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AN ALTERNATIVE PARADIGM TO EVALUATE THE ACQUISITION OF THE VAN HIELE LEVELS

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This article presents an alternative way of analyzing the van Hiele level of students' geometrical reasoning. We evaluate the students' answers, taking into account the van Hiele level they reflect and their mathematical accuracy. This gives us a description of how accomplished the students are in applying the procedures associated with each of the van Hiele levels and allows us to determine the students' degree of acquisition of the van Hiele levels. In this way we obtain a clearer picture of the students' geometrical reasoning than with the traditional assignment of one van Hiele level to the learners. An example of the application of this method is provided: We describe a test that evaluated students' ability to reason in three-dimensional geometry, some responses of students (9 eighth-grade pupils and 41 future primary school teachers), and the classification of their responses using our method.

Approximately 25 years ago, the van Hieles proposed a model of the development of geometric thinking that identified five differentiated levels of thinking, ordered so that the students moved sequentially from one level of thinking to the next as their capability increased (van Hiele, 1957, 1986; van Hiele-Geldof, 1957). In the last 10 years there has been a growing interest in the van Hieles' model of the development of geometric thinking (Fuys, Geddes, & Tischler, 1988; Gutiérrez & Jaime, 1989; Hoffer, 1983; Senk, 1985). An important focus of research has been on ways to determine students' level of thinking. The main goal of this article is to present an alternative method to evaluate the students' van Hiele level of reasoning, thus offering a way of identifying those students who are in transition between levels.

The way the student's level of reasoning is ascertained plays an important role in research related to the van Hiele model. Most researchers have determined a student's van Hiele level for a topic following assessment criteria based on the number of right answers to a written test (Gutiérrez & Jaime, 1987; Mayberry, 1983; Usiskin, 1982) or on the thinking level shown by the student in each activity during an interview (Burger & Shaughnessy, 1986; Fuys et al., 1988). In both cases, the respective criteria have assigned each student to one van Hiele level.

Although most students show a dominant level of thinking when answering open-ended questions, a large number of them clearly reflect in their answers the presence of other levels, and there are some students whose answers show two consecutive dominant levels of reasoning simultaneously (Usiskin, 1982; Burger & Shaughnessy, 1986; Fuys et al., 1988). Burger and Shaughnessy and Fuys et al. suggested that these students were in transition between two levels, but their approaches to the problem have been different. Burger and Shaughnessy sought a consensus in the evaluators' opinions; Fuys et al. assigned a student to Level 1-2 to indicate that the student clearly used both Levels 1 and 2 of reasoning for an activity.

A PARADIGM FOR DETERMINING THE DEGREE OF ACQUISITION OF VAN HIELE LEVELS

Degrees of Acquisition of Levels

The previous considerations have led us to conclude that the van Hiele levels are not discrete, and we need to study in more depth the transition between levels. Our proposal in this sense is based on the following two arguments: (a) To have a more complete view of the current geometrical reasoning of students, we should take into account their capacity to use each one of the van Hiele levels, rather than assign a single level. (b) Continuity in the van Hiele levels means that the acquisition of a specific level does not happen instantaneously or very quickly but rather can take several months or even years.

We have quantified the acquisition of a level of thinking by representing it with a segment graduated from 0 to 100. However, it is possible to identify several distinct ways of reasoning during the acquisition of a level; consequently, it is also convenient to divide this continuous process into five periods characterized by the qualitatively different ways in which the students reason. These periods represent fundamental differences in the *degree of acquisition* of a given level.

The proposed division of each van Hiele level into periods does not imply that the progress through van Hiele levels is not continuous. Assigning a numerical value to the degree of acquisition of a level could be useful to researchers. However, in order to plan for instruction, it is necessary to have qualitative measures so that we can differentiate between students to assign them to appropriate learning activities. Figure 1 shows both the quantitative and qualitative interpretations of the process of acquiring a level. The specific values that we have assigned to the limits are, to some extent, subjective.

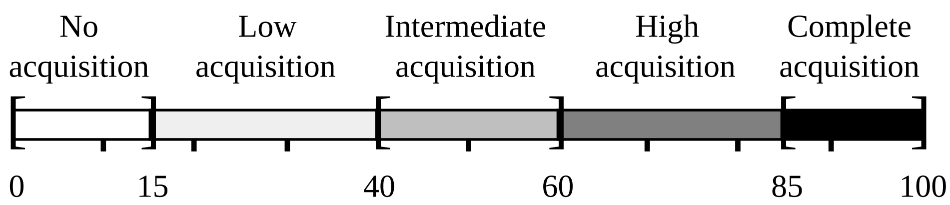

Figure 1. Degrees of acquisition of a van Hiele level.

Initially, students are not conscious of the existence of, or need for, thinking methods specific to a new level. They have *no acquisition* of this level of reasoning.

Once the students have begun to be aware of the methods of thinking at a given level and of their importance, they try to use them. Nevertheless, because of their lack of experience, the students simply make some attempts to work on this level, with little or no success in solving the activities, and they return to the lower level of reasoning. They have a *low degree of acquisition* of the level.

As the students' experience grows, they enter a period of *intermediate degree of acquisition* of the new level. They already use the methods of the level more often, continuously, and accurately. Nevertheless, the lack of mastery of those methods makes the students fall back on the methods of the lower level when they encounter special difficulties in their activities, although afterwards they try to go back to the higher level. Therefore, the reasoning during this time is characterized by frequent jumps between the two levels. The confusing answers reported by Burger and Shaughnessy (1986) and Fuys et al. (1988) correspond to this degree of acquisition of a level.

With more experience, the students' reasoning is progressively strengthened. They reason in the usual way corresponding to this level, but they make some mistakes or sometimes go back to the lower level. This is therefore a period in which the students have reached a *high degree of acquisition* of the level, but it is still not complete.

Finally, the students attain *complete acquisition* of the new level when they have complete mastery of this way of thinking and use it without difficulties.

Assessment of Levels and Degrees of Acquisition

In order to assign students to a specific degree of acquisition within each van Hiele level, we propose an assessment procedure consisting of a series of open-ended items and criteria for evaluating students' responses to each item. For each item students are assigned a numerical score that is related to the scale used to determine the degrees of acquisition. By averaging the scores assigned to items that measure each particular level, a student is assigned to a degree of acquisition within each level. The following paragraphs give a detailed explanation of such a procedure.

We start with the assumption that it is more important to observe the students' type of reasoning than their ability to solve certain problems correctly in a set time. Furthermore, a partially correct (or even a totally incorrect) answer may also afford us information. An incorrect answer may, by itself, give us a negligible amount of information, but the case is different when it is considered in conjunction with other answers. In scoring each response, we take into account both the van Hiele levels reflected by the answers and the mathematical accuracy. However, we do not give the same value to a completely incorrect answer as to a partially incorrect one or to a correct one. We make an evaluation of each answer that takes into account the thinking level(s) reflected as well as its mathematical accuracy and completeness. Specifically: (a) Each answer is classified according to the van Hiele level of thinking it reflects following the descriptors of the levels. Answers evidencing two consecutive levels are assigned to the higher level because they indicate a certain degree of acquisition of that level. (b) Next, each answer is assigned to one of a number of types of answer, depending on its mathematical accuracy and on how complete the solution to the activity is. To determine which type an answer belongs to, it is necessary to consider it from the point of view of the van Hiele level it reflects, since an answer can be adequate according to the criteria of a given thinking level but not valid according to the criteria of a higher level. Any reply to an open-ended item may be assigned to one of the following types:

Type 0. No reply or answers that cannot be codified.

Type 1. Answers that indicate that the learner has not attained a given level but that give no information about any lower level.

Type 2. Wrong and insufficiently worked out answers that give some indication of a given level of reasoning; answers that contain incorrect and reduced explanations, reasoning processes, or results.

Type 3. Correct but insufficiently worked out answers that give some indication of a given level of reasoning; answers that contain very few explanations, inchoate reasoning processes, or very incomplete results.

Type 4. Correct or incorrect answers that clearly reflect characteristic features of two consecutive van Hiele levels and that contain clear reasoning processes and sufficient justifications.

Type 5. Incorrect answers that clearly reflect a level of reasoning; answers that present reasoning processes that are complete but incorrect or answers that present correct reasoning processes that do not lead to the solution of the stated problem.

Type 6. Correct answers that clearly reflect a given level of reasoning but that are incomplete or insufficiently justified.

Type 7. Correct, complete, and sufficiently justified answers that clearly reflect a given level of reasoning.

Answers of Types 0 and 1 indicate no level. Nevertheless, there is a difference between them, because Type 1 answers indicate that a specific level has not been attained. The numerical value that we assign to both types is the same, and consequently, from this point of view, both types could be joined into a single one; but they are qualitatively different. For this reason we think that it may be helpful to differentiate them.

Answers of Types 2 and 3 point to the beginning of the acquisition of a level. In both cases answers are very incomplete and generally very short. Because of their incompleteness, they do not allow the evaluator clearly to identify a level from the student's reasoning. The evaluator will only be able to identify vague traces or flashes of that level of reasoning.

Type 4 indicates answers for which the student uses two levels of reasoning, but neither of the levels is clearly predominant. This kind of answer characterizes students who are in an intermediate phase of transition between two levels, because they are aware of the convenience of using thinking methods of the higher level but they cannot dispense with the methods of the lower level. As to their mathematical accuracy and completeness, this kind of answer has characteristics similar to those of Types 5, 6, and 7.

Types 5 and 6 correspond to answers reflecting clearly the student's use of a predominant specific level of reasoning, although sometimes a lower level can appear. These answers reflect an advanced phase in the transition between two levels, with differing degrees of acquisition of the higher level, because according to Burger and Shaughnessy (1986), the acquisition of a thinking level depends on its reasoning methods and on the appropriate development of the mathematical concepts. Therefore, an incomplete acquisition of a level can be observed (a) when the student uses reasoning methods of this level imperfectly and sometimes needs to resort to methods from the lower level, or (b) when the student is unable to complete the answer or to realize that it is not correct.

Finally, Type 7 indicates that the student has fully acquired a given level, since he or she is able to solve the whole activity using only methods of reasoning characteristic of that level.

The eight types of answers reflect the various degrees of acquisition of the van Hiele levels of thinking defined above. Types 0 and 1 indicate no acquisition, Types 2 and 3 indicate low acquisition, Type 4 indicates intermediate acquisition, Types 5 and 6 indicate high acquisition, and Type 7 indicates complete acquisition of the level.

Thus, we can assign a vector (l, t) to each answer of a test, where l is the van Hiele level reflected in the answer and t is the type of answer (l is empty when t is zero). The types of answers are quantified in terms of the scale of acquisition of the reflected level of reasoning; Table 1 shows the numerical weight assigned to each type, according to the interpretation above and Figure 1. The degree of acquisition of a van Hiele level by a student is determined by calculating the arithmetic average of the weights of the vectors (l, t) for all the items that could have been answered at that level. In the next sections, an application of this method of evaluating the acquisition of the van Hiele levels will be shown, using a test of three-dimensional geometry.

Table 1
Weights of Different Types of Answers

Type	0	1	2	3	4	5	6	7
Weight	0	0	20	25	50	75	80	100

METHOD

Spatial Geometry Test

The Spatial Geometry Test was designed to evaluate the van Hiele level of students' thinking in three-dimensional geometry. Five different versions of the test were administered to pilot groups of primary school students and to future primary school teachers. For each version, one of the team members redesigned any faulty items, which the other two members then validated. After each experience we proceeded to analyze the students' answers and to modify or eliminate items. The version previous to the one now in use was sent to three experts to obtain outside validation.

Measurement of van Hiele Levels

We have based the test design on specific three-dimensional geometry descriptors for the van Hiele Levels 1 to 4, as follows:

Level 1 (Recognition). Solids are judged by their appearance. The students consider three-dimensional objects as a whole. They recognize and name solids (prisms, cones, pyramids, etc.), and they distinguish a given solid from others on a visual basis. The students do not explicitly consider the components or properties in order to identify or to name a solid; on the contrary, they use reasoning of the type “it looks like...” or irrelevant attributes.

Level 2 (Analysis). The students identify the components of solids (faces, edges, etc.), and the solids are bearers of their properties (parallelism, regularity, etc.). They describe in an informal way three-dimensional shapes by means of their properties. They are not able to logically relate the properties to each other, nor can they logically classify solids or families of solids. The students are able to discover properties of the solids by experimentation.

Level 3 (Informal deduction). The students are able to logically classify families of solids (classes of prisms or rounded solids, regular polyhedra, duality, etc.). Definitions (necessary and sufficient conditions) are meaningful for students, and they are able to handle equivalent definitions for the same concept. They can give informal arguments for their deductions, and they can follow some formal proofs given by the teacher or the textbook, but they are only able to carry out simple inferences by themselves.

Level 4 (Formal deduction). The students understand the role of the different elements of an axiomatic system (axioms, definitions, undefined terms, and theorems). They can also perform formal proofs.

Activities

The nine items of the Spatial Geometry Test were grouped into five activities. Activities 1 and 2 focused on the observation and manipulation of the polyhedra pictured in Figure 2; each student was given the six solids (made of cardboard) and allowed to manipulate them. In Activity 3 the students used a given set of properties in order to identify a solid described by these properties. In Activities 4 and 5 the students had to make logical deductions. Now we shall describe the test activities in detail.

Activity 1 had four parts. For each part the students were asked to select the solids (from those in Figure 2) that had the given property. The properties were (a) being a pyramid, (b) having each face parallel to another face, (c) having at least one plane of symmetry, and (d) having three faces joined at each vertex.

Activity 2 required the students to “complete a chart writing the differences and similarities between a cube and each of the solids A, B, C, and I.”

Activity 3 had two parts. First, students were asked to draw a solid (different from those of the collection) that satisfied the following conditions and to describe it with words.

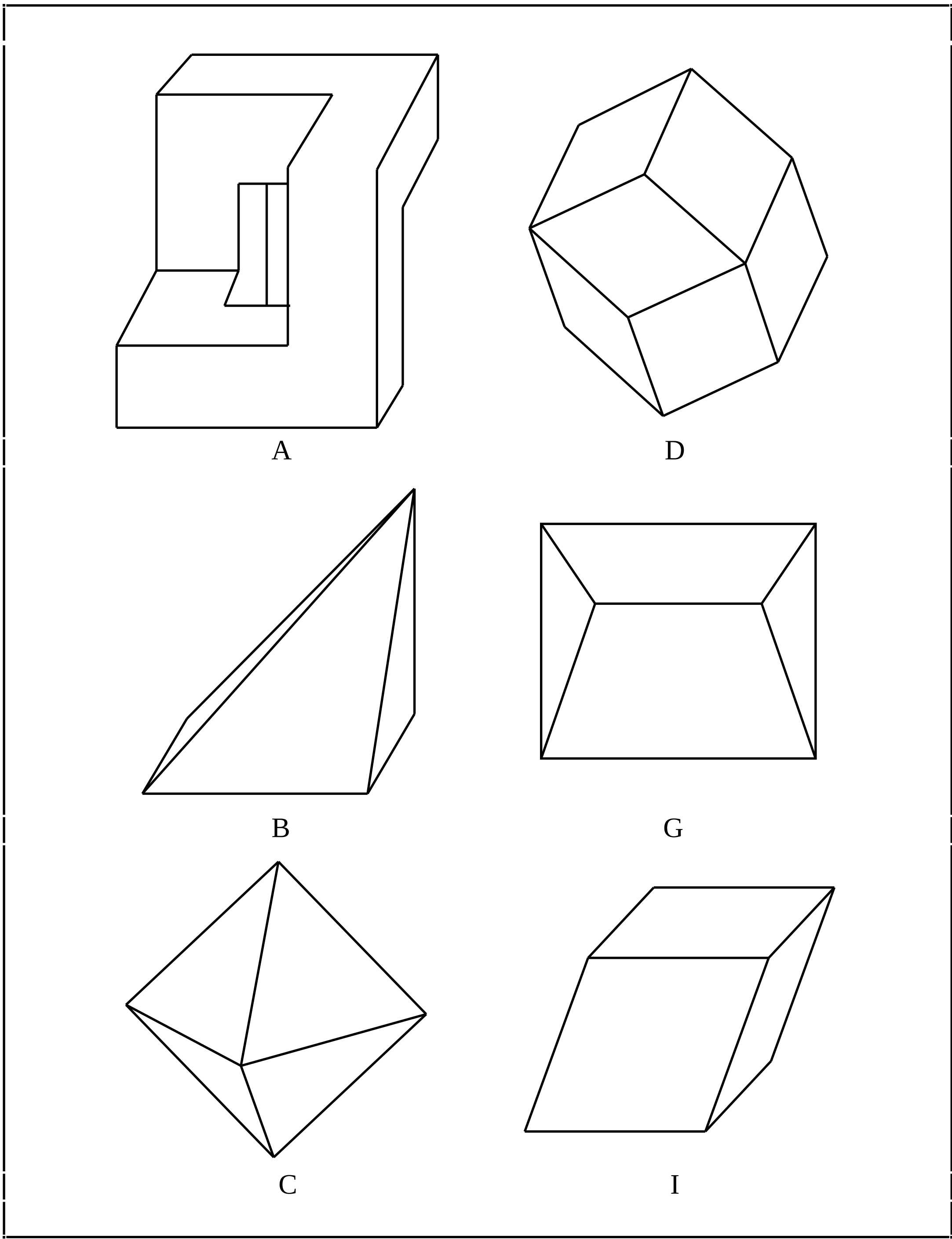

Figure 2. The solids manipulated by the students.

1) It has exactly 8 short equal edges and 4 long equal edges.
2) There are exactly 3 different-sized angles formed by the edges.
3) At least 2 of the short edges are parallel.
4) Every face is parallel to another face.
5) All the long edges are parallel.

Then, the students were asked to identify the smallest set of the conditions that determined the figure and to justify their answer.

Activity 4 required students to check whether the following implication is true or false and to prove their answer: “If a solid has a central point that bisects every segment going through that point and whose ends are on the solid’s surface, then each of the solid’s faces is parallel to the face opposite it.”

Activity 5 was stated as follows: “A corner of a room is usually formed by 3 rectangles. Is it possible to build corners formed by 3, 4, 5, 6, or 7 equilateral triangles? Give a proof of your answer.”

Items in Activities 1 and 2 provided a measure of both van Hiele Levels 1 and 2, because they could be answered by paying attention to visual qualities of the solids or to their geometric properties. Item 3a assessed Levels 2, 3, and 4, because the students could look for a solid satisfying the conditions (a) as if they were independent, (b) by relating them in an informal way, or (c) by deducing new simpler properties to help in the search for the solid. And Items 3b, 4, and 5 assessed Levels 3 and 4, because students could explain them in an informal or a formal way when proving their answers.

Sample

The test was administered to a sample of 50 students who had not been given any specific instruction in spatial geometry in the recent past. The students represented the three following groups:

A) Twenty future primary school teachers, specialists in science, in their third year at a teacher training college (ages 21-22). The students all attended the same mathematics class for the year.

B) Thirteen future primary school teachers, specialists in kindergarten, in their third year at a teacher training college (ages 21-22) and eight future primary school teachers, specialists in modern languages, in their first year at a teacher training college (ages 19-20). We joined these two subgroups into a single group because both had similar mathematical background and motivation, which differed from those of the students included in Group A. The students in each subgroup all attended the same mathematics class for the year.

C) Nine eighth-grade pupils in the same classroom of a state primary school (ages 13-14).

The students in Groups A and B had studied mathematics in primary school, lower secondary school, and the first year at the teacher training college. Group A students (and some belonging to Group B) had also studied mathematics in upper secondary school. The students of Groups A, B. and C all had the same previous studies of three-dimensional geometry, just the little spatial geometry that is included in primary school. This instruction usually consists of recognizing the various sorts of solids (prisms, pyramids, regular polyhedra, cylinders,...); studying the solid’s elements and components (faces, edges, height, radius,...); calculating surface and volume; and sometimes studying symmetry of solids.

Procedures

For each group the test was administered at the same time to all the students in the group. The test was administered from January to March during the second term. The students were allowed to take as much time as they needed, and it took most of them about 1 hour to answer.

The students were given a booklet containing the questions and blank spaces to answer them and drawing devices (straightedge, set square, compass, and protractor). For Activities 1 and 2 each student also had a set of cardboard solids (Figure 2). The students were not given any specific directions about how to answer the questions on the test.

The tests were evaluated in several phases. First, each of the three members of our team scored all the tests and assigned a vector (l, t) to each answer. Then we compared the assignments of the three researchers: When there was disagreement on the assignments for an answer, there was a discussion to agree on a single assignation for that answer. Scoring the pilot tests helped to clarify the criteria for the evaluation, and therefore few discrepancies were found when scoring the final test. During the process of evaluating the tests and while preparing this article, we modified some of our points of view on some elements of the method of evaluation and on the meaning of some types of answers. When this happened, we scored again the items that were affected by the change of criteria and analyzed the new results, in order to modify our conclusions if necessary.

Once we assigned each answer to a van Hiele level l and a type of answer t, the degree of acquisition of a level obtained by the student was determined by quantifying the vectors (l, t) corresponding to all the questions that could have been answered at that level. We summarize the process by means of an example taken from the Spatial Geometry Test: Table 2 shows the vectors (l, t) obtained after marking the test of a hypothetical student. Table 3 shows the weights assigned to the nine items.

Table 2
Vectors of a Student's Test

	Items								
	1a	1b	1c	1d	2	3a	3b	4	5
Level (l)	2	2	2	1	2	3	—	4	3
Type (t)	7	6	5	7	7	4	0	2	1

Table 3
Weights of the Student's Answers

	Items									
Level	1a	1b	1c	1d	2	3a	3b	4	5	Average
1	100	100	100	100	100	—	—	—	—	100
2	100	80	75	0	100	100	—	—	—	76
3	—	—	—	—	—	50	0	100	0	37
4	—	—	—	—	—	0	0	20	0	5

We assumed the hierarchical structure of the van Hiele Levels; then, since Item 3a could have been answered at Levels 2, 3, or 4, the fact that it had been answered in the example at Level 3 and Type 4 implies complete acquisition of Level 2 (weight = 100), intermediate acquisition of Level 3 (weight = 50), and no acquisition of Level 4 (weight = 0). The arithmetical averages of the values in the rows gave the student's degrees of acquisition of each van Hiele level: Acquisition of Level 1 was complete (average = 100), of Level 2 was high (average = 76), of Level 3 was low (average = 37), and there was no acquisition of Level 4 (average = 5).

RESULTS

First, we present and comment on some examples of the students' answers in order to illustrate the different possible answers to an item, depending on the level or the type of answer.

The following are three answers to Item 2 illustrating different response types for van Hiele Level 2. For the first student, the similarities between Solid 1 and a cube were "In both solids the faces are parallelograms and both have six faces." And the differences were "The angles in [Solid] 1 are not right." The answer showed reasoning characteristic of Level 2 (it described the parts of the solids) and Type 3 because it was correct but very incomplete. The answer of the second student showed reasoning of Type 4: The student said that Solid 1 and a cube were alike "only because both solids have parallel faces and all the edges are the same [in length]," and they differed "because they don't have the same shape." Some parts of the answer only referred to the solids' shapes (Level 1), whereas sometimes the student paid attention to their components or properties (Level 2); also, when describing the other solids included in this activity, the student mixed arguments of Levels 1 and 2. The third student gave an answer of Type 6 to the same item, responding that Solid 1 and a cube were similar because "both have the same number of faces, the faces are parallel in pairs, and three faces join at each vertex," and they were different because "the faces are not squares but rhombuses and the angles are not right."

The following are some answers to Item 4 showing different kinds of thinking. One student gave an answer that belongs to Level 3 and Type 7: "Wrong. Because a sphere verifies the first part of the statement, but it doesn't have faces parallel in pairs." This response was assigned to Level 3 because the student showed a lack of logical formal reasoning and to Type 7 because, according to the student's thinking level, the answer was correct. Another student answered, "True. As the faces are parallel, any segment going from one face to another will pass through a central point that divides the segment into two equal parts." The student simply repeated the reciprocal statement, so he or she had not attained Level 4, but one cannot infer a lower level of thinking from this answer, so it was assigned to Level 4 and Type 1. And, finally, another student gave an answer assigned to Level 4 and Type 6: "True. If we choose the central point of a face and we draw a perpendicular straight line going through the center of the solid, this line goes just to the center of the opposite side. For this reason they are parallel. If we draw the straight line from a vertex through the center of the solid, it goes to the opposite vertex." This answer was assigned to Level 4 and Type 6 because the proof was quite formal and correct but the student skipped a part of the explanation (about perpendicularity).

Table 4 summarizes the various degrees of acquisition of the van Hiele levels of reasoning attained by the students in the three groups. Looking at this table, it can be seen that complete acquisition of van Hiele Level 2 in spatial reasoning prevailed for the Group A students (most of them having some acquisition of Level 3); complete acquisition of Level 1 prevailed for the Group B students (although most of them had a high acquisition of Level 2); and for most of the Group C students Level 1 had not been highly or completely acquired.

Table 4
Number of Students Attaining Degrees of Acquisition of Each van Hiele Level

		Degree of acquisition				
Group	van Hiele Level	No acquisition	Low	Intermediate	High	Complete
A	1	0	0	0	0	20
A	2	1	0	3	6	10
A	3	2	3	6	6	3
A	4	13	7	0	0	0
B	1	0	0	1	2	18
B	2	0	3	4	13	1
B	3	9	6	5	1	0
B	4	16	5	0	0	0
C	1	0	2	4	2	1
C	2	3	4	2	0	0
C	3	9	0	0	0	0
C	4	9	0	0	0	0

Within each group of students, the higher the level, the lower the degrees of acquisition, which agrees with the hierarchical structure of the van Hiele levels. This result confirms that the test is globally valid; but a complete confirmation of its validity can only be obtained by taking into account the degrees of acquisition of the four levels for each student. An examination of the results obtained shows that for 46 of the 50 students (92%), the degrees of acquisition follow a decreasing order for all four levels (for l = 1, 2, and 3, the degree of acquisition of Level l is greater than the degree of acquisition of level ($l + 1$), or both are equal to 100 or to 0). This accords with the hierarchical structure of the van Hiele model and represents a factor of validation of the test and of the criteria used to measure the acquisition of the levels.

Of the four students whose responses did not fit the hierarchical model, three were in Group A and one in Group B. Figure 3 represents the students' degrees of acquisition of the van Hiele levels. They did not follow a decreasing order, because their degree of acquisition of Level 2 was lower than it should be. The occurrence of results like these should not be surprising, unless they were very frequent (in which case, the validity of the test or the marking criteria should be revised). Previous research has also found students who answer higher level items better than lower level ones (Gutiérrez & Jaime, 1987; Mayberry, 1983; Usiskin, 1982). This kind of result could be related to the teaching method used with the students.

Within our sample, we identified six profiles of students' reasoning that correspond to various stages in the process of a student's intellectual development. The characteristics of the profiles that we have identified are shown in Table 5. The successive profiles represent a sequence ranging from those students who had completely acquired the van Hiele Levels 1, 2, and 3 (Profile 1) to those who were

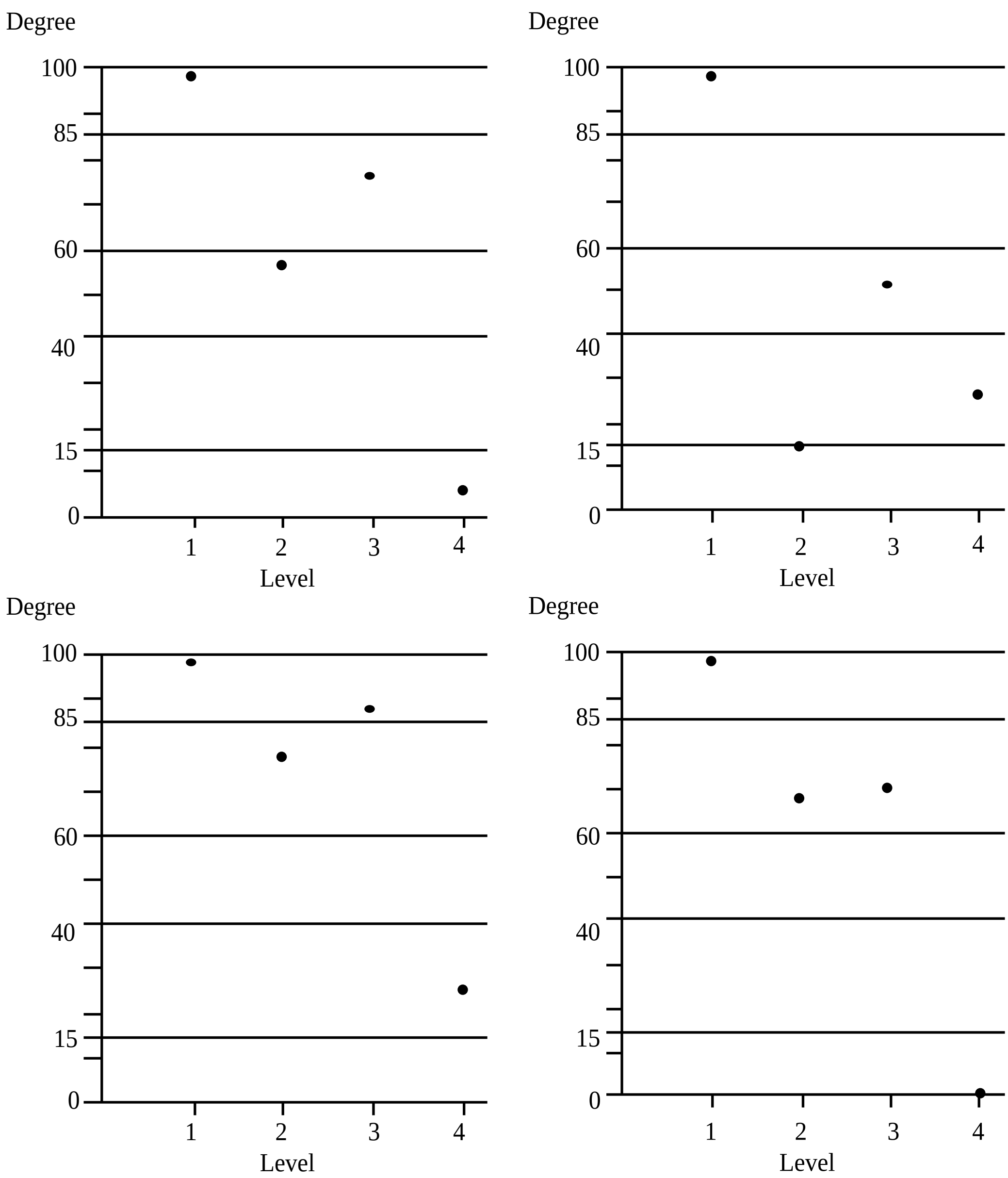

Figure 3. Unusual behavior patterns.

beginning to acquire the first level (Profile 6). Of the 50 students, 42 fit perfectly into the profiles listed, 7 students fit some profile except for one level, and 1 student did not fit appropriately any profile. Table 6 shows the number of students in each group fitting each profile.

Table 5
Characteristics of Observed Profiles of the Acquisition of van Hiele Levels

	Degree of acquisition			
Profile	Level 1	Level 2	Level 3	Level 4
1	Complete	Complete	Complete	≤ Low
2	Complete	Complete	≤ High	≤ Low
3	Complete	High	≤ Intermediate	≤ Low
4	Complete	≤ Intermediate	≤ Low	No Acquisition
5	High or Intermediate	≤ Low	No Acquisition	No Acquisition
6	Low	No Acquisition	No Acquisition	No Acquisition

Table 6
Distribution of Students' Degrees of Acquisition According to the Profiles

	Group		
	A	B	C
Profile 1	2	0	0
Profile 2	9[a]	1	0
Profile 3	6[c]	13[ab]	0
Profile 4	2	4	1
Profile 5	0	3[a]	6[a]
Profile 6	0	0	2
Fit no profile	1	0	0

[a]One of the students did not fulfill the conditions of Level 2.
[b]One of the students did not fulfill the conditions of Level 3.
[c]Two students did not fulfill the conditions of Level 3.

CONCLUSIONS

We have presented a method for the evaluation of the van Hiele level of the students' thinking based on a flexible interpretation of the van Hiele theory, and we have exemplified it by applying it to a test of reasoning in solid geometry. The total number of students to whom the Spatial Geometry Test has been admininstered was large enough to draw conclusions, even though Group C was small. We believe that the results obtained and the differences noted among the different kinds of students indicate that the proposed method of evaluation of the van Hiele levels is coherent and should be studied further.

The way of evaluating the thinking levels explained in this article allows for the possibility that a student can develop two consecutive levels of reasoning at the same time, although what usually happens is that the acquisition of the lower level is more complete than the acquisition of the upper level. In fact, we observed that not all students used a single level of reasoning, but some of them used several levels at the same time, probably depending on the difficulty of the problem. This does not imply a rejection of the hierarchical structure of the levels but rather suggests that we should better adapt the van Hiele theory to the complexity of the human reasoning processes; people do not behave in a simple, linear manner, which the assignment of one single level would lead us to expect.

Another interesting result is the students who showed a better acquisition of Level 3 than of Level 2. It is necessary to go more deeply into the study of this problem in order to determine whether it is caused by faults in the test, limitations in the method of evaluation, or the teaching methods used in the classroom.

Although we have based this article on a written and manipulative test of three-dimensional geometry, the method of determining the degrees of acquisition of the van Hiele levels and the conclusions that can be obtained with regard to the students' reasoning can be applied to any geometric topic and to clinical interviews. Moreover, the method can also be used in all topics where the van Hiele model can be applied.

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