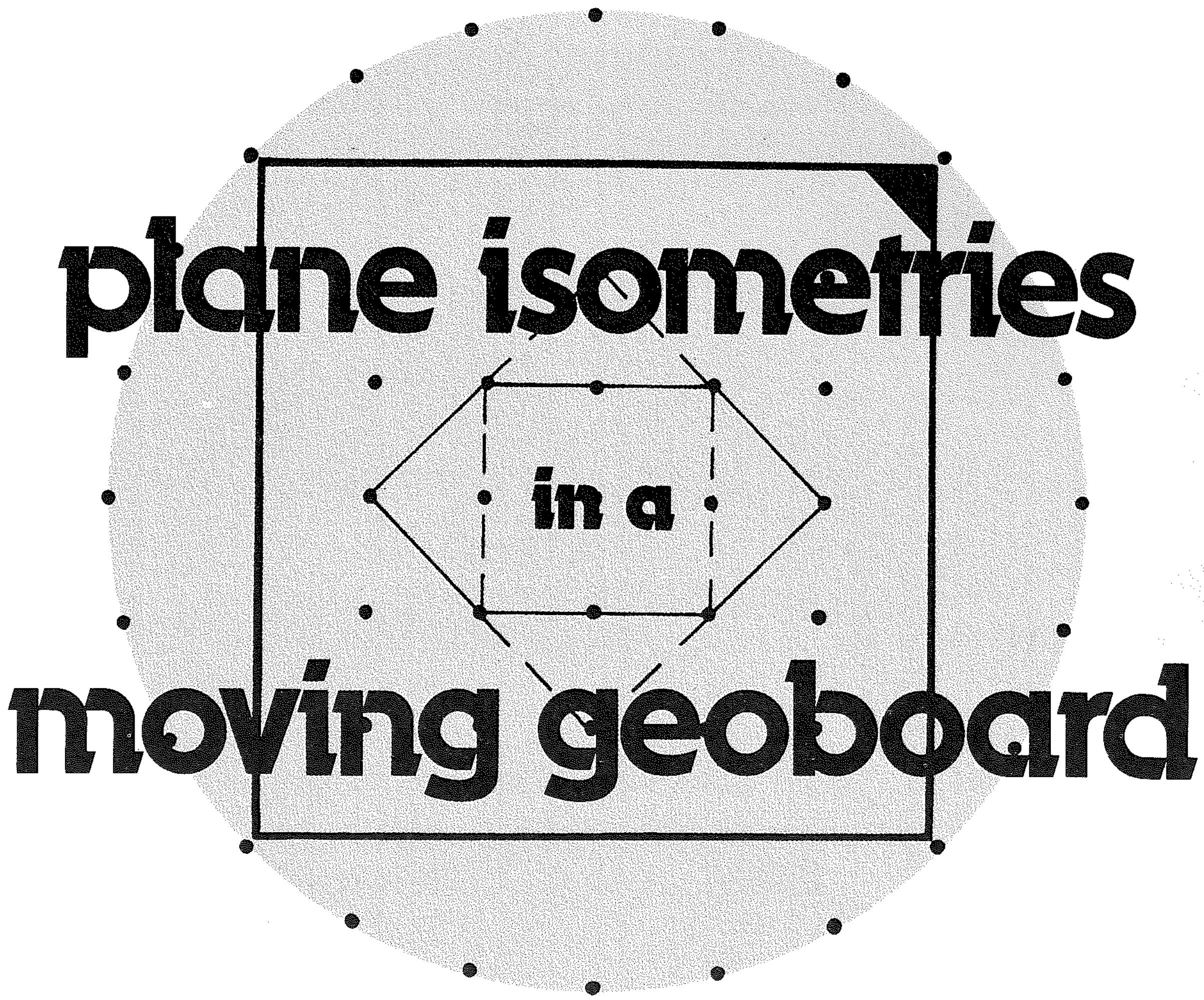
plane isometries in a moving geoboard



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*In this article a new kind of geoboard is described, whose special characteristic is its mobility and whose main use is in the learning of the plane transformations. Some activities are proposed for using this "**Gyratory Geoboard**" in Primary and Secondary Grades.*

The geoboard is a material widely used in the teaching of mathematics; this is fundamentally because it admits a large variety of activities which include almost all the geometry learned in the primary schools of any country. Also some non-geometrical themes may be undertaken with a geoboard, such as divisibility, topology or fractions. Other reasons for its great diffusion are the facts that it is a very cheap material and is easy to build, even by children; it is amusing for the students, they may easily draw perfect geometrical figures and mis-shape them in order to go quickly from one shape to another.

At present there are some different models of geoboards: ones with square nets, triangular or circular ones; the one with the biggest variety of uses is the square geoboard, while the circular or the triangular ones are used in some particular themes and as a complement of the square geoboard (see Harkin[1] and Bennett[2]).

In this paper we describe a new model of geoboard, that we call a "*Gyratory Geoboard*", which we have developed thinking of facilitating the movement of the net of the traditional geoboards. In designing blocks of activities in geometry for the pupils of the Spanish Basic General Education (aged from 6 to 14), we considered the possibility of teaching symmetries and rotations by using the geoboard; from here it appeared the convenience of easily moving the geoboard to make possible the rotation of the figures built on it a determined number of degrees. The result is the Gyratory Geoboard, which broadly is a traditional geoboard based on an axis.

With the Gyratory Geoboard the pupils will get accustomed to recognise the figures in non-standard positions, which should be a constant objective in the teaching of geometry in the primary and secondary grades. Moreover, the activities carried out with a fixed geoboard acquire a new dimension and also it is possible to approach some new themes such as the plane isometries, which we shall comment on later.

In the following sections we describe the Gyratory Geoboard and we propose some sets of activities for the teaching of plane rotations and central symmetry. When we say a "*5 × 5 geoboard*" we refer to a square geoboard with 25 nails and with a "circular 24-pin geoboard" we refer to the one with 24 nails in the circumference and another nail in the centre.

Description of the Gyratory Geoboard

To make a Gyratory Geoboard we use the same materials as for any other geoboard. The Gyratory Geoboard is composed of two pieces (made of wood usually) placed one on the other. The lower one, which stays immobile, is a circular 24-pin geoboard in which the central nail, longer than the others, is the axis of rotation for the upper piece. This second piece is a square inscribed in the cited circumference with a hole in the centre through which the axis enters (Figure 1); in this way the upper piece turns freely and the nails in the base allow us to measure the angle of rotation. The moving piece may support a square, circular or triangular geoboard (Figure 2) but making sure that the axis of rotation is a point of the net and to be as high as the other nails in the upper geoboard (Figure 3).

With respect to the size, we think that 30 cm for the diameter of the circumference in the base (therefore 21.2 cm for the side of the upper moving piece) gives us a suitable Gyratory Geoboard.

Obviously the main use of the nails in the base is to count degrees and to measure angles (two adjoining nails form a central angle of 15°). Then it is useful to mark a corner of the moving piece; thus the pupils will easily make rotations only by counting the nails which the marked corner touches.

In the following activities we use a Gyratory Geoboard with a base of 24 nails, but it is also possible to use other numbers of nails for obtaining other objectives (for example, a base with 36 nails it is necessary for making rotations of 10°).

Activities

Now we begin the description and comments on some sets of activities whose objective is to introduce the concepts of rotation and central symmetry on the plane by means of manipulations with a Gyratory Geoboard. These activities may be included in a bigger block of activities for the teaching of the plane similarities; for activities in other similarities, see Barson[3], ATM[4], SMP[5], Farrell[6], Lund[7] and Cech, Tate[8]).

We use the 5×5 geoboard and the circular 12-pin geoboard. The following activities are directed to pupils of the Intermediate Cycle in the Spanish Basic General Education (aged from 8 to 11) and they only are a model of the many possibilities offered by this new geoboard.

First of all we introduce axial symmetry from the reflection in a mirror, in a similar way as the authors referred above. These activities may be done on a standard geoboard but since the pupils will later use the Gyratory Geoboard, we think it convenient to use this one from the beginning.

Afterwards we begin the study of rotations by means of activities whose main goals are the concepts of rotation, centre of rotation and invariance under a rotation. Because of the structure of the Gyratory Geoboard, the centre of the geoboard must be taken as the centre of rotation.

First the pupils do easy activities designed to make them become familiar with the new geoboard, by making some rotations. Also they discover with these activities that the angle from a nail to the next is of 15° and they may measure angles by counting nails.

By drawing a segment on the geoboard we ask the pupils to rotate it until the segment returns to its original position. (This activity has been proposed for an immobile geoboard, see Barson[3] and Farrell[6], but the manipulations are more difficult than on a Gyratory one).

A-1. Copy on your Gyratory Geoboard a segment of the Figure 4. When you rotate the geoboard, the segment changes its position. Keep on rotating it until the segment returns to its initial position. How many degrees have you rotated the geoboard? Repeat the activity with every segment in the figure. Have you ever needed the same angle?

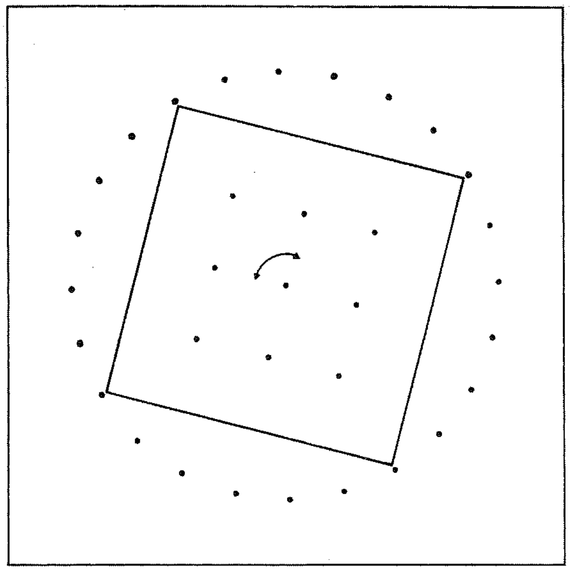

Fig. 1

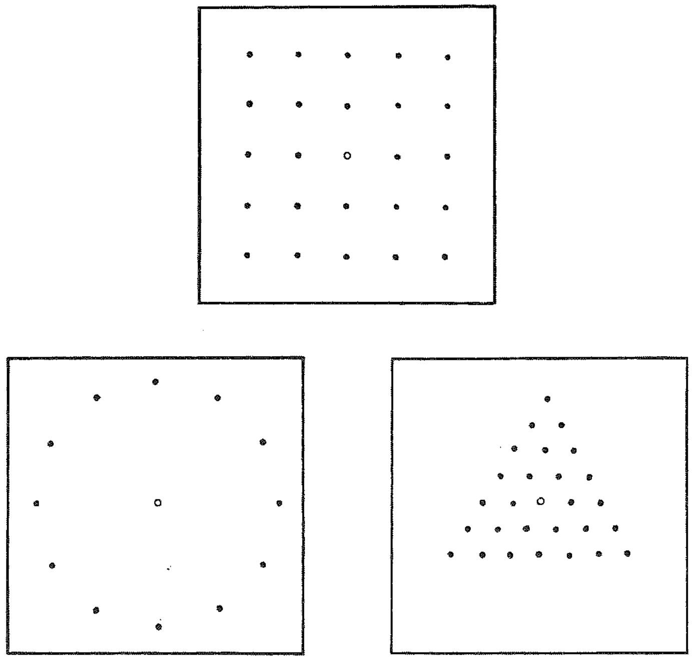

Fig. 2

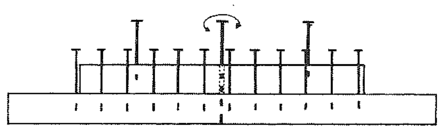

Fig. 3

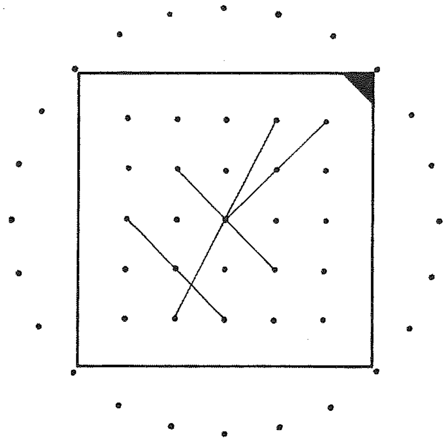

Fig. 4

With the previous activity children discover that some segments need 360° and other ones only need 180° for turning to the initial position. Now they have the basis of the concept of centre of symmetry and symmetric points with respect to a centre. This concept may be strengthened with the following activities:

A-2. Build in your geoboard some symmetric segments with respect to (3,3).

A-3. Tell your teacher if the following pairs of points are symmetric with respect to (3,3): (2,2) and (4,4); (1,2) and (3,4); (2,4) and (5,1); (2,5) and (4,1).

A-4. Look for the symmetries of the following points with respect to (3,3): (1,2), (2,4), (2,1), (3,3), (1,1).

In a similar way as the pupils have worked with axial symmetry, now they will carry out activities in search of the symmetries of a polygon with respect to a centre, which will direct children to the invariant polygons under a rotation and to the centre of symmetry of a polygon. Some of the activities (that may easily be completed by the reader) are:

B-1. Make in your Gyratory Geoboard the polygon of the Figure 5 and show it to another child. Ask him to look away while you rotate the geoboard 90°, 180°, 270° or 360°. When the child looks again at the geoboard, will he know how many degrees the geoboard has been rotated?

B-2. Complete the polygon of the Figure 6 so that it has a centre of symmetry in (3,3). Next try to complete it so that, moreover, it remains invariant for a rotation of 90°. Later try to complete the polygon in a way that it remains invariant for a rotation of 180° but it does not for a rotation of 90°.

Similar activities to B-1 and B-2 above done on a circular Gyratory Geoboard are useful to strengthen the concepts and to give variety to the exercises because we may work with different angles of 90° or 180°.

This theme is completed with activities that look for the centre and the axes of symmetry of the most usual polygons and that point out the relation of intersection between axes and centre.

Apart from the completion of these activities, the pupils are introduced to learning about the plane isometries and they have discovered the most important properties from their own efforts and research, which is the best way of learning. In a later stage of learning, these children will be in a position to comprehend and formalise the isometries and they will be able to use geoboards for looking for examples and for checking their conjectures.

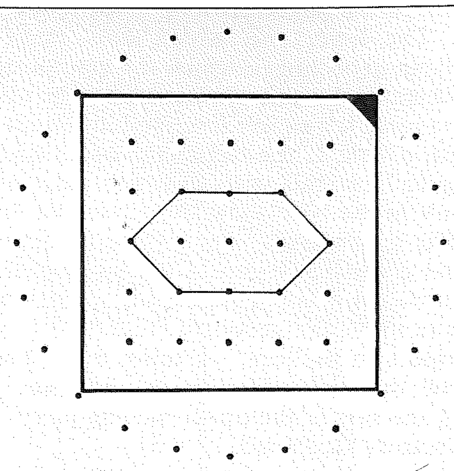

Fig. 5

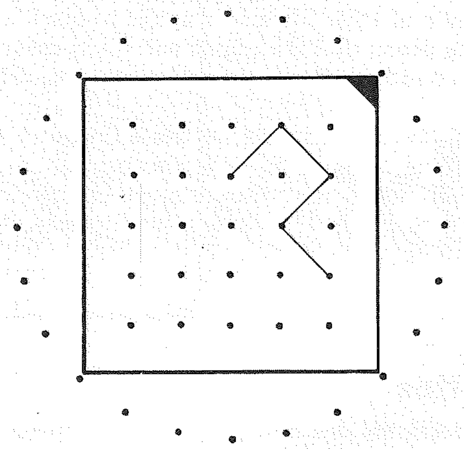

Fig. 6

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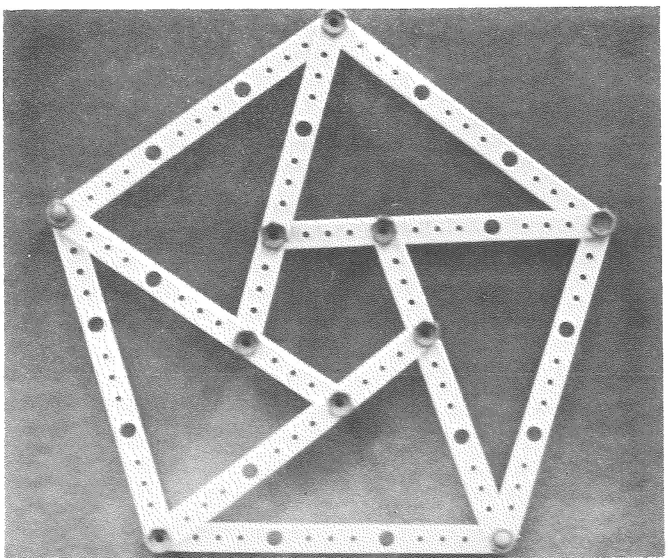

An Impossibility?

This construction was made by a student on a P.G.C.E. course at the University of Leeds.