RESEARCH ON GEOMETRICAL THINKING

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The CERME-4-Working Group on Geometrical Thinking was a rather small group of about a dozen persons interested in Geometry and its teaching and learning from primary education to secondary education and teacher training. In all, seven papers were accepted before the conference and served as a basis for intensive discussions.
Continuing the work at the CERME-3-conference (for a report on this work see http://www.dm.unipi.it/~didattica/CERME3/proceedings/Groups/TG7/TG7_introduction_cerme3.html, the Working Group started with the presentation of a special framework for Geometry and its teaching and learning by looking into geometrical paradigms. This framework was already presented at the CERME-3 conference (see the "geometrical approaches in the paper of Kuzniak and Rauscher). This description was meant to give some common ground for the discussions in the group. After this introductory session, the Working Group focussed on primary education. Papers from Italian colleagues gave an excellent opportunity to look deeply into geometrical concepts held by young students (see the papers by Medici et al. and Marchini et al.), but also into the role of specific tools for the teaching and learning of Geometry at that age level (see the paper by Vighi). Naturally, this debate also included Geometry teaching and learning at secondary level, what gradually brought us into issues more linked to the Geometry curriculum for grades 5 and above. Hamiti and Xhevdet presented ideas for the implementation of a new Geometry curriculum in the recent new primary school curriculum in Kosovo, showing the way current theoretical frameworks and approaches for teaching Geometry have influenced the curriculum development and replaced the traditional Geometry curriculum in Kosovo. The paper by Jones et al. gave a description of geometrical reasoning in Chinese and Japanese classroom (mainly from the perspective of teachers), but also trying to be specific about students' geometrical thinking. The analysis of students' reasoning went nicely together with the paper by Markopoulos and Potari. In addition to this, the paper from Greece also opened a window on spatial Geometry by analysing dynamic transformations of solids.
The second part of the paper by Kuzniak and Rauscher rounded off the travel through Geometry and its teaching and learning by analysing problems and potentials of Geometry in in-service teacher training. At the end of the seven sessions, the Working Group even had time for a general closing debate and the preparation of ideas for the report of the group in the final plenary at CERME-4 and this summary. For detailed information on the individual papers the reader is nevertheless directed to these papers.
Looking back on the discussions of the Working Group during the last session, we came up with four major issues: For research, it was obvious that existing

frameworks (like the ones from Piaget or the famous van-Hiele-levels) are helpful to analyse only some aspects of the variety of data in research on geometry teaching and learning. Additional research categories are needed and new local theories are needed to better analyse and understand the data collected in recent empirical studies. At present, we are not in a position to offer a unified theory to completely cover the richness and diversity of the data on the teaching and learning of Geometry. Theoretical innovations visible in the work of Kuzniak and Rauscher as well as Markopoulos and Potari are only indications of this trend, while the paper of Vighi (by looking into a simple artefact like squared paper) reminds us of the importance of the tools (and their use) for the teaching and learning of Geometry.

If one wants **change in teaching Geometry**, for instance because of the necessity of defining an adequate curriculum, one faces a dilemma closely linked to the epistemology of the knowledge to be taught. By its very "nature", Geometry is organised around wide conceptual networks with far-reaching relations inside the area, but also implying links to other mathematical and extra-mathematical areas, especially cultural ones. In contrast to this, school teaching usually oversimplifies such wide networks, particularly in the Geometry lessons. This seems to be at least one reason for the poor learning often occurring in our classrooms. Some of the papers linked to this Working Group can also be read as examples of more open approaches to teaching and learning Geometry.

Implications for the educational **policy** are most obvious, but the Working Group wants to especially mention one issue here: Textbooks are crucial instruments of teaching and learning. According to the research results available now, they are the most important teaching and learning tool even in the age of new technologies like computers and software, especially Dynamic Geometry Software (DGS). Nevertheless, textbooks available at present seldom meet the expectations of the members of the WG - both on choice of content and variety of teaching approaches.

Finally, the Working Group looked into **teacher training**. The participants took for granted that there is an urgent need for training future and practicing teachers with respect to Geometry taught at school. The situation for Geometry seems to be particularly difficult because of the poor knowledge of the teaching force within this mathematical area - and additional training should include both content, i.e. Geometry as a sub-domain of Mathematics, as well as the "Didactics" of Geometry, for instance the theoretical background supporting the organisation of Geometry curricula and specific suggestions and innovations why, how and what to teach in Geometry lessons.