good position to influence teachers’ practice. Furthermore, depending on the way textbooks include some specific contents, the impact of the related research results, for instance problem solving, shall be more or less significant.

Following the two examples of official curricula from Spain and Malaysia shown before, we look now at some textbooks corresponding to those curricula.

The Spanish official curriculum points that problem solving shouldn’t be taught by scheduling a segment of classes to teach it, but it should be integrated into the other topics. The application of this methodological guideline can be seen looking at the index of the textbook.

In lesson 1, the four Polya’s phases are introduced. Then, from lesson 2 to the last lesson, different heuristic strategies of problem solving are studied by using them to solve some problems, based on the contents of the lesson.

Let’s see the detailed contents of two lessons:

- In the first lesson, the four Polya’s phases are explained. Then, a problem is solved in the textbook showing how to put to work each phase, and finally another problem is stated, to be solved by the students in the same way.

- The other pages of this textbook devoted to problem solving have all them the same structure: A heuristic is presented, a problem is solved by using this heuristic and the phases, and finally other problems are stated, to be solved by the students using the same heuristic.

In this page, the heuristic is *make a drawing*. First the heuristic is explained. Then a problem is solved: “Sergio and Juan collect mail stamps. …” Next the problem is solved in the textbook with the help of some drawings. Finally three problems are stated, all them having the same mathematical structure as the problem solved.

This is a page from a Malaysian textbook for Primary School year 5. It shows the application of Polya’s phases by the textbook, where the usual labels for the phases have been changed to other specific labels that are more meaningful for teachers and students. If we compare this page with the one shown before from a Spanish textbook, we can recognize a high similarity among them.

Returning to the case of the NCTM Principles and Standards, an interesting question is whether they are influencing the textbooks published in USA. Given the relevance of NCTM, most textbooks claim to be aligned with the Principles and Standards. Such statement is far from being obvious in some cases, other textbooks, particularly those produced by research projects funded by the NSF, do really take into consideration the Principles and Standards and, therefore some of the research results included into the Standards. It can be observed that textbook published ten years ago were aligned with the 1989 Curriculum and Evaluation Standards, but more recent editions of those textbooks have been changed to align with the new Principles and Standards.

Anyway, given the particular characteristics of the system followed by many American schools to select the textbooks, where teachers, parents, school administrators, and other members of the community are involved in the decision, authors have to consider their perspectives and opinions when writing their textbooks, apart from considering also, if possible, the research results and the NCTM recommendations. Thus, the textbooks authors and publishers have to walk the thin line connecting their vision of school mathematics, research results, and the related traditions embedding the American society. Usually, successful innovations are those that do not depart too much from current practices and beliefs, so for the impact of new research results in ordinary classrooms to become feasible, the authors have to make short steps, and promote small changes or innovations better than big ones.

To conclude this review of the impact of research results on textbooks, we'll show you very quickly what may be one of the most clear examples we can find of such impact. Many of you, specially if more than 20 years ago you already were more than 20 years old, know a research carried out by the Shell Centre of the University of Nottingham, in UK, in the late 1980s, that resulted in the publication of "The language of functions and graphs", a very successful teaching material to introduce functions to secondary school students based on a phenomenological approach to functions. Here you can see some fragments of activities from this publication side by side to fragments of pages in two Spanish textbooks for 3rd grade in secondary school, from the lesson where functions are taught for the very first time.

The Spanish government translated into Spanish the Shell Centre publication, helping to make it accessible to Spanish teachers and textbooks writers. Those textbooks you have been looking at are examples of use of research findings both to didactically organize the presentation of the mathematical contents and as part of the contents themselves.

5.1. Routes of research results to impact on students' learning: Teachers' training

Agudelo-Valderrama, in a recent issue of ESM, describes the beliefs of a sample of Colombian teachers and the factors they consider as influencing their teaching practices. Among the factors mentioned by the teachers are students, families, mathematical contents, etc., but research is not one of those factors. Other authors have identified the same problem and mentioned some possible reasons. For instance, Ruhama Even stated that, while research has flourished in the last decades, many teachers are not familiar with it, for several reasons like texts written in foreign languages, or the common belief among teachers that research results are irrelevant to their ordinary practice because researchers do not produce rules for action that can be put to immediate use. Even proposed to implement adequate in-service courses including among their aims to approach teachers to the mathematics education research.

Teachers are the weaker link in the chain between research results and students, since they need help to implement changes and innovations in their classes. We, the mathematics education community, are aware of it, but unfortunately educational authorities are too often blind to this need. It is too usual that a new official curriculum implements important methodological and didactical changes but almost no money is devoted to train teachers in the new methods and contents nor to provide them with teaching materials adopting the changes and novelties and showing how to implement them in the classrooms.

We found in Italy an example showing how in-service training courses can modify teachers' perception about and use of research results.

Continuing with the issue of impact of research results on teachers' teaching, Chile is a case showing how the government implemented a strategy to elaborate an innovative didactical proposal based on research results, and brought it to the schools in the form of teaching materials.

This program, called "Reading-Writing-Mathematics Campaign" (LEM) was aimed to support Primary Schools in the implementation of a new mathematics curriculum. It started in 2003 with an agreement between the government and the university of Santiago de Chile, that was extended to other 6 universities. Until now this campaign has helped 776 schools in seven regions of the country from a total of about 10.000 primary schools in Chile.

The researchers produced 16 teaching units for the first four grades of Primary School. Based on Chevallard's Anthropological Theory of Didactics and Brousseau's Theory of Didactical Situations, each teaching unit is a proposal to organize the teaching of some *nuclear learning* in each grade for about 6 hours of class in 2 weeks. In the teaching units are explicit:

The *mathematical tasks* to be posed to the pupils.

The *procedures* expected to be used by students at the beginning and at the end of the learning process.

The *mathematical knowledge* that justify the procedures used by students.

The *didactical variables* that will be manipulated to change the conditions of accomplishment of the mathematical tasks. And

A proposed *teaching methodology* to help the teachers lead a process in which pupils explore, construct their own procedures, and justify them, articulating the knowledge newly acquired with that they already owned.

The authors of this program also took care of designing an adequate process of diffusion of the proposal to promote its transference to, and appropriation by school teachers. To help in this process, the figure of the *Consulting Teachers* was created. They are out-standing school teachers that are qualified by the university teams and are in contact with them. Under de conduction of a consulting teacher, the teachers of the schools study the teaching units, and apply them, with the observation and feedback of the consulting teacher.

A main weakness observed by this program is that primary school teachers have a poor knowledge of mathematical contents to be taught and mathematics education contents to be used, so inservice courses are necessary.

5.2. Routes of research results to impact on students' learning: Teachers' beliefs

Apart from our own analysis of the field to identify cases of impact of research on teachers' teaching and students' learning, we decided that it should be useful to hear the voices of teachers. We designed a questionnaire intended to be answered by mathematics teachers teaching kindergarden, primary or secondary school, and teacher training courses. We distributed the questionnaire electronically by means of distribution lists, societies of teachers, the ICME11 web page, and asking personally to other colleagues to send it to groups of teachers they knew. The aim was to reach as many teachers as possible from as many countries as possible. The content of the questionnaire focused on the impact of results from mathematics education research. Let's see the distribution of answers to the main questions:

Number of answers: 234
Number of countries: 28
Number of years of practice
Educational levels taught

Note that most teachers have taught in several levels, so the figures in this table add more thsn 234.

Questions about the usefulness of in-service training activities:

3.	Have you attended any in-service activities during last 15 years?
	Yes – 226/234 (96,6%)
	No answer – 8/234 (3,4%)
5.	Have the teachers or presenters showed applications of research results?
	Yes – 199/226 (88%)
	No – 26/226 (11,5%)
	No answer – 1/226 (0,5%)
6.	Have you applied some of those research results to your classes?

Yes – 189/199 (95%)

No – 10/199 (5%)

8. Has their inclusion in your classes been positive?

Yes – 187/189 (98,9%)

No – 2/189 (1,1%)

Questions about the usefulness of publications:

9. Have you read magazines/journals or books (except textbooks or their teacher guides) during last 15 years?

Yes – 224/234 (95,7%)

No – 9/234 (3,9%)

No answer – 1/234 (0,4%)

12. Do those publications inform on research results?

Yes – 169/224 (75,5%)

No – 45/224 (20,1%)

No answer – 10/224 (4,4%)

13. Have you applied some of those results from research to your classes?

Yes – 169/169 (100%)

15. Has the use of those research results in your classes been positive?

Yes – 165/169 (97,6%)

No – 4/169 (2,4%)

Questions about the impact of research results on teaching and learning:

16. Do you note some influence of research results in:

- the official curriculum for the grades you teach?

Yes – 130/234 (55,6%)

No – 99/234 (42,3%)

No answer – 5/234 (2,1%)

- the programs for the mathematics classes in your centre?

Yes – 141/234 (60,3%)

No – 88/234 (37,6%)

No answer – 5/234 (2,1%)

- the textbook you have used in your classes?

Yes – 137/234 (58,6%)

No – 94/234 (40,2%)

No answer – 3/234 (1,2%)

18. Referred only to the grades where you have taught mathematics, do you believe that the results from research on Mathematics Education are necessary to improve the quality of teaching and learning mathematics?

Yes – 137/234 (58,6%)

No – 94/234 (40,2%)

No answer – 3/234 (1,2%)

19. How do you believe is the influence of such research results on the daily activity in your classes of mathematics? (mark with an X)

Null – 6/234 (2,6%)

Small or sporadic – 73/234 (31,2%)

High or frequent – 110/234 (47%)

Very high or permanent – 38/234 (16,2%)

No answer – 7/234 (3%)

After having processed the replies received, we have some concern about the validity of the sample, we mean its representativeness of the collective of ordinary teachers. Most teachers who answered the questionnaire received it from distribution lists or teachers associations, so they might be only representative of teachers having some interest in being up to date, attending conferences or courses, reading mathematics education publications, and experimenting in their classes. On the other side, the answers of these more informed teachers to questions about the impact of research results on curricula, textbooks, or classes are more reliable than those from less informed teachers who are not so knowledgeable of research findings.

We wonder what answers would have got from ordinary teachers not related to associations or distribution lists, but they were out of our reach. Anyway, the answers to the questionnaire demonstrate that research results have produced a significant impact on teaching of those informed teachers active and interested in being up to date and in adopting new proposals to improve their students' learning of mathematics.

6. Conclusions.

For reasons having to do with the great diversity of national contexts and, not less important, the great variety of languages, we have not pretended to give you information on the worldwide connection among research results and ordinary mathematics classrooms, but to show you some cases helping to decide whether such connection exists or not, and to give some clues on whether it is generalized or just anecdotal. We hope we have helped to define the problem of measuring the impact of research findings on teaching and learning of mathematics, and to raise some questions to be answered in the future.

As a global answer to the question we posed at the beginning of our report, we can conclude that results from research on mathematics education are impacting on students' learning of mathematics. We have shown several cases that, not pretending to be representative, show that there is a worldwide tendency to take into consideration research results to improve teaching and learning. Of course, we could also have shown you other cases where well known research results have been absolutely ignored. However, granted that there is an impact on students learning, there are not adequate tools to measure the extent of such influence.

The impact or influence of research results on students' learning is a function of several variables. Variables having to do with the resistance of the mediators to any change, with the easiness or difficulty for researchers to influence each mediator, with the *distance* between the mediator and the students, i.e., their more or less direct influence on students' learning, and having to do with the geographical extent of application of a research result.

In some cases, research results may take many years to reach students. We have two examples from the USA: Cognitively Guided Instruction (CGI) and Simcalc. They were research programs that later became part of professional development and now are beginning to show impact on students' learning. But, these programs started in the 1980 with Carpenter and Kaput. So, it is a long way between research and impact. Not too many lines of inquiry resist that long to show impact.

In other cases, the time required has been much shorter, for instance in the advances related to educational software; DGS and CAS are two example. The impact of research on those topics is very different from some countries (or even regions) to others, but the existence of a relative impact is quite clear.

A third case are research results that have been absorbed by the educational system and they are permanently influencing teaching and learning although they have lost the character of innovations. Some examples may be Piaget's theories and constructivism, or, in a more specific level, Dienes' multibase blocks. Constructivism is permanently present in curricula since several decades ago. Dienes' blocks are part of the didactical tools proposed by most primary school textbooks to help learn arithmetic.

Official curricula should be a main way for research results to impact on teacher's teaching and student's learning. However, we cannot ignore that the possibility of such influence is conditioned not only by intrinsic characteristics of the research but, mainly, by political or social circumstances.

Anyway, the impact of research results included in official curricula may be diverse. When innovations derived from research are implemented in an official curriculum, it may happen that some authors of textbooks, schools or teachers don't wish or don't know how to adopt the innovations. In those cases, they are converted in “ritual” or “decorative” elements of the school curriculum and textbooks more than in really integrated parts of the teachers’ teaching and students’ learning.

Teachers and textbooks are the actors in the educational system directly in contact with students, so they are the main mediators between research results and students. Research results willing to reach students’ learning shall succeed only if textbooks propose them to teachers in an adequate way, and teachers integrate them into their ordinary daily teaching practice. However, the leap from the statement that research results have impacted curricula or textbooks to the statement that they have impacted students’ learning is quite large and difficult to evaluate.

Implementation of any change in an educational system, specially changes dealing with teachers’ habits, tend to need a long way and many years to be a generalized change, so researchers shouldn’t expect quick results for research to actually impact practice, and they have to take care not only of producing useful results, but also of disseminating efficiently the information.