GERMAN NATURAL GAS SEASONAL EFFECTS ON FUTURES HEDGING

Vanesa García Seligrat

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Tutor: Dr. Hipòlit Torró

Universidad Complutense de Madrid
Universidad del País Vasco
Universidad de Valencia
Universidad de Castilla-La Mancha

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QFB, University of Castilla –La Mancha, Spain

Tutor: Dr. Hipòlit Torró
University of Valencia
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Abstract

In a context of strong increases in volatility in energy markets it is useful to design hedging instruments to manage the risks of such increases. In natural gas markets, as well as other energy markets, changes in the spot price are partially predictable due to the existing seasonalities in weather, demand and storage levels. We find a strong seasonal pattern in spot price returns and, volatility, been winter volatility significantly higher than summer volatility. We propose to follow Martínez and Torró (2015) extending the study developed there to the Germans GASPOOL and NCG natural gas markets.

We follow the approach of Ederington and Salas (2008). The minimum variance hedge is based on the predictive power of the base (futures price minus spot price) explaining unexpected changes in spot prices. When considering the partial predictability of changes in spot prices there is considerably improvement in attained risk reductions as Ederington and Salas (2008) obtained. We find that long hedges achieve greater hedging performance than short hedges and there is not benefit to be gained by the use of more complex hedging estimation (BEKK) over the simpler OLS model.

Keywords: Hedging effectiveness, achievable risk reduction, naïve strategy, OLS strategy, BEKK.
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1. Introduction

The European Union has the purpose to achieve natural gas liberalization. For the last years, the EU has been trying to implement different codes with the object to achieve a market integration and an effective competition. Heather (2015) analyzes the development of the gas hub and observes that “the process of transformation towards liberalized gas markets is not progressing at the same rate across Europe and that there is still a lot to be done, especially in Eastern Europe”. Thus, Heather (2015) establishes that “Europe is not one homogenous gas market, neither in terms of infrastructure nor in political desire to change, and that even within each area there can be many levels of development” and the most developed part of Europe in terms of liberalized gas hubs is the North-West.

In the past few years, the number of liquid markets have increased, currently the most liquid market in Europe is NBP\(^1\) followed by TTF\(^2\), ZEE\(^3\), NCG\(^4\) and GPL\(^5\). Heather (2012) establishes “For a hub to develop to become a price reference it needs to have amongst other attributes, depth, liquidity and transparency and to be able to readily attract a significant number of market participants”.

Nowadays, Germany has two market areas and two hubs which started trading in 2009 and which is the object of this work namely NetConnect Germany (NCG) and Gaspool (GPL). Heather (2012) examines the purpose of German Market unite into one Market Area and he says: “If the German market cannot unite into one Market Area, this could be a major stumbling block preventing a German hub from developing further”.

Among the motivations of this work, it should be included the possibility of conducting a similar study for Spain. On 18 September 2014, the Spanish Council of Minister submitted to the Parliament a draft bill in order to implement a Gas Hub and since then, there have been taken different measures in order to fulfil the requirements of the European Network Code.

---

\(^1\) Britain’s National Balancing Point Hub.
\(^2\) Dutch Title Transfer Facility Hub.
\(^3\) The Belgian Zeebrugge Hub.
\(^4\) The German NetConnect Germany Hub.
\(^5\) The German Gaspool Balancing Services Hub.
Few years ago, there was not an organized market in the Iberian Peninsula, the traded gas were negotiated in bilateral transaction on the Over – The –Counter Market (OTC). Most of the transactions consisted of deliveries of the virtual balancing point (the so-called AOC). The volume of traded gas on the OTC market have to be communicated to the system MS –ATR platform, ENEGAS, that allows us to know the volume and the number of operations of purchase and sale but not the price.

The law 8/2015 dated 21 May, establishes the creation of an organized market and creates MIBGAS S.A. the company that will operate the new gas market. The Royal Decree 984/2015 dated 30 October, regulates the gas organized market with standardized products and third party access to the natural gas system facilities. The resolution of 4 December 2015 published in the BOE establishes the rules of the market, the adhesion contract and the resolutions of the organized gas market.

The negotiations in MIBGAS began on 16 December 2015 and the regulated contracts are: Within-Day, Day-Ahead, Balance of Month and Month-Ahead. Finally, October 1st, 2016 is the date that will really represent the real launch of the Spanish gas hub.

The main object of this work is to analyze German natural gas seasonal effects on futures hedging and to compare different hedging strategies in order to choose the optimal hedging strategy. In a context of strong increases in volatility in energy markets it is useful to design hedging instruments to manage the risks of such increases. In this work we find out a strong seasonal pattern in volatility of basis, spot and futures returns, being winter volatility significantly higher than summer volatility. A relevant fact in the natural gas market is that changes in the spot are partially predictable due to the seasonal pattern, demand and storage levels. In this work, following Ederington and Salas (2008), we use the base (futures minus spot prices) as explicative variable of the unexpected changes in the spot prices. We find that the basis has predictive power for explaining unexpected spot price changes. Nevertheless, the basis has less ability to forecast futures price changes.

Along the work, we extend the study developed by Martínez and Torró (2015) to the Germans GPL and NCG gas markets. We use four strategies: naïve, ‘OLS without basis’, ‘OLS with basis’ and BEEK; then we compare the risk reduction achieved by

---

6 OLS: Ordinary Least Squares.
each hedging strategy and we detect a positive duration effect in hedging effectiveness. Furthermore, we analyze the attained risk reduction with the standard and Ederington and Salas (2008) approaches. We find that the standard approach tends to underestimate the risk reduction and overestimate the riskiness of unhedged position when changes in the spot are partially predictable. Finally, we can affirm that our results are similar to those obtained by Martínez and Torró (2015).

This work is organized as follows. In the next section, we review the existing literature. Section three and four describe the estimation methodology and the data, respectively. Section five presents a preliminary analysis for the full sample. Section six presents the empirical results. Finally, section seven contains a brief conclusion.

2. Literature review.

The literature modelling time-varying volatility is abundant since the development of the ARCH\textsuperscript{7} by Engle (1982) and the GARCH\textsuperscript{8} by Bollerslev (1986). Bekaert and Wu (2000) in their paper entitled “Asymmetric volatility and Risk in Equity Market” introduce asymmetries in the GARCH and conclude that “negative shocks increase conditional covariances substantially, whereas positive shocks have a mixed impact on conditional covariances”. Other authors who modelling time-varying volatility are Efimova and Selertis (2014) and Hendry and Sharma (1999).

The literature modelling conditional covariance is less extensive. Bollerslev et al.(1988) proposed the VECH model, in their paper entitled “A Capital Asset Pricing Model with Time-varying Covariances” the univariate GARCH is extended to the vectorized conditional -variance matrix. Bollerslev (1990) propose a multivariate time series with time varying conditional variances and covariances but constant conditional correlation, that is to say, the constant correlation model (CCORR). Engle and Kroner (1995) propose a class of MGARCH (the BEKK), it practically ensures that $H_t$ will be positive definite. These three models are most used to model conditional variance.

\textsuperscript{7} ARCH: Autoregressive Conditional Heteroscedastic (mean zero, serially uncorrelated processes with nonconstant variances conditional on the past, but constant unconditional variances).

\textsuperscript{8} GARCH: Generalization of the ARCH (stationarity conditions and autocorrelation are derived).
Yang (2001) in his paper entitled “M-GARCH Hedge Ratios and Hedging Effectiveness in Australian Futures Markets” analyzes the traditional model, the VAR model, the VECM and the DVEC multivariate GARCH and obtains that the GARCH time varying hedge ratios provide the greatest portfolio risk reduction.

Bonga-Bonga and Umoetok (2015) estimate the hedging ratio in an emerging equity market and they conclude that for daily hedging periods the most effective method is the traditional model estimated by OLS; however, for weekly and monthly hedges, the VECM and multivariate GARCH models are more effective.

Cotter and Hanly (2008) in their paper entitled “Hedging Effectiveness under Conditions of Asymmetry” obtain that the OLS model produce the best hedging effectiveness and suggest that “there is little economic benefit to be gained by the use of more complex hedging estimation models over the simple OLS model irrespective of the characteristics of the return distribution”.

A relevant fact in gas prices is the presence of jumps, the natural gas prices are being affected by economic, politic, geopolitical factors and weather conditions. Mu (2004) estimates a GARCH model from the U.S. natural gas market and he uses the deviation of temperature (weather surprise) as a proxy for the demand shocks and a determinant of the conditional volatility of natural gas futures returns. He concludes that this proxy has a significant effect on the conditional volatility of natural gas prices.

Nick and Thoenes (2014) show that “gas natural price is affected by temperature, storage and supply shortfalls in the short term, whereas the long-term development is closely tied to both crude oil and coal prices”.

Henaff et al. (2013) argue a seasonal pattern and the existence of spikes, “the demand for natural gas heating in cold periods of the year produces a seasonal behavior for prices during winter periods, while unpredictable changes in weather can cause sudden shifts in gas prices”. They also affirm that “most of the positive spikes happen during the winter months of January and February and the summer month of June, which can be explained by the occurrence of an unpredicted cold front or heat wave”. They in their futures model incorporate seasonality in the futures curve and in their spot model describe the existence of spikes.
Efimova and Selertis (2014) affirm that “the natural gas market is influenced in a much larger extent by fundamental factors, such as predictable fluctuations in demand driven by weather variables, storage and transportation conditions, and seasonal production and consumption patterns”. They introduce seasonal dummy variable in the mean equation and obtain distinct fluctuations in the natural gas price, the price is higher in winter and summer than in the transition seasons.

The naïve strategy involves taking futures market positions equal in magnitude but of opposite sign to their position in spot market. Ederington (1979) use the standard minimum variance hedge ratio to analyze the GNMA and T- Bill futures markets as instrument for such hedging. He concludes that “appear to be more effective in reducing the price change risk over long (four – week) than over short (two – week) periods”.

Fama and French (1987) propose two views of the basis: firstly, the Theory of Storage; and secondly, in the alternative view propose express the basis as the sum of an expected premium and an expected change in the spot price. They obtain that in eight out of the regressions the basis $F(t,T) – S(t)$ has reliable information about the future change in the spot price $S(T) – S(t)$ for most maturities $(T-t)$.

Ederington and Salas (2008) extend the Ederington (1979) approach to the case when the changes in the spot are partially predictable. They propose to use the basis (futures-spot spread) as the information variable to approximate the expected spot price change. They conclude that “incorporating measures of the expected change in the spot price, into the regression results in substantially lower estimates of the riskiness of hedged and unhedged positions, and substantially higher estimates of the risk reduction achievable through hedging”. Therefore, when spot price returns can be partially forecasted his approach produces the more efficient estimates.

Martínez and Torró (2015) apply this new approach to European gas markets and establish that it enables a significant improvement for hedging strategies and obtain that unexpected shocks in spot prices can be partially anticipated using the information contained in the basis (between 10%-30%). They also conclude that hedging effectiveness improves as increase the duration of the hedge. They find a strong seasonal pattern in the volatility of spot and futures price returns, which have been significantly higher in winter than in summer. Martínez and Torró (2015) also use the
BEKK with the asymmetric extension, the seasonal covariance model and the seasonal-basis covariance model but their results show that the better statistical performance of the GARCH does not imply a better hedging strategy performance.

3. Methodology

1.1 Traditional model

In our analysis, we define the conventional minimum variance hedge ratio in a one-period model and an economic agent who is committed to a given a position in the spot market at the beginning of the period or t. To reduce the risk exposure, the agent may choose to hedge at time t in the futures market with the same underlying asset.

We denote $\Delta S$ and $\Delta F$ as spot and futures variations respectively which formulas are presented in table 1 following Martínez and Torró (2015). At t+1, that is, at the end of the period, the result of the hedger is calculated as follows:

$$x_{t+1} = \Delta S(t) - b_t \Delta F(t, T)$$  \hspace{1cm} (1)

Where $b_t$ is the hedging ratio which indicates the positions to be taken in futures. If $b_t$ is positive (negative), short (long) positions are taken in futures. The hedger will choose $b_t$ to minimize the risk associated with the random result $x_{t+1}$. As Martínez and Torró (2015) do, we are also agree with Alexander et al. (2013) methodology who argue “…for assets with prices that can jump, log returns can be highly inaccurate proxies for percentage returns even when measured at the daily frequency.” This is the reason why we use realized returns instead log returns and our hedging analysis is based on profit and loss (P&L).

We follow the procedure of Martínez and Torró (2015) and we use the variance conditional on the available information to compute the risk of a hedge strategy:

$$Var[x_{t+1}|\psi_t] = Var[\Delta S(t) - b_t \Delta F(t, T)|\psi_t]$$  \hspace{1cm} (2)
The optimal hedge ratio has been defined as the amount of futures position per unit spot position such that the hedged portfolio variance is minimized and it can be obtained by minimizing eq. (2).

\[ b_t = \frac{\text{cov}(\Delta S(t), \Delta F(t, T)|\psi_t)}{\text{var}(\Delta F(t, T)|\psi_t)} \]  

(3)

In this equation second moments are conditioned to the information set available at the beginning of the hedging period, \( \psi_t \). The equation 3 can be estimated from a linear relationship between spot and futures returns if we use an unconditional probability distribution. Thus is, estimating the linear relationship appearing in eq. (1) by ordinary least squares (OLS henceforth) but adding an intercept and white noise:

\[ \Delta S(t) = \alpha + b\Delta F(t, T) + \epsilon(t) \]  

(4)

Where \( b_t \) is the unconditional definition of the optimal hedge ratio of the eq. (3) (Ederington, 1979) estimated with OLS.

### 1.2 Alternative model

In the framework of Ederington and Salas (2008) this approach has been adapted to the case where spot price changes are partially predictable and futures prices are unbiased estimators of future spot prices. In this context, Ederington and Salas (2008) show that the riskiness of the spot position is overestimated and the achievable risk reduction is underestimated. If we consider this approach, the unexpected result of the hedge in eq.(1) can be reformulated as:

\[ x_{t+1} = (\Delta S(t) - E[\Delta S(t)|\psi_t]) - b'_t \Delta F(t, T) \]  

(5)

We reformulate the risk of the hedge strategy in Eq. (2) as:

\[ \text{Var}[x_{t+1}|\psi_t] = \text{Var}[(\Delta S(t) - E[\Delta S(t)|\psi_t]) - b'_t \Delta F(t, T)|\psi_t] \]  

(6)
and the minimum variance hedge ratio are obtained after minimizing eq. (6) is:

\[ b'_t = \frac{\text{cov}((\Delta S(t) - E[\Delta S(t)|\psi_t]), \Delta F(t,T)|\psi_t)}{\text{var}(\Delta F(t,T)|\psi_t)} \tag{7} \]

Changes in the spot price are partially predictable in many markets, Fama and French (1987) have shown that \( F_t - S_t \) has predictive ability for futures changes in spot price. Ederington and Salas (2008) following Fama and French (1987) approach propose futures price minus spot price namely basis at the beginning of the hedge as the variable information to approximate the expected spot price change. They propose the alternative estimation:

\[ \Delta S_{t+s} = \alpha' + \beta' \Delta F_{t+s} + \lambda Z_t + \epsilon'_{t+s} \tag{8} \]

“If \( \hat{\lambda}Z_t = E_t(\Delta S_{t+s}) \), the variance of the residuals from this equation (eq.8) provides an unbiased estimate of variance of a position hedged using hedge ratio \( \hat{\beta}' \).

We obtain an unconditional estimate of the hedge ratio in eq. (7) by estimating the following liner regression using OLS which is equivalent that proposed by Ederington and Salas (2008) where \( Z_t = (F_t - S_t) \).

\[ \Delta S(t) = \alpha' + b' \Delta F(t,T) + \lambda (F(t,T) - S(t)) + \epsilon(t) \tag{9} \]

Where \( \lambda(F(t,T) - S(t)) \) is used to estimate \( E[\Delta S(t)|\psi_t] \). So the expected change in the spot is perfectly approximated with the product between the basis at the begging of the hedge and its estimated coefficient \( \hat{\lambda}(F(t,T) - S(t)) = E[\Delta S(t)|\psi_t] \).

### 1.3 GARCH model: BEKK

The asymmetric version of the BEKK is estimated to obtain conditional estimates of the second moments. The input to the BEKK are the residuals obtained by the estimating a model in means. Firstly, we propose the same vector error correction as Martinez and Torro (2015), introducing the lagged value of the basis (can be seen as an error
correction term when spot and futures prices are co-integrated, see Lien, 1996) in the model:

\[ \Delta^k S(t) = \gamma_1 + \gamma_{10} (F(t,T_k) - S(t)) + \sum_{\tau=1}^{p} \gamma_{11\tau} \Delta^k S(t-\tau) + \sum_{\tau=1}^{p} \gamma_{12\tau} \Delta^k F(t-\tau,T_i) + \epsilon_{1,t+k} \]  

(10)

\[ \Delta^k F(t,T_i) = \gamma_2 + \gamma_{20} (F(t,T_k) - S(t)) + \sum_{\tau=1}^{p} \gamma_{21\tau} \Delta^k S(t-\tau) + \sum_{\tau=1}^{p} \gamma_{22\tau} \Delta^k F(t-\tau,T_i) + \epsilon_{2,t+k} \]

Where \( \Delta S \) and \( \Delta F \) are computed as table 1 shows; the parameters to estimate are the gammas; \( p \) is the lag of the VAR and is chosen by minimizing the Hannan and Queen (1979) information criteria with the object to eliminate any pattern of autocorrelation. The vector of residuals \( \epsilon_{t+k} \) obtained by estimating the VAR by OLS\(^9\) is saved and used as observable data to estimate the multivariate GARCH. The number of parameters to estimate in the second part is reduced with this procedure\(^{10}\), also decreases the estimation error, and enables a faster convergence in the estimation procedure.

Engle and Kroner (1995) proposed a class of MGARCH model called the BEKK, we estimate this model introducing asymmetries following Glosten et al. (1993) approach. The two – dimensional asymmetric BEKK model can be written in its compacted form as:

\[ H_t = C'C + B'H_{t-1}B + A'\epsilon_{t-1}\epsilon_{t-1}' + G'\eta_{t-1}\eta_{t-1}' + G \]  

(11)

Where \( C, A, B \) are 2x2 matrices of parameters; \( H_t \) is the 2x2 conditional covariance matrix, the diagonal elements of \( H_t \) are variance terms and elements outside the

---

\(^9\) Engle and Granger (1987): “least squares standard error will be consistent estimates of the true standard errors”.

\(^{10}\) Engle and Ng (1993) estimate the unexpected return at time \( t \) (\( \epsilon_t \)) that is treated as a collective measure of news at time \( t \), then use these unexpected return as a observable data in the ARCH and GARCH models. Kroner and Ng (1998) follow a similar procedure, they estimate four multivariate GARCH models (BEKK; FARCH; VECH and CCORR) in two steps, first they estimate the mean equation to get the residuals and in the second step they estimate the conditional covariance matrix parameters using maximum likelihood.
diagonal are covariances; and \( \varepsilon_t \) and \( \eta_t \) are 2x1 vectors containing the shocks and threshold terms series. The unfolded covariance model is written as follows:

\[
\begin{bmatrix}
    h_{11t} & h_{12t} \\
    h_{12t} & h_{22t}
\end{bmatrix} =
\begin{bmatrix}
    c_{11} & c_{12} \\
    0 & c_{22}
\end{bmatrix}
\begin{bmatrix}
    h_{11t-1} & h_{12t-1} \\
    h_{12t-1} & h_{22t-1}
\end{bmatrix}
\begin{bmatrix}
    b_{11} & b_{12} \\
    b_{21} & b_{22}
\end{bmatrix}
+ \begin{bmatrix}
    a_{11} & a_{12} \\
    a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
    \varepsilon_{1t-1}^2 & \varepsilon_{1t-1}\varepsilon_{2t-1} \\
    \varepsilon_{2t-1} \varepsilon_{1t-1} & \varepsilon_{2t-1}^2
\end{bmatrix}
+ \begin{bmatrix}
    g_{11} & g_{12} \\
    g_{21} & g_{22}
\end{bmatrix}
\begin{bmatrix}
    \eta_{1t-1}^2 & \eta_{1t-1}\eta_{2t-1} \\
    \eta_{2t-1} \eta_{1t-1} & \eta_{2t-1}^2
\end{bmatrix}
\begin{bmatrix}
    a_{11} & a_{12} \\
    a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
    \varepsilon_{1t}^2 & \varepsilon_{1t}\varepsilon_{2t} \\
    \varepsilon_{2t} \varepsilon_{1t} & \varepsilon_{2t}^2
\end{bmatrix}
\begin{bmatrix}
    a_{11} & a_{12} \\
    a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
    \eta_{1t}^2 & \eta_{1t}\eta_{2t} \\
    \eta_{2t} \eta_{1t} & \eta_{2t}^2
\end{bmatrix}
\]

(12)

Where \( h_{ij} \) for all \( i,j \in \{1,2\} \) are the conditional second moment series; \( c_{ij}, b_{ij}, a_{ij} \) and \( g_{ij} \) for all \( i,j \in \{1,2\} \) are parameters; \( \varepsilon_{1t} \) and \( \varepsilon_{2t} \) are the unexpected shock series obtained from eq.(10); and \( \eta_{1t} = \max(0,-\varepsilon_{1t}) \) and \( \eta_{2t} = \max(0,-\varepsilon_{2t}) \) are the Glosten et al (1993) dummy series capturing negative asymmetries from the shocks. The parameters of the BEKK model are estimated by maximizing the conditional log-likelihood function:

\[
L(\theta) = -\frac{TN}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T} \left( \ln|H_t(\theta)| + \varepsilon_t^2 H_t^{-1}(\theta) \varepsilon_t \right)
\]

(13)

where \( \theta \) denotes the vector of all the parameters to be estimated; \( T \) is the number of observations; and \( N \) is the number of equations in the system. The log-likelihood function is estimated using the BFGS algorithm via quasi-maximum likelihood estimation (QMLE).

### 1.4 Hedging effectiveness

The hedging effectiveness is computed following Ederington (1979) approach, that is, the hedging effectiveness is measured by the percentage reduction in variance of the hedged position relative to the unhedged position (\( b=0 \)):

\[
\frac{Var(U) - Var(H)}{Var(U)} = 1 - \frac{Var(H)}{Var(U)}
\]

(14)
Where, \( \text{Var}(U) \) is the variance of unhedged position and \( \text{Var}(H) \) is the variance of hedged position.

The variance of unhedged position is computed following the standard and Ederington and Salas (2008) approaches respectively:

\[
\text{Var}[\Delta^k S(t)]
\]

\[
\text{Var}[\Delta^k S(t) - \hat{\lambda}(F(t, T_k) - S(t))]
\]

In section 6 we compare four hedging strategies labeled:

- Naïve strategy: a hedge where futures positions have the opposite sign to the position held in the spot market but the same size (\( b_t = 1 \) for all \( t \)).
- OLS without basis: obtained after estimating equation (4).
- OLS with basis: obtained after estimating equation (9).
- BEKK: obtained after estimating the BEKK model equations (10) to (13).

The variance of hedged positions is computed with the same procedure as we use to compute the variance of the unhedged positions, that is to say, with the standard and Ederington and Salas (2008) approaches respectively:

\[
\text{Var}[\Delta^k S(t) - \hat{b}_t \Delta^k F(t, T_i)]
\]

\[
\text{Var}[\Delta^k S(t) - \hat{b}_t \Delta^k F(t, T_i) - \hat{\lambda}(F(t, T_k) - S(t))]
\]

Also, in a preliminary study in section 5, we analyze if the adjusted \( R^2 \) is a good measure of the achievable risk reduction. We compare the adjusted \( R^2 \) with the hedging effectiveness computed as described.

4. Data

We obtained natural gas spot and futures prices from EEX and ICE with observations at daily intervals. We use data from October 2007 to June 2015. In the case of weekly frequency data, the prices are taken on Wednesday or the previous trading day if not
tradable, and rollovers are taken the last Wednesday of the month or second to last Wednesday if the last trading day of the month is Wednesday.

As we have said in section 3, we use realized returns computed as shown in table 1. In the last column are reported the basis (spread) used to approximate the expected spot price change.

Table 1: Types of hedges

This table displays the type of hedges; basis approximating; and $\Delta^k S(t)$ and $\Delta^k F(t,T)$ (spot and futures returns, respectively). Where $k = w, m$ indicates one week and one month, respectively.

<table>
<thead>
<tr>
<th>Hedging period</th>
<th>Data Frequency</th>
<th>Spot return</th>
<th>Futures return</th>
<th>Basis approximating $E[\Delta^k S(t)\mid \psi]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 week</td>
<td>Weekly</td>
<td>$\Delta^w S(t) = S(t+1\text{week}) - S(t)$</td>
<td>$\Delta^w F(t,T) = F(t+1\text{week},T) - F(t,T)$</td>
<td>$F(t,T) - S(t)$</td>
</tr>
<tr>
<td>1 month</td>
<td>Monthly</td>
<td>$\Delta^m S(t) = S(t+1\text{month}) - S(t)$</td>
<td>$\Delta^m F(t,T) = F(t+1\text{month},T) - F(t,T)$</td>
<td>$F(t,T) - S(t)$</td>
</tr>
</tbody>
</table>

The time series for the spot and futures with maturity one month are exhibits in figure 1. Spot and futures present a seasonal pattern with prices higher in winter and lower in summer. The highest prices in winter (at the end of 2008, January 2009, February 2012 and March 2013) can be explained by the dispute about gas prices and transit between Russia and Ukraine combined with the cold winter, higher storage costs and demand’s increases.

Figure 1: Spot and futures natural gas prices

Spot price (-----) and the first to delivery futures prices (—— ). a)GPL spot and first to delivery futures contract. b) NCG spot and first to delivery futures contract.
A relevant fact is that in June 2014 Russia halted its natural gas supplies to Ukraine; nevertheless the prices didn’t increase due to the warm summer and the sufficient storage levels.

Annex I displays the basic statistics of spot and futures prices differences. In most cases, as we can see, spot mean values are positives but not significantly different from zero; nevertheless, the spot mean values are negative but not significantly for monthly returns from October 2007 to June 2015 in the NCG. Futures mean values are negatives and significantly\(^{11}\) different from zero in all cases. Futures mean values varying between -0.13 for weekly returns and -0.577 for monthly returns, it could be said that the futures market is in contango. We contrast the null hypothesis of median equality between futures and spot time series with the Kruskal–Wallis test. The null hypothesis is not rejected in all cases except for weekly spot and futures returns in both Hubs. Also we contrast the null of variance equality, the Levene test is rejected in all cases except for monthly spot and futures returns in both Hubs. The standard deviation is always higher in spot returns.

All the time series analyzed in Annex I have significant excess of Kurtosis and one out of the total time series analyzed has significant excess of Skewness. The Jarque-Bera test is used to contrast the null hypothesis of normality distribution, we reject this null in

\(^{11}\) At 5% of significance level. Henceforth, we do the contrasts at 5% significance level.
all cases. Thus, we reject the normality distribution hypothesis in all the time series analyzed. The Ljung–Box test with twenty lags detects significant autocorrelation in four out of the total time series analyzed: weekly spot and futures returns in both Hubs. Also it was detected a significant heteroscedasticity in five out of the total time series analyzed.

Annex II displays the correlation between spot and futures returns. In the case of NCG, the highest correlation is obtained by spot and futures for weekly returns and the lowest by spot and futures for monthly returns. However, the highest correlation in the GPL is obtained by spot and futures for monthly returns, and the lowest by spot and futures for weekly returns.

We regress $\Delta S$ and $\Delta F$ on basis approximating respectively to analyze the Ederington and Salas (2008) approach. The results for weekly and monthly periods are presented in tables 2 and 3, respectively. The basis has predictive power for explaining unexpected spot price changes that improves as hedging length increases (between 3.56% and 10.90%). As tables 2 and 3 show the basis has less ability to forecast futures price changes. Ederington and Salas (2008) get the same results and argue that “futures changes in the spot price, $S$, are more predictable that changes in the futures price, $F$”. These results also coincide with the results of Martínez and Torró (2015) who associate the futures prices results with the martingale hypothesis.

Table 2: Basis as a predictor of the change in spot and futures prices (weekly data).

This table displays the results of the regression between spot and futures changes appearing in the second column on the basis defined in the third column for weekly data. The t-statistic computed with Newey- West standard error are reported between bracketed [ ]. The numbers between parenthesis (.) are p-values.

<table>
<thead>
<tr>
<th></th>
<th>Dependent variable</th>
<th>Basis</th>
<th>Intercept</th>
<th>Basis coefficient</th>
<th>Adjusted $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NCG</strong></td>
<td>$\Delta S(t)$</td>
<td>$F(t,T_1)-S(t)$</td>
<td>-0.079027 [0,2210]</td>
<td>0.209284 [2,614545]</td>
<td>0.051015</td>
</tr>
<tr>
<td></td>
<td>$\Delta F(t,T_1)$</td>
<td>$F(t,T_1)-S(t)$</td>
<td>-0.092447 [-1.870274]</td>
<td>-0.126107 [-3.296245]</td>
<td>0.035603</td>
</tr>
<tr>
<td><strong>GPL</strong></td>
<td>$\Delta S(t)$</td>
<td>$F(t,T_1)-S(t)$</td>
<td>-0.053053 [0.4181]</td>
<td>0.196609 [2.405027]</td>
<td>0.040996</td>
</tr>
<tr>
<td></td>
<td>$\Delta F(t,T_1)$</td>
<td>$F(t,T_1)-S(t)$</td>
<td>-0.085803 [-1.870302]</td>
<td>-0.146479 [-3.916151]</td>
<td>0.049877</td>
</tr>
</tbody>
</table>
Table 3: Basis as a predictor of the change in spot and futures prices (monthly data).

This table displays the results of the regression between spot and futures changes appearing in the second column on the basis defined in the third column for monthly data. The t-statistic computed with Newey- West standard error are reported bracketed [ ]. The numbers between parenthesis (.) are p-values.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Basis</th>
<th>Intercept</th>
<th>Basis coefficient</th>
<th>Adjusted $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta S(t)$</td>
<td>$F(t,T_1)-S(t)$</td>
<td>-0.487489 [—1.512640] (0.1340)</td>
<td>0.455368 [3.330335] (0.0013)</td>
<td>0.109085</td>
</tr>
<tr>
<td>$\Delta F(t,T_1)$</td>
<td>$F(t,T_1)-S(t)$</td>
<td>-0.316021 [-1.248565] (0.2151)</td>
<td>-0.201346 [-1.740112] (0.0853)</td>
<td>0.030636</td>
</tr>
<tr>
<td>$\Delta S(t)$</td>
<td>$F(t,T_1)-S(t)$</td>
<td>-0.354093 [-1.350912] (0.1802)</td>
<td>0.448036 [4.371599] (0.0000)</td>
<td>0.098930</td>
</tr>
<tr>
<td>$\Delta F(t,T_1)$</td>
<td>$F(t,T_1)-S(t)$</td>
<td>-0.347880 [-1.445215] (0.1519)</td>
<td>-0.186964 [-1.785938] (0.0776)</td>
<td>0.018451</td>
</tr>
</tbody>
</table>

Figure 2 shows seasonal basis, we can see a seasonal pattern being the basis positive in winter and negative in summer. It stands out the cold winter of 2008-2009 in which the bases are superior to 3 and the beginning of the summer 2013 with basis less than -2. There are several authors who explain this pattern; we, as Martínez and Torró (2015), explain the positive bases in winter for the cold weather, so the demand increases, the storage levels decreases and storage cost increases. In summer, the opposite situation takes place producing a negative basis, nevertheless, in some cases the basis is positive, which may be due to the increasing number of cooling systems.

Figure 2: Seasonal basis

This figure shows seasonal basis. 13-week moving average basis approximating at NCG (——) and GPL (--------). The vertical lines separate winter (October to March) and summer (April to September) seasons.
Figure 3 displays spot and futures prices volatility for NCG and GPL respectively, the seasonal pattern is similar to that of the basis, winter volatility is higher than summer volatility. The peaks in the volatility figures correspond to large declines in the market, that is to say, the peaks correspond with the dispute between Russia and Ukraine, the
Libyan war in 2011, the withheld Russian exports in 2012. The peak in 2013 can be due to the cold winter of 2013.

We divided the year in two seasons for weekly spot and futures returns and basis approximating in order to analyze the seasonal effects (see Annex III). The two seasons, as in the EEX and ICE markets are: summer from April to September and winter from October to March. The null hypothesis of median equality of the Kruskal - Wallis test is not rejected in all cases for spot and futures returns and for basis approximating. The Levene test contrasts the null hypothesis of variance equality between summer and winter and is rejected in all cases. The winter volatility is significantly higher than summer volatility for spot and futures returns and basis approximating, therefore we can affirm that there is a strong seasonal pattern.

5. Preliminary Analysis

In a preliminary study we estimate the traditional model and the alternative model (see equations 4 and 9, respectively) for weekly and monthly hedges for the full sample. Tables 4 shows the results of both models for weekly and monthly hedges, columns 2 and 3 display the hedge ratio estimated for the traditional model and the alternative model, respectively. Columns 4 and 5 report the Newey –West standard errors for the estimations without and with basis explicative variable. For monthly hedges, the alternative model appears to be more efficient; nevertheless, for weekly hedges the traditional model.

Columns 8 and 9 report the ratio of variances without and with basis approximating of unhedged and hedged positions\(^\text{12}\), we obtain the same results as Ederington and Salas (2008), when changes in the spot price are anticipated, the traditional model tends to overstate the variance. The overestimate ranges for unhedged position from 9% for monthly hedges to 1.40% for weekly hedges and for hedged position from 108.44% for monthly hedges to 36.33% for weekly hedges. The overestimate of the variance increases as the hedging duration increases. These results confirm the proposition 3 and

\(^{12}\) These ratios are computed as follow, in the case of unhedged position we compute the variance with \(Z_t\) and without \(Z_t\) (see equations 15 and 16, respectively) and, then, we divide the variance with \(Z_t\) by the variance without \(Z_t\). In the case of hedged position, the procedure is the same, but in this case we use equations 17 and 18, respectively.
4 of Ederington and Salas (2008) who propose that “when changes in the spot price are anticipated, the traditional model will tend to overstate the variance of hedged and unhedged positions”.

The estimated percentages reduction in variances (adjusted $R^2$) are reported in columns 10 and 11 for the traditional model and the alternative model, respectively, in all cases the traditional model under–estimates the percentage reduction in variance. The hedging effectiveness is computed following Ederington and Salas (2008) approach. The results are displayed in Columns 12 and 13, respectively, in all cases the traditional model tends to underestimate the hedging effectiveness. Furthermore, we find a positive duration effect in hedging effectiveness. If we compare the adjusted $R^2$ as a predictor of the risk reduction, we can conclude that the adjusted $R^2$ is not a good measure of the hedging effectiveness because in all cases tends to overestimate the hedging effectiveness.

Table 4: Preliminary results: all sample weekly and monthly hedges.

This table displays: the minimum variance hedge ratio estimates, the efficiency of the estimated hedge ratio, the riskiness of hedged and unhedged positions, the percentage reduction in the variance achievable by hedging (adjusted $R^2$ and Ederington and Salas (2008) approach) for the traditional model (eq.4) and the alternative model (eq. 9). The hedges are estimated with all sample size (data from the beginning of our sample to June 2015). In this table k and m indicate weekly and monthly hedges respectively.

<table>
<thead>
<tr>
<th></th>
<th>Minimum variance hedge ratio estimates (b)</th>
<th>Newey-West standard errors for hedge ratio estimates</th>
<th>Coefficient of the information variable $Z_t(k)$</th>
<th>Ratio of variance estimates (with basis/without basis)</th>
<th>Estimated percentage reduction in variance</th>
<th>Hedging effectiveness (Ederington and Salas, 2008, approach)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without basis</td>
<td>With basis</td>
<td>Without basis</td>
<td>With basis</td>
<td>Coefficient</td>
<td>T value</td>
</tr>
<tr>
<td>NCG k1</td>
<td>0.97643 (0.0000)</td>
<td>1.07958 (0.0000)</td>
<td>0.04607</td>
<td>0.05084</td>
<td>0.34542 (0.0000)</td>
<td>6.25369</td>
</tr>
<tr>
<td>GPL k1</td>
<td>1.00342 (0.0000)</td>
<td>1.13410 (0.0000)</td>
<td>0.06147</td>
<td>0.07566</td>
<td>0.36282 (0.0000)</td>
<td>5.61974</td>
</tr>
<tr>
<td>NCG m1</td>
<td>0.88085 (0.0000)</td>
<td>1.06800 (0.0000)</td>
<td>0.16091</td>
<td>0.09177</td>
<td>0.67040 (0.0000)</td>
<td>6.6233</td>
</tr>
<tr>
<td>GPL m1</td>
<td>0.88427 (0.0000)</td>
<td>1.03079 (0.0000)</td>
<td>0.10863</td>
<td>0.06883</td>
<td>0.64078 (0.0000)</td>
<td>7.23395</td>
</tr>
</tbody>
</table>

The results obtained in this preliminary analysis are the same that the conclusions of Ederington and Salas (2008) and, therefore, we agree with their approach. All the implications of this preliminary analysis are suitable for empirical methodology carried out in the next section.
6. Results

In this section, firstly we estimate the models with data from the beginning of our samples until November 2010, that is, we estimate the models for the first three years. In this part (in-sample) the hedging strategies are compared ex-post. In the second part (out-of-sample), we estimate the models for the first three years and the estimated parameters are then used to construct hedges for the subsequent three years period, then we move ahead one observation and use the parameters using data through the second first three years to construct a hedge for the three following years and so on. Therefore in the ex-ante study results are compared using forecasted hedge ratios.

The ex-post and ex-ante hedging ratios for the traditional and the alternative model are displayed in figure 4 (one week period) and figure 5 (one month period), the pattern is similar in both cases. The ex-ante hedging ratios moves around ex- post hedging ratios in both cases, also the hedging ratios estimated with the alternative model are situated above the hedging ratios estimated with the traditional model.

Figure 4: Weekly hedging ratios estimated with traditional and alternative models

a) GPL traditional and alternative hedging ratios. b) NCG traditional and alternative hedging ratios. The vertical line separates the ex-post and ex-ante (three years moving window) periods. The traditional hedging ratios estimated with eq. 4 (——) and alternatives hedging ratios estimated with eq. 9 (-------).
Figure 5: Monthly hedging ratios estimated with traditional and alternative models

a) GPL traditional and alternative hedging ratios. b) NCG traditional and alternative hedging ratios. The vertical line separates the ex-post and ex-ante (three years moving window) periods. The traditional hedging ratios estimated with eq. 4 (—) and alternatives hedging ratios estimated with eq. 9 (-----).
Furthermore, figure 6 shows the hedging ratios for the BEKK, as in the previous figures the *ex-ante* hedging ratios moves around *ex-post* hedging ratios. In these figures it can be seen that conditional hedging ratio values move around linear regression based hedge ratios.

*Figure 6: Weekly hedging ratios estimated with BEKK*

a) GPL BEKK hedging ratios. b) NCG BEKK hedging ratios. The vertical line separates the *ex-post* and *ex-ante* (three years moving window) periods.
Also we have tested the equality in mean between summer and winter with Anova test for the *ex-ante* periods, the results are displayed in Annex IV. The null of mean equality is not rejected (at 5% of significance level) in four out of ten cases analyzed, only is rejected in the case of the alternative model for weekly hedges, the estimated hedging ratios are significantly higher in summer. The null of median equality contrasted with Kruskal-Wallis test only is rejected in the same cases as the Anova test, that is to say, is rejected for the hedging ratios estimated with the alternative model. Equality in variance contrasted with Levene test only can be rejected for weekly and monthly hedging ratios estimated with the traditional model in the NCG, the winter volatility is significantly higher. We also analyze the risk reduction achieved by seasons for the *ex-ante* period, in the case of weekly hedges (see Annex V, table 17) we obtain results similar to those reported in table 5 for both seasons; nevertheless, for monthly hedges (Annex V, table 18), the attained risk reduction in winter is higher to those reported in table 6 and the achievable risk reduction in summer is lower.

The hedging effectiveness are displayed in table 5 and 6 for weekly and monthly hedges. In the *ex-ante* periods for weekly hedges, the largest risk reductions are obtained with the ‘OLS without basis’ strategy and variance computed with Ederington and Salas (2008) approach. The worst outcomes are obtained for the BEKK; nevertheless, in the *ex-post* periods the largest risk reductions are produced with this model in both Hubs. Anyway, the most relevant results are those corresponding to the
ex-ante periods because in the ex-post periods we do not consider only the information available until $t$. In the ex-post period the ‘OLS with basis’ strategy and the BEKK produce the poorest results in the NCG and GPL, respectively.

Table 5: Hedging effectiveness in weekly hedges.

This table reports the risk reduction achieved by each hedging strategy: naïve ($b=1$); OLS without basis (eq.4); OLS with basis (eq.9); and BEKK (eq.12). The in-sample results are computed for the first six months and then a moving window of six months is used to compute the out-of-sample results. The risk reduction achieved is computed using eq.(14). The unhedged spot position variance is computed using eq.(15) and eq.(16) in the standard and Ederington and Salas (2008) approaches, respectively. The variance of each hedging strategy is computed using eq.(17) and eq.(18) in the standard and Ederington and Salas (2008) approaches, respectively.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>in sample</td>
<td>out of sample</td>
<td>in sample</td>
<td>out of sample</td>
</tr>
<tr>
<td>A.1 Hedging one-week spot risk in GPL</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spot variance (no hedged)</td>
<td>3,082018</td>
<td>3,140703</td>
<td>1,324045</td>
<td>1,208424</td>
</tr>
<tr>
<td>Risk reduction (%)</td>
<td>55,307</td>
<td>68,3379</td>
<td>33,3477</td>
<td>45,1441</td>
</tr>
<tr>
<td>Naïve strategy ($b=1$)</td>
<td>55,307</td>
<td>68,3379</td>
<td>33,3477</td>
<td>45,1441</td>
</tr>
<tr>
<td>OLS without basis</td>
<td>55,4048</td>
<td>69,0508</td>
<td>33,3648</td>
<td>45,1975</td>
</tr>
<tr>
<td>OLS with basis</td>
<td>54,3948</td>
<td>70,0418</td>
<td>31,647</td>
<td>44,4154</td>
</tr>
<tr>
<td>BEKK</td>
<td>58,7945</td>
<td>75,2596</td>
<td>28,0558</td>
<td>42,5905</td>
</tr>
<tr>
<td>A.2 Hedging one-week spot risk in NCG</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spot variance (no hedged)</td>
<td>2,963743</td>
<td>2,871649</td>
<td>1,050719</td>
<td>0,990873</td>
</tr>
<tr>
<td>Risk reduction (%)</td>
<td>53,9041</td>
<td>66,9372</td>
<td>39,1045</td>
<td>51,0693</td>
</tr>
<tr>
<td>Naïve strategy ($b=1$)</td>
<td>53,9041</td>
<td>66,9372</td>
<td>39,1045</td>
<td>51,0693</td>
</tr>
<tr>
<td>OLS without basis</td>
<td>53,9104</td>
<td>67,0669</td>
<td>39,499</td>
<td>51,2411</td>
</tr>
<tr>
<td>OLS with basis</td>
<td>53,3419</td>
<td>67,6536</td>
<td>38,1895</td>
<td>50,7632</td>
</tr>
<tr>
<td>BEKK</td>
<td>52,7934</td>
<td>70,9006</td>
<td>30,2856</td>
<td>43,2112</td>
</tr>
</tbody>
</table>

Results comparing the attained risk reduction with the standard and Ederington and Salas (2008) approaches show that the standard approach tends to underestimated the achievable risk reduction, for weekly hedges the attained risk reduction is underestimated with the standard approach between 11.74% and 14.53% in the ex-ante periods; in the ex-post period these values are higher, between 13.03% and 18.11%.

In the ex-ante periods, the best outcomes of table 6 for one month hedge periods are obtained with the naïve strategy; furthermore, the achieved risk reduction of ‘OLS’ and naïve hedges are quite similar when we compute the variance with Ederington and Salas (2008) approach. In this period, the standard approach underestimates the attained risk
reduction between 14.41% and 19.85%. The naïve strategy produces the poorest outcomes when we compute the variance with the standard approach.

The biggest achieved risk reduction in the \textit{ex-post} periods are obtained with ‘OLS with basis’ strategy, the standard approach underestimate the risk reduction between 22.48% and 29.14%.

\textbf{Table 6: Hedging effectiveness in monthly hedges.}

This table is computed as table 6, but using monthly data frequency and only three hedging strategies (naïve, OLS without basis and OLS with basis).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>October 2007- October 2010</td>
<td>7,664527</td>
<td>7,479363</td>
<td>6,146954</td>
<td>5,186982</td>
</tr>
<tr>
<td>November 2010- June 2015</td>
<td>67,0613</td>
<td>93,2554</td>
<td>46,5294</td>
<td>62,9739</td>
</tr>
<tr>
<td></td>
<td>69,3457</td>
<td>91,8275</td>
<td>46,5771</td>
<td>60,9851</td>
</tr>
<tr>
<td></td>
<td>67,8654</td>
<td>93,3445</td>
<td>46,2734</td>
<td>62,0778</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>October 2007- October 2010</td>
<td>9,326139</td>
<td>8,963394</td>
<td>6,195252</td>
<td>5,201321</td>
</tr>
<tr>
<td>November 2010- June 2015</td>
<td>63,9457</td>
<td>90,9582</td>
<td>33,7378</td>
<td>53,1465</td>
</tr>
<tr>
<td></td>
<td>64,2956</td>
<td>89,6443</td>
<td>34,5486</td>
<td>52,1483</td>
</tr>
<tr>
<td></td>
<td>62,4372</td>
<td>91,5779</td>
<td>32,7348</td>
<td>52,5819</td>
</tr>
</tbody>
</table>

From these results we can deduce that hedging performance improves as hedging length increases. As to whether the variance computed following Ederington and Salas (2008) approach produces better results than variance computed with standard approach the results are conclusive. For weekly hedging and monthly hedging the Ederington and Salas (2008) approach produces better outcomes. Furthermore, when changes in the spot are partially predictable, the usual estimates of the riskiness of unhedged positions are biased upward in most cases.

Results comparing the traditional model with the alternative model are inconclusive because for monthly hedges the alternative model improves the hedging effectiveness but not for weekly hedges in which the traditional model produce better outcomes.
Figures 4 and 5 represent the hedging ratios for weekly and monthly for the traditional and the alternative models, the hedging ratios estimated with the alternative model are above the hedging ratios estimated with the traditional model. Also we have contrasted the equality in mean between both hedging ratios, the Anova test is rejected in the case of one week and one month hedging period (see table 8), the hedged ratios estimated with the alternative model are significantly higher. This implies that a more efficient hedge ratio estimate will not mean an improvement in the performance of the hedging strategy. The null of variance equality contrasted with Levene test is rejected in all cases, the volatility of hedging ratios estimated with the alternative model is significantly higher.

**Table 7: Anova test (OLS with basis and OLS without basis)**

This table reports the mean and volatility for the traditional model (eq.4) and the alternative model (eq.9). Where, w and m indicate weekly and monthly hedges, respectively. This table also displays the Anova test and Levene test and its p-values between brackets (·).

<table>
<thead>
<tr>
<th></th>
<th>OLS Without basis</th>
<th>OLS With basis</th>
<th>Anova test</th>
<th>OLS Without basis</th>
<th>OLS With basis</th>
<th>Levene tet.</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPL w1</td>
<td>0.9870</td>
<td>1.0613</td>
<td>939.7780</td>
<td>(0.0000)</td>
<td>0.0348</td>
<td>0.0493</td>
</tr>
<tr>
<td>NCG w1</td>
<td>0.9588</td>
<td>1.0643</td>
<td>1141.4890</td>
<td>(0.0000)</td>
<td>0.0350</td>
<td>0.0336</td>
</tr>
<tr>
<td>GPL m1</td>
<td>1.0397</td>
<td>1.1849</td>
<td>0.018321</td>
<td>(0.0000)</td>
<td>0.1044</td>
<td>0.1297</td>
</tr>
<tr>
<td>NCG m1</td>
<td>0.9547</td>
<td>1.1426</td>
<td>149.6534</td>
<td>(0.0000)</td>
<td>0.0785</td>
<td>0.0825</td>
</tr>
</tbody>
</table>

The largest risk reduction is obtained for monthly hedges, in the case of the GPL coincides with the biggest correlation but not in the case of NCG. Therefore, we can say that the correlations are not a really interesting point to keep in mind when we choose the optimal hedging strategy.
7. Conclusions

In this work we have analyzed the gas natural market of Germany in a similar way to Martínez and Torró (2015) in their paper entitled “European natural gas seasonal effects on futures hedging”. Also we have followed Ederington and Salas (2008) approach, that is, we have adapted the standard minimum variance hedge ratio to the case where spot prices are partially predictable.

The European Union has been trying to implement different codes with the object to achieve natural gas liberalization but this process is not progressing at the same rate across Europe. Furthermore, in Spain there has been taken different measures in order to fulfil the requirements of the European Network Code and there have been established the date 1st October, 2016 for launch the Spanish gas hub.

Firstly, we have analyzed the power of the basis (spot minus futures prices) for explain the unexpected change in the spot and futures prices. We find that the basis have predictive power for explaining unexpected spot price changes that improves as hedging length increases (between 3.56% and 10.90%). However, the basis has less ability to forecast futures price changes.

Secondly, we have estimated the traditional and the alternative models for data from the beginning of our samples through June 2015 (all sample) and we have compared both outcomes and if the adjusted $R^2$ is a good measure of the hedging effectiveness. Our conclusions have been the same as Ederington and Salas (2008), the traditional model underestimate the achievable risk reduction in all cases. Furthermore, the adjusted $R^2$ in all cases tends to overestimate the risk reduction.

Thirdly, we have realized the study in two sub-periods, *ex-ante* and *ex-post*, and we have computed the hedging effectiveness with the standard and Ederington and Salas (2008) approaches. Depending on the hedging duration, the achievable risk reduction attains values between 42.59% and 62.79% in the *ex-ante* period. The largest risk reductions for both hedging periods are obtained with the Ederington and Salas (2008) approach. Therefore, we agree with Ederington and Salas (2008), they conclude that more efficient estimates can be obtained using his approach when spot price returns can
be partially forecasted. Furthermore, the riskiness of unhedged position is overestimated with the standard approach when changes in the spot are partially predictable.

Fourthly, we have estimated the hedging ratios with the BEKK for a week hedging period; nevertheless we can affirm that in the *ex-ante* period the better statistic performance of the BEKK does not imply an improvement in the hedging performance. In this framework we have used the effectiveness measure proposed by Ederington (1979); nevertheless, Lien (2005 b) affirm that “Ederington (1979) hedging effectiveness is only useful for measuring the risk reduction effect of the OLS hedge ratio….A strict application of this measure almost always leads an incorrect conclusion stating that the OLS hedge ratio is the best hedging strategy”

Fifthly, if we compare the hedging effectiveness between one week and one month hedging period, we conclude that hedging effectiveness increases as hedging period increases.

Finally, we have analyzed a seasonal pattern in spot and futures prices. We find a strong seasonality in the volatility of basis approximating and spot and futures returns, the winter volatility is significantly higher than summer volatility. The highest volatility coincides with the Russian-Ukrainian gas dispute of 2009, the Libyan civil war in 2011, the withheld Russian exports in 2012 and the cold winter of 2013.
8. References.


Royal Decree 948/2015. BOE, Spain, 31st October, 2015.

Resolution 4th December, 2015. BOE, Spain, 9th December 2015.

Annex I: Statistics of spot and futures prices differences.

This Annex I displays the summary statistics of spot and futures prices returns. The median and variance equality between $\Delta S(t)$ and $\Delta F(t, T_i)$ are tested by Kruskal-Wallis and Levene statistics test. Skewness and Kurtosis mean the skewness and Kurtosis coefficients and have asymptotic distribution of $N(0,6/T)$ and $N(0,24/T)$ under normality, respectively ($T$ is the sample size). The nulls hypothesis test whether the skewness and Kurtosis coefficient are equal to zero. The normal distribution hypothesis is tested by Jarque-Bera test, its statistic is computed as $T[\text{Skewness}^2/6 + (\text{Kurtosis})^2/24]$ and has an asymptotic $\chi^2$ distribution under the normal distribution hypothesis. $Q(20)$ and $Q^2(20)$ are Ljung-Box test for twentieth order serial correlation in the differentiated and its squared series, respectively. Marginal significance levels of the statistical are displayed as [.].

Table 8: One week variations statistic.

<table>
<thead>
<tr>
<th></th>
<th>NCG</th>
<th></th>
<th>GPL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.14377 (0.00290)</td>
<td>0.00081 (0.99040)</td>
<td>-0.13031 (0.00710)</td>
<td>0.00305 (0.96580)</td>
</tr>
<tr>
<td>Median</td>
<td>-0.13150</td>
<td>-0.02500</td>
<td>-0.10000</td>
<td>0.00500</td>
</tr>
<tr>
<td>Kruskal-Wallis</td>
<td>4.74542 (0.02940)</td>
<td>5.73650 (0.01660)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.D.</td>
<td>0.96058</td>
<td>1.34793</td>
<td>0.96484</td>
<td>1.42445</td>
</tr>
<tr>
<td>Levene</td>
<td>16.04191 (0.00010)</td>
<td>15.28325 (0.00010)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.32124 (0.00880)</td>
<td>-0.04672 (0.70324)</td>
<td>-0.34349 (0.00590)</td>
<td>-0.44622 (0.00027)</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.82283 (0.00000)</td>
<td>4.02925 (0.00000)</td>
<td>3.49745 (0.00000)</td>
<td>5.43163 (0.00000)</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>135.65730 (0.00000)</td>
<td>263.39850 (0.00000)</td>
<td>205.96800 (0.00000)</td>
<td>492.56320 (0.00000)</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.51000</td>
<td>6.40000</td>
<td>3.92000</td>
<td>6.03000</td>
</tr>
<tr>
<td>Minimum</td>
<td>-4.45000</td>
<td>-6.25000</td>
<td>-4.55000</td>
<td>-7.85000</td>
</tr>
<tr>
<td>Q(20)</td>
<td>31.94900 (0.04400)</td>
<td>42.65700 (0.00200)</td>
<td>492.56320 (0.00000)</td>
<td></td>
</tr>
<tr>
<td>Q²(20)</td>
<td>228.51000 (0.00000)</td>
<td>97.86500 (0.00000)</td>
<td>97.86500 (0.00000)</td>
<td></td>
</tr>
</tbody>
</table>

Table 9: One month variations statistic.

<table>
<thead>
<tr>
<th></th>
<th>NCG</th>
<th></th>
<th>GPL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.57710 (0.01000)</td>
<td>-0.00330 (0.99080)</td>
<td>-0.53757 (0.01780)</td>
<td>0.00682 (0.98010)</td>
</tr>
<tr>
<td>Median</td>
<td>-0.33500</td>
<td>-0.02000</td>
<td>-0.27200</td>
<td>0.00000</td>
</tr>
<tr>
<td>Kruskal-Wallis</td>
<td>2.93685 (0.08660)</td>
<td>2.05962 (0.15120)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levene</td>
<td>1.71820 (0.19160)</td>
<td>2.39969 (0.12310)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>-2.00907</td>
<td>-1.20812 (0.00000)</td>
<td>-1.08736 (0.00003)</td>
<td>-0.54398 (0.03720)</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.3894 (0.00006)</td>
<td>6.32187 (0.00000)</td>
<td>2.77240 (0.00000)</td>
<td>4.59131 (0.00104)</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>29.42830 (0.00000)</td>
<td>154.05110 (0.00000)</td>
<td>42.13767 (0.00000)</td>
<td>13.94210 (0.00094)</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.20000</td>
<td>8.07000</td>
<td>3.53000</td>
<td>6.45000</td>
</tr>
<tr>
<td>Q(20)</td>
<td>20.39100 (0.43400)</td>
<td>28.78800 (0.09200)</td>
<td>20.96600 (0.39900)</td>
<td>30.47600 (0.06200)</td>
</tr>
<tr>
<td>Q²(20)</td>
<td>33.22500 (0.03200)</td>
<td>26.11600 (1.00000)</td>
<td>29.69300 (0.07500)</td>
<td>73.77000 (0.99500)</td>
</tr>
</tbody>
</table>
Annex II: Correlations.

This Annex displays correlation matrix of the spot and futures prices variations for weekly and monthly data. For a sample size of $T$ observations, the asymptotic distribution of the $\sqrt{T}$ times the correlation coefficient is a zero-one normal distribution.

Table 10: Weekly correlations.

<table>
<thead>
<tr>
<th></th>
<th>NCG</th>
<th>GPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta S(t)$</td>
<td>1.000000</td>
<td>0.676270</td>
</tr>
<tr>
<td>$\Delta F(t,T_1)$</td>
<td>0.910953</td>
<td>0.684259</td>
</tr>
<tr>
<td>$\Delta F(t,T_1)$</td>
<td>0.695837</td>
<td>0.965219</td>
</tr>
</tbody>
</table>

Table 11: Monthly correlations.

<table>
<thead>
<tr>
<th></th>
<th>NCG</th>
<th>GPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta S(t)$</td>
<td>1.000000</td>
<td>0.676270</td>
</tr>
<tr>
<td>$\Delta F(t,T_1)$</td>
<td>0.965219</td>
<td>0.694191</td>
</tr>
<tr>
<td>$\Delta F(t,T_1)$</td>
<td>0.676270</td>
<td>1.000000</td>
</tr>
<tr>
<td>$\Delta F(t,T_1)$</td>
<td>0.965219</td>
<td>1.000000</td>
</tr>
</tbody>
</table>
Annex III: Summer and winter mean and volatility.

Tables 12, 13 and 14 report, for weekly frequency data, the mean and volatility (standard deviation) of basis approximating, spot and futures returns in winter (October to March) and summer (April to September), respectively. These tables also report the values of different tests (Anova, Kruskal – Wallis, and Levene); p-values are reported between brackets ( ).

Table 12: Summer and winter mean and volatility: basis approximating.

<table>
<thead>
<tr>
<th></th>
<th>Summer mean</th>
<th>Winter mean</th>
<th>Equality</th>
<th>Summer volatility</th>
<th>Winter volatility</th>
<th>Equality</th>
</tr>
</thead>
<tbody>
<tr>
<td>NCG m1</td>
<td>0.445169</td>
<td>0.342230</td>
<td></td>
<td>1.169456</td>
<td>1.746067</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.479788 (0.4885)</td>
<td>1.169456</td>
<td></td>
</tr>
<tr>
<td>GPL m1</td>
<td>0.359732</td>
<td>0.233377</td>
<td>1.011832 (0.3145)</td>
<td>1.082261</td>
<td>1.863577</td>
<td>5.616831 (0.0183)</td>
</tr>
</tbody>
</table>

Table 13: Summer and winter mean and volatility: spot.

<table>
<thead>
<tr>
<th></th>
<th>Summer mean</th>
<th>Winter mean</th>
<th>Equality</th>
<th>Summer volatility</th>
<th>Winter volatility</th>
<th>Equality</th>
</tr>
</thead>
<tbody>
<tr>
<td>NCG m1</td>
<td>0.032027</td>
<td>-0.044754</td>
<td></td>
<td>1.212755 (0.2708)</td>
<td>1.052900</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.212755 (0.2708)</td>
<td>1.052900</td>
<td></td>
</tr>
<tr>
<td>GPL m1</td>
<td>0.026377</td>
<td>-0.038874</td>
<td>0.353252 (0.5523)</td>
<td>1.013616</td>
<td>1.699086</td>
<td>18.13975 (0.0000)</td>
</tr>
</tbody>
</table>

Table 14: Summer and winter mean and volatility: futures.

<table>
<thead>
<tr>
<th></th>
<th>Summer mean</th>
<th>Winter mean</th>
<th>Equality</th>
<th>Summer volatility</th>
<th>Winter volatility</th>
<th>Equality</th>
</tr>
</thead>
<tbody>
<tr>
<td>NCG m1</td>
<td>-0.073470</td>
<td>-0.210104</td>
<td>2.322121 (0.1267)</td>
<td>0.782429</td>
<td>1.046811</td>
<td>12.75851 (0.0004)</td>
</tr>
<tr>
<td>GPL m1</td>
<td>-0.074464</td>
<td>-0.203344</td>
<td>2.886805 (0.0893)</td>
<td>0.858225</td>
<td>1.033792</td>
<td>7.211672 (0.0076)</td>
</tr>
</tbody>
</table>
Annex IV: Seasonal hedging ratios.

This Annex III reports the mean, median and volatility (standard deviation) of weekly and monthly hedging ratios (tables 15 and 16, respectively) estimated using OLS with basis, OLS without basis and BEKK in winter (October to March) and summer (April to September). This table also reports the values of different test (Anova, Kruskal – Wallis, and Levene); p-values are reported between brackets (\(.)\).

### Table 15: Seasonal hedging ratios (one week period).

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean Summer</th>
<th>Mean Winter</th>
<th>Median Summer</th>
<th>Median Winter</th>
<th>Kruskal-Wallis Summer</th>
<th>Kruskal-Wallis Winter</th>
<th>Levene Summer</th>
<th>Levene Winter</th>
</tr>
</thead>
<tbody>
<tr>
<td>NCG OLS with basis</td>
<td>1.1173</td>
<td>1.0977</td>
<td>12.4751</td>
<td>1.1114</td>
<td>1.0991</td>
<td>5.7858</td>
<td>0.0331</td>
<td>0.0458</td>
</tr>
<tr>
<td>NCG OLS without basis</td>
<td>0.9896</td>
<td>0.9887</td>
<td>0.0446 (0.8329)</td>
<td>0.9897</td>
<td>0.9934</td>
<td>0.0004</td>
<td>0.0201</td>
<td>0.0383</td>
</tr>
<tr>
<td>GPL OLS with basis</td>
<td>1.0743</td>
<td>1.0526</td>
<td>35.5963 (0.0000)</td>
<td>1.0729</td>
<td>1.0565</td>
<td>23.2989 (0.0000)</td>
<td>0.0241</td>
<td>0.0281</td>
</tr>
<tr>
<td>GPL OLS without basis</td>
<td>0.9590</td>
<td>0.9559</td>
<td>0.4313 (0.5121)</td>
<td>0.9575</td>
<td>0.9650</td>
<td>0.8356 (0.3555)</td>
<td>0.0301</td>
<td>0.0383</td>
</tr>
<tr>
<td>NCG BEKK</td>
<td>1.0994</td>
<td>1.0861</td>
<td>0.0844 (0.7716)</td>
<td>1.0645</td>
<td>1.0250</td>
<td>0.3756 (0.5400)</td>
<td>0.3392</td>
<td>0.3195</td>
</tr>
<tr>
<td>GPL BEKK</td>
<td>1.0429</td>
<td>1.0386</td>
<td>0.0044 (0.9470)</td>
<td>0.9584</td>
<td>1.0023</td>
<td>0.2341 (0.6285)</td>
<td>0.4456</td>
<td>0.4711</td>
</tr>
</tbody>
</table>

### Table 16: Seasonal hedging ratios (one month period).

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean Summer</th>
<th>Mean Winter</th>
<th>Median Summer</th>
<th>Median Winter</th>
<th>Kruskal-Wallis Summer</th>
<th>Kruskal-Wallis Winter</th>
<th>Levene Summer</th>
<th>Levene Winter</th>
</tr>
</thead>
<tbody>
<tr>
<td>NCG OLS with basis</td>
<td>1.1631</td>
<td>1.1407</td>
<td>0.8842 (0.3520)</td>
<td>1.1485</td>
<td>1.1645</td>
<td>0.3333 (0.5637)</td>
<td>0.0761</td>
<td>0.088</td>
</tr>
<tr>
<td>NCG OLS without basis</td>
<td>0.9433</td>
<td>0.9676</td>
<td>1.0240 (0.3168)</td>
<td>0.9447</td>
<td>0.1108</td>
<td>0.8996 (0.3294)</td>
<td>0.0387</td>
<td>0.1108</td>
</tr>
<tr>
<td>GPL OLS with basis</td>
<td>1.2120</td>
<td>1.2028</td>
<td>0.0664 (0.7978)</td>
<td>1.2418</td>
<td>1.2048</td>
<td>0.3099 (0.5777)</td>
<td>0.1297</td>
<td>0.1170</td>
</tr>
<tr>
<td>GPL OLS without basis</td>
<td>1.0379</td>
<td>1.0719</td>
<td>1.5921 (0.2134)</td>
<td>1.0672</td>
<td>1.0722</td>
<td>0.9392 (0.3325)</td>
<td>0.0923</td>
<td>0.0941</td>
</tr>
</tbody>
</table>
Annex V: Hedging effectiveness by seasons.

This table reports the out-of-sample seasonal risk reduction achieved by each hedging strategy: naïve (b=1); OLS without basis (eq.4); OLS with basis (eq.9). The risk reduction achieved is computed using eq.14. The unhedged spot position variance is computed using eq.15 and eq.16 in the standard and Ederington and Salas (2008) approaches, respectively. The variance of each hedging strategy is computed using eq.17 and eq.18 in the standard and Ederington and Salas (2008) approaches.

Table 17: Hedging effectiveness by seasons in weekly hedges.

<table>
<thead>
<tr>
<th>Hedging one-week spot risk in NCG</th>
<th>Summer</th>
<th>Winter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot variance (no hedged)</td>
<td>0.823672 0.637815</td>
<td>1.2958 1.3356</td>
</tr>
<tr>
<td>Risk reduction (%)</td>
<td>35.0645 49.0606</td>
<td>40.2479 51.1292</td>
</tr>
<tr>
<td>Naïve strategy (b=1)</td>
<td>35.0713 49.0997</td>
<td>40.919 51.4230</td>
</tr>
<tr>
<td>OLS without basis</td>
<td>33.9879 48.0845</td>
<td>39.3744 51.0515</td>
</tr>
<tr>
<td>OLS with basis</td>
<td>27.0058 47.2313</td>
<td>34.7994 43.9998</td>
</tr>
<tr>
<td>OLS without basis</td>
<td>27.2154 47.6201</td>
<td>34.8029 44.0237</td>
</tr>
<tr>
<td>OLS with basis</td>
<td>23.6109 43.7349</td>
<td>33.6555 43.9277</td>
</tr>
</tbody>
</table>

Table 17: Hedging effectiveness by seasons in monthly hedges.

<table>
<thead>
<tr>
<th>Hedging one-month spot risk in NCG</th>
<th>Summer</th>
<th>Winter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot variance (no hedged)</td>
<td>4.8571 5.2051</td>
<td>7.1941 5.0534</td>
</tr>
<tr>
<td>Risk reduction (%)</td>
<td>32.5452 30.4874</td>
<td>33.4222 73.3254</td>
</tr>
<tr>
<td>Naïve strategy (b=1)</td>
<td>34.9972 32.8347</td>
<td>33.4609 69.5186</td>
</tr>
<tr>
<td>OLS without basis</td>
<td>27.9137 26.1844</td>
<td>34.1364 75.3742</td>
</tr>
<tr>
<td>OLS with basis</td>
<td>47.1909 46.9436</td>
<td>45.7611 74.2634</td>
</tr>
<tr>
<td>OLS without basis</td>
<td>45.838 45.7161</td>
<td>46.7821 72.2300</td>
</tr>
<tr>
<td>OLS with basis</td>
<td>41.6839 40.7387</td>
<td>48.1674 76.6109</td>
</tr>
</tbody>
</table>