10-18
Geometry
Van Hiele

ANALYZING AND DESCRIBING STUDENTS' THINKING IN GEOMETRY: CONTINUITY IN THE VAN-HIELE LEVELS

Michael Shaughnessy and William Burger--Oregon State University
Angel Gutierrez and Adela Jaime--Universidad de Valencia
David Fuys--Brooklyn College

This Symposium is dedicated to our colleague Bill Burger, researcher, mentor, and friend. We all miss him very much.

While the results of first efforts in van Hiele research generally confirm the validity of the model for describing students reasoning processes in geometry, several unanswered questions have emerged. Is there a way to describe a students' progress through the van Hiele levels as a continuum, so that the model accounts for students who are acquiring more than one level at a given point in their geometric development? Can some combination of clinical and traditional methodologies be used to devise a reliable, yet flexible and valid, test for measuring students' van Hiele levels? This symposium will be a research-workshop on some new approaches to assessing van Hiele levels. Participants will actually become co-researchers with the presenters, investigating these two questions during the symposium.

Overview

The van Hiele model has provided a framework for investigating children's and adolescent's thinking in geomtry (the levels), and also has suggested a pedagogical model for teaching geometric concepts (the phases). Within the past decade, research based on the model indicates that the description of thought processes in geometry is a fertile area for the interaction of psychologists and mathematics educators alike (Usiskin, 1982; Mayberry 1983; Shaughnessy & Burger 1985; Senk 1985, 1989; Burger & Shaughnessy 1986; Crowley 1987, 1990; Fuys et. al. 1988; Wilson, 1990; Gutierrez et. al., in press). Thus, the van Hiele model provides a particularly useful framework to investigate the crossroads between theory and practice in teaching and learning geometry. There is concurrent interest among both teachers and researchers on the potential usefulness of the model for providing both diagnostic information about students' thinking in geometry, and also prescriptive information about how to redesign the geometry curriculum to facilitate students' geometric development.

The first early work on researching the van Hiele levels focused on attempting to identify the existence of these reasoning levels in students, to validate the model, to

describe level indicators of reasoning, and to use the five pedagogical phases in teaching experiments to help move students through the levels of reasoning. In all of this work, tasks were developed to allow students to reason in a geometric environments. Some of these tasks were purely paper and pencil tasks, some were interview tasks of a more open ended nature. The results of these first efforts generally confirmed the validity of the model for describing students reasoning processes in geometry (Usiskin, 1982; Burger & Shaughnessy, 1986; Fuys et. al. 1988). However, several unanswered questions emerged from this first series of research efforts. Among them are two that we wish to address in this symposium.

First, the van Hiele levels do not appear to be entirely discrete. Several of the researchers mentioned above found that students often flip-flopped between levels from one task to another, or even within the same task. Also, many students seem to have a "preferred level of reasoning" on certain tasks. That is, they may prefer to respond in an analytical way when they are perfectly capable of verifying some argument by deduction (second Level preferred over higher levels), or they may respond purely visually when they could just as well have talked about properties of shapes or relationships among those properties had they been required to do so (first level preferred over higher levels). Thus, the process of determining a students' van Hiele level is much more complicated than just assigning a single level on a few tasks. There are also task variables and content knowledge variables, so that students who reason at a level on one task do not necessarily exhibit that same level of reasoning on a subsequent task. This raises the question: Is there a way to describe a students' progress through the van Hiele levels as a continuum, so that the model accounts for students who are acquiring more than one level at a given point in their geometric development? This view presupposes that their are passages between the levels, and that students can reason partially at one level, and partially at another.

A second question that has been researched more recently is the problem of devising a suitable test to assess van Hiele levels, the development of valid, reliable tasks. Both pencil and paper tests and clinical interview tasks have demonstrated certain strengths and weaknesses in van Hiele research. The former may sacrifice detail and/or reliability for convenience and speed. The latter while lending itself well to detailed probing can prove cumbersome and time consuming to administer to large numbers of students. Perhaps some combination of both methodologies is needed to devise a reliable, yet flexible and valid, test for the van Hiele levels.

In this symposium we will focus on describing the continuous development of students' passage through the van Hiele levels. Gutierrez and Jaime (in press) have developed a method of analyzing students' written responses to geometric tasks that includes a first attempt to quantify the passage between levels. After an introductory phase which recaps the attempts of several projects (both in the USA and in Spain) to identify students' van Hiele levels, the participants in this symposium will be put to work in a research-workshop. Participants will be given the opportunity to analyze and discuss students' responses on tasks, both pencil and paper tasks and audio taped tasks, while learning about and using the analysis approach of Gutierrez and Jaime.

After group discussion, the presenters will share their own interpretations of the task results with the participants. The symposium will conclude with suggestions for merging the best parts of both methodolocial approaches—interview and paper and pencil—for researching students' continuous passage through the levels. This approach has recently proved quite valuable in obtaining a more accurate picture of a students' van Hiele levels.

Conduct of the Symposium

The symposium will evolve in three phases.

Phase 1.

Mike Shaughnessy and Bill Burger

Shaughnessy and Burger have planned the introduction and overview for the symposium. Prototype instances where students are between van Hiele levels on certain tasks will be presented. Sample student responses to particular tasks will be used to introduce the dilemma one faces when a student appears to be between levels. Responses to tasks presented in taped interviews (developed in the USA) and pencil and paper tasks (developed in Spain) will be considered. The interview tasks are similar to the pencil and paper tasks, but not all are identical. Some general comparisons of the two different methodologies for determining van Hiele levels will be mentioned. (Time: about 30 minutes)

Angel Gutierrez and Adela Jaime

Gutierrez and Jaime will provide a brief background on their research, and desribe their scheme of "levels and types" for quantifying the passage of students between van Hiele levels. In this scheme, a students' response to a particular task is assigned both a van Hiele level and a "type" of answer. The types reflect both the strength and clarity of the predominant van Hiele level on that task, and also the mathematical completeness and accuracy of the response. The types are quantified to indicate how complete a student's aquisition of a level is. This scheme will also makes allowances for task variables and the potential range of thinking levels that may be used to answer a particular question. Each task can be pre assigned a potential "range of levels" of response. Thus, a question could be answered at, say, van Hiele levels 2,3, or 4 (using 1-5 numbering), and if a student answers it at level 3, a complete aquisition of level 2 is assumed, a partial aquistion of level 3 (depending on the "type" of answer) is assumed, and no aquisition of level 4 is inferred on that task. Using the "types"it becomes possible to quantify a students responses and to talk about a student's "degree of aquisition" of each of the four (1-4) van Hiele levels.

Gutierrez and Jaime will give specific examples of students' responses that they have coded by levels and types, and the corresponding degree of acquisition of the levels, in order to provide the necessary information for the second phase of the symposium in which the participants themselves will "do" some van Hiele research using this scheme. (Time: about 30 minutes)

Phase 2.

Participants will be given taped responses of a student's work on a geometric task(s). The participants will be asked to evaluate the "type and level" of the student on each task. The participants will work in pairs on this activity, first noting their own responses, and then interacting with their partner.
(Time: about 30 minutes)

In the second part of phase 2, the participants will share the results of their pair-wise analysis with the large group. The symposium organizers will also share their own analyses of the same task(s). (Time: about 30 minutes)

Phase 3

David Fuys

David Fuys will play the role of reactor. His remarks will be partly devoted to methodological considerations, with special attention to a "marriage" of certain aspects of the interview and paper & pencil methodologies. When post hoc structured interviews are administered to students after they have answered paper and pencil van Hiele tasks, the in depth probing allowed in the interview format may help to clarify a students' true acquisition of the van Hiele levels. Fuys will discuss examples of students where this was indeed the case. He will also reflect on the process of attempting to quantify the passage between van Hiele levels, and in particular, the process in which the participants have engaged during phase 2.
(Time: about 30 minutes)

The final part of the symposium will be devoted to open discussion about the process of researching van Hiele levels, focusing on the method of quantifying a student's passage between and through the continuum of levels. (Time: abut 30 minutes)

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