Globality versus locality of the van Hiele levels of geometric reasoning

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ABSTRACT. The theory proposed by P.M. and D. van Hiele have given rise to a model of teaching and learning whose three main characteristics are discreteness and hierarchy of levels and usefulness of the theory for prediction.

We present the results of an empirical research which deals with the hierarchical structure and the predictive property of the model. The comparison of results obtained by administering three tests (on polygons, measurement and solids) to a group of preservice elementary teachers allows us to formulate the following conclusions:

a) Levels 1 to 4 form a hierarchy, but level 5 has some particularities that need an in deep investigation.

b) There is not relation between the individual results in the different tests, so the assessment of pupils' level in a topic cannot predict their level in other topic.

INTRODUCTION

The theory developed by P.M. van Hiele and D. van Hiele-Geldof is currently being considered with great interest in Mathematics Education. This theory is based on the definition of several thought levels, through which students progress while learning mathematics, each level being characterized by a specific form of reasoning, a vocabulary and a kind of knowledge. Briefly, the thinking abilities acquired by students in the different levels are:

Level 1 (recognition): Students recognize the objects and mathematical concepts by their physical aspect and in a global way, without distinguishing explicitly their components nor mathematical properties.

Level 2 (analysis): Students recognize the components and mathematical properties of an object or concept. They are able to establish relationships among objects and/or components, but only in an experimental way. They can not establish logical relationship nor make formal descriptions.

Level 3 (classification): Students make logical relationship between mathematical properties and they are able to follow simple deductive reasoning, but they still do not understand the function of the elements of a mathematical axiomatic system (axioms, definitions, proofs, etc.) and, therefore, they do not know how to handle them.

Level 4 (deduction): Students understand and make deductive reasoning, since they already understand the function of axioms, hypothesis, definitions, etc. However, students still have not acquired a global insight of axiomatic systems and they do not understand the need for rigorous reasoning.

Level 5 (rigor): Students understand the need for rigorous reasoning, they are able to write abstract proofs in different axiomatic systems, and to analyze and compare two axiomatic systems.

More information about the characteristics of van Hiele levels can be found in Usiskin (1982), Hoffer (1983), Fuys, Geddes (1984), Burger, Shaughnessy (1986), and van Hiele (1986). The greater interest of van Hiele theory is the possibility of using it as framework to build a teaching model for geometry, where each level carries a form of activities, a language and an organization of the learning process helping students to reach next level. Examples of programs based on the van Hiele levels can be found in the Soviet Union, Holland and USA (Freudenthal (1973), Wirszup (1976), Mathematics Resource Project (1978), Hoffer (1983) and Fuys, Geddes (1984)).

If we analyze the theoretical structure of van Hiele model, there are three characteristic that have to be deeply studied: The discreteness of the levels, that is the way students move from a level to the next one, their hierarchical organization, and their globality, that is students' capacity to transfer their level of reasoning from a mathematics topic to another one. During last years important research on each of the above mentioned characteristics have been carried out (Usiskin (1982), Mayberry (1983), Fuys, Geddes (1984) and Burger, Shaughnessy (1986)), whose main results are summarized in Senk (1985). In spite of such intense research, it has not been possible to determine in a satisfactory way the validity of any of such characteristics of the model.

THE STUDY

In this paper we present the results of a study aimed to evaluate the validity of two theoretical characteristics of the van Hiele levels: Their hierarchical structure, and their globality with respect to several fields of geometry.

Up to now, most experiences developed to determine the properties of van Hiele theory have been based on problems of plane geometry, almost in every case related to polygons. The first part of our research has consisted on designing three tests based on the three more important fields of geometry: Plane geometry (mainly polygons), geometric measurement (length, surface and volume) and space geometry (mainly polyhedrons).

The comparison of the levels reached by each student in the three tests has permitted us to observe the correlation among them and, therefore, to decide on the globality or locality of the van Hiele levels. On the other hand, the independent analysis of each test allowed us to evaluate the hierarchy of the levels.

We have designed three tests based on the structure of the test in Usiskin (1982). Each test consists of 25 multichoice items, 5 items measuring each van Hiele level, with 5 possible answers for each item, only one of them being correct. To make more reliable the comparisons of the results, the same grammatical structure has been used in all the three tests. Respect to the mathematical content of the items, it is easy to write models of problems valid for both plane (polygons) and space geometry (polyhedrons), since they have similar conceptual structures and internal relationships, but the field of measurement has a much more simple conceptual organization, making it difficult to maintain the structural similarity with the items in the two other tests.

On the other hand, the design of tests based on mathematical concepts different from those related to polygons is useful to open new research directions on the van Hiele model of reasoning to use it in the teaching of more geometry topics.

THE SAMPLE

The tests have been administered to 563 pupils from the three courses of the Primary Teacher Training School of the University of Valencia. Table 1 shows the number of students that answered the different sets of tests.

	Test P	**Test M**	**Test S**
Test P	409	276	232
Test M	---	392	241
Test S	---	---	318
Tests P, M and S	193		

P = plane geometry; M = measurement; S = space geometry

Table 1

The answering of the tests took place in three different sessions, with at least three days between one and the next, though in most cases this interval between the administration of two tests was of 1 or 2 weeks. The students had free time to answer the tests (average of 30 to 45 minutes).

RESULTS AND CONCLUSIONS

It is evident the importance for the results and conclusions in the research of the criteria adopted by researchers to assign the minimal level to a student and to determine when to assign a level or the next one. Usually, the criterion applied has been that a student pass a level when 2/3 of the items corresponding to the level are correctly answered. If we apply this criterion to our tests, a student pass a level when 3 or 4 out of the 5 items corresponding to the level are correctly answered. Our previous knowledge of our pupils and other studies carried out with similar kind of students (Mayberry (1983) and Matos (1985)) induced us to suppose that most students in our sample would be in levels 2 and 3 (the first level in van Hiele model is level 1; we assign to level O those students who do not fit the criterion for level 1). For this reason, we have used two criteria of assignment of students to levels to guarantee (Usiskin (1982), p. 23):

a) That, for the lower levels, students are not assigned to a level under their real level. Therefore we use the criterion 3/5.
b) That, for the higher levels, students are not assigned to a level over their real level because they give correct answers at random. Therefore we use the criterion 4/5.

We have defined two different criteria of assigned of students to the levels: The criterion 33344 (that is, students pass levels 1, 2, and 3 if they correctly answer 3 of the 5 items, and they pass levels 4 and 5 if they correctly answer 4 of 5 items) and the criterion 33444. The only difference among both criteria is the assignation of students to level 3. Furthermore, when a pupil passes levels 1 and 2 and fails level 3, he has been assigned to level 2, independently of the results obtained in levels 4 and 5. Table 2 shows a summary of the levels obtained by students in the sample.

	Test P		**Test M**		**Test S**	
Level	**33344**	**33444**	**33344**	**33444**	**33344**	**33444**
0	4.40	4.40	24.74	24.74	45.28	45.28
1	11.98	11.98	20.92	20.92	44 97	44.97
2	24.45	57.95	25.00	41.58	1.89	6.29
3	56.72	23.47	22.96	8.67	7.55	3.14
4	1.71	1.71	2.30	1.02	0.00	0.00
5	0.73	0.49	4.08	3.06	0.31	0.31

Table 2: Distribution (%) of the pupils by levels

To analyze these results we have calculated several statistical coefficients. To check the hierarchy of the levels, the coefficient of reproductivity R from Guttman Scalographic Analysis has been used. Coefficient R evaluates the quantity of students that have failed a level but have passed a higher one. It has been used in an effective way in previous research projects, like Mayberry (1983) or the C.S.M.S. Project of the Chelsea College (Hart and others (1981)). Table 3 contains the values of R obtained from the answers to the whole tests.

	Criteria	
Test	**33344**	**33444**
P	0.940	0.939
M	0.868	0.861
S	0.853	0.903

Table 3: Coefficient R for levels 1 to 5

	Criteria	
Test	**33344**	**33444**
P	0.976	0.990
M	0.959	0.966
S	0.858	0.935

Table 4: Coefficient R for levels 1 to 4

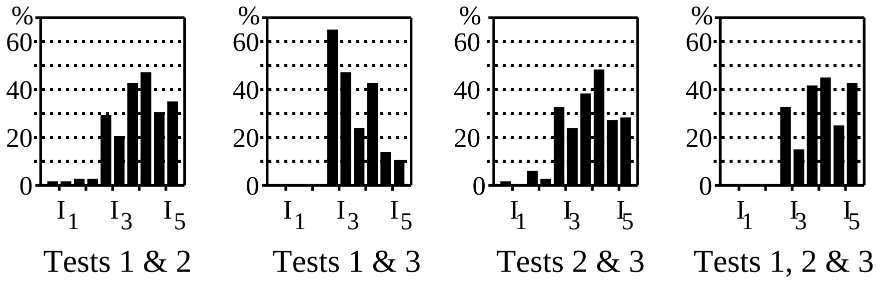
Usually, a hierarchy is considered as valid when coefficient R is not lower than 0.90 (Mayberry (1983)) or to 0.93 (Hart (1981)). Therefore, according to table 3, the hierarchy of van Hiele levels in measurement and solid geometry should be rejected. However, a more detailed analysis of the results clearly shows a significant influence of items in level 5 in the previous values of coefficient R: Many students passed level 5 but failed a lower level, so producing errors in the hierarchy. Table 4 contains the values of R obtained from the answers only to the items for levels 1 to 4.

From these results we can conclude that van Hiele levels 1 to 4 form a hierarchy, but that level 5 has some special features that should be studied in detail in order to re-state its characteristics or to consider the convenience of eliminating it from the model, as suggested by van Hiele himself (1986, p. 47).

To evaluate the globality of reasoning in the different van Hiele levels, we have calculated two coefficients. The Leik consensus coefficient C measures the degree of dispersion of the levels reached by a person in the different tests, while the Kruskal coefficient γ measures the correlation among the answers of the whole sample to two tests.

Coefficient C varies between 0 and 1, with C = 0 indicating disparity (maximum dispersion), C = 0,5 indicating randomness, and C = 1 indicating concordance among the answers (no dispersion). We have grouped the values of C in several intervals: I_1=[0, 0.15[, I_2=[0.15, 0.30[, I_3=[0.30, 0.70[, I_4=[0.70, 0.85[, and I_5=[0.85, 1]. The graphics below show the percentage of students in each interval.

For each interval, the left column corresponds to criterion 33344 and the right column corresponds to criterion 33444.



Coefficient γ varies between -1 and +1 its meaning being the usual for correlation coefficients. Table 5 shows the values of γ for each pair of tests.

	Criteria	
Test	**33344**	**33444**
P y M	0.46	0.52
P y S	0.45	0.40
M y S	0.41	0.44

Table 5: Coefficient γ

Coefficient C and coefficient γ indicate that:

1) There is not divergence among the different tests.
2) There is certain degree of concordance among the results of the tests, but this is not sufficient to support the hypothesis of the globality of the van Hiele levels.

Both our experience and the others cited before present very low percentages of students in levels 4 and 5, therefore we believe that they are not valid enough to raise conclusions with respect to these levels. A research direction that should be explored in the future is to design experiences with groups of students most of them reasoning in levels 4 and 5. Very likely, in this case positive results about the globality of van Hiele levels would be obtained, since students reasoning in levels 1 to 3 have a fragmented local vision of mathematics that inhibits their transfer of knowledge and reasoning skills from an area of mathematics to another. On the contrary, those students that have reached levels 4 and 5 have a more global vision of mathematics that facilitates their transfer. Our hypothesis respect to this question is that levels 1, 2 and 3 are of a local nature, so they do not allow students to have a global reasoning, while levels 4 and 5 are of a global nature and they allow students to have a global reasoning.

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