GROUP 2:
TOOLS AND TECHNOLOGIES

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TOOLS AND TECHNOLOGIES

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1. Tools and technologies in the didactics of mathematics

The thematic group discussed the role of tools and technologies in mathematics education on the basis of contributions of the nine attached presentations which cover various tools (including a range of programs and — interestingly enough — one non-computational tool consisting of semi-transparent mirrors). These spanned very different levels of schooling and topics: from primary school to university level, from numeration decimal system to calculus and geometry.

The case of the semi-transparent mirrors points to a general issue which emerged from the group: that focussing on the roles of computational tools, and how they mediate learning, is a special case of a focus on tools more generally. More important still, a primary outcome of this kind of focus is that it centres our attention on the ways in which tools mediate knowledge construction, and therefore, on quite general questions concerning mathematical learning — which are themselves independent of specific tools, whether computational or not.

It is useful to distinguish three embedded levels when analysing the use of tools in mathematics education:

- the level of the interactions between tool and knowledge

Tool <-------> Knowledge

- the level of interactions between knowledge, tool and the learner

Tool <-------> Knowledge

Learner

- the level of integration of a tool in a mathematics curriculum and in the classroom

Tool <-------> Knowledge

Learner

Teacher

2. Tools and knowledge

A key issue at the level of interactions between tool and knowledge concerns the question of how the tool mediates knowledge and how this process of mediation actually changes knowledge itself and its use. A paradigmatic example of this kind of change, which led to a lively discussion within the group, was provided by the following task:

The task began by asking for the enumeration of the various possibilities for the number of intersection points of four straight lines in the plane. It then continued:

1. *In how many points can the angle bisectors of a quadrilateral intersect question*
   Use a dynamic geometry environment and choose a quadrilateral. Construct its angle bisectors.
2. *By moving the vertices, can you obtain all possibilities which you listed in part 1?*
   Point out all the possibilities you found
   Which ones are missing ? Explain why.
   In a few sentences, write down your explanation.
3. *In how many points can the angle bisectors of a triangle intersect question?*
   Justify your answer.

One way of looking at this task (not the way of those of us who only read one part at a time!) is that it leads to a proof of the fact that the angle bisectors of a triangle intersect all in a point. But using a dynamic geometry environment affords a different (and more general) question concerning quadrilaterals. We do not propose to spoil this question for the reader by providing answers here, but the group was surprised to find that it led to a completely new way to think about an old question (and its answer) — put briefly, we came to see that the bisectors intersect in one point because they cannot intersect in three points!

Such a perspective simply would not arise in a paper and pencil environment, since in this case, there is no empirical evidence of the fact that the four angle bisectors of a quadrilateral cannot intersect in three points. The use of a tool in this example not only changes the way of exploring the question but even the meaning of the property: instead of appearing as a beautiful fact specific to a triangle, it becomes the by-product of a more general property. In one sense at least, it is not only the approach to the mathematical goal that changes, but the mathematical goal itself.

Thus some important questions for teaching of mathematics arise about a new epistemology of mathematics created by the use of technology. In particular it seems that modelling is more relevant in the computer era than it was before (cf. problems given in the paper of Belousova & Byelyavtseva). The nature of proof is also very much subject to change by the use of technology: mathematics might become the science of modelling rather than a fundamental science taught for its internal structure and specific ways of developing knowledge (we did not discuss the changes that are happening to mathematics itself — that is for another conference perhaps!).

A further category of changes through the mediation of the tool deals with the new behaviour of the objects due to the mediation. A very good example is given by the behaviour of points in a dynamic geometry environment. In a static environment, there is no reason to question the behaviour of objects when some basic elements are moved because there is no such possibility. But now it is natural to ask what could be or should be the trajectory of a point on a segment AB when one of its endpoints is dragged? There is no answer in Euclidean classical geometry because the question is meaningless. Thus the mediation of this geometry in a dynamic geometry environment actually creates objects of a new kind. The “danger” for the user is that the new nature of the object may be not visible. They may be transparent for the users who may believe that the objects are identical to those with which they are familiar.

The designers of tools specifically devoted to mathematics are thus faced with decisions about the behaviour (or properties) of the new objects they create. The group discussed the effect of these choices not only in geometry (cf. Jones and Dreyfus & al. papers) but also in calculus (cf. Gélis & Lenné paper). The changes made to knowledge by the use of the tool inevitably lead us to address the question of the meaning constructed by the user and in particular by the learner when using the tool.

3. Interactions between tool and learner

Tools are mostly used in mathematics teaching for their potential to foster learning. Integrating a spreadsheet into mathematics teaching, a CAS or a dynamic geometry environment is not primarily aimed at learning how to use them: it is essentially intended to improve the learning of mathematics by creating a context giving sense to mathematical activity. But there might be some distance between what the learner constructs from the use of the tool and the expectations of the teacher. Some papers of the group investigate the extent to which different environments may lead to different kinds of learning by observing strategies developed by students in both environments (cf. Price and Hedren papers about counting and calculating strategies).

Approaching the understandings constructed by the students when using tools and technologies (or in other words *emerging from the use of the tool*), and the evolution of these understandings, was seen by the group as a key issue of research. The group expressed the need for empirical research focusing on solution processes and the underlying constructed meanings. An example of such an investigation is proposed in Ainley & al. paper in the construction of a formula by 8-9 year-old children interacting with a spreadsheet. The tool and/or technology in this type of task is viewed as facilitating by its feedback the pupil awareness of some inconsistency in their data. It has to be noted that tools may be used as catalysts for making students aware of the erroneous character of their strategies or answers. By using very different tools as semi-transparent mirrors or computer software, it might happen that what the students observe as a result of their actions differs from what they expected. From a cognitive psychological point of view, we might consider this a source of a cognitive perturbation or ‘internal conflict’ which may lead to a cognitive progress.

4. Tools and technologies in the curriculum

One of the key issues for teachers is how to design tasks based on tools or technologies in which real questions for the learner emerge from the use of the tool, in which the tool is relevant and gives a new dimension to the task. Some participants of the group stressed that there is a danger in asking the students to solve simple tasks with a complex tool: a fascination may be created by the discovery of the tool in itself and the pleasure of using it. This led the group to distinguish two types of use of tools and technologies: as functional for their own sake or as used for a didactical purpose. Thus we were led to focus on a further distinction: between tools created for a specific teaching purpose or more "universal" tools like spreadsheets or interactive dynamic geometry environments. The latter do not involve a teaching agenda while the former ones may be based on a pedagogical strategy. The paper by Rakov & Gorokh gives some examples of use of such general tools for teaching at university level and how it is possible to use several tools for solving the same problem from different perspectives. Similarly semi-transparent mirrors are not designed for specific teaching interventions — an example of an extensive curriculum in geometry designed around their use is presented in the paper by Zuccheri.

5. The papers

You will find below the papers on which the work of the group was based. Their authors had a short time after the conference to modify their contributions on the basis of the group's discussion. We hope that they will generate questions and reactions from readers. We are open to any kind of exchanges, especially by e-mail. The list of the e-mail addresses of the participants of the group is given below. We encourage readers to copy this list as it stands, and create (and add to!) an informal mailing list which can continue the work of the group, and discuss future possibilities for collaboration and communication.

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