PERFORMANCE MEASURES UNDER PARAMETRIC AND SEMIPARAMETRIC MODELS. WHAT IS THE BEST APPROACH?

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Contents

Abstract

In this work we implement two approaches of modelling PMs for both the Sharpe Ratio and Quantile Ratio families to select equities. A parametric (P) approach based on an asymmetric GARCH and a semiparametric (SP) approach based on the CAViaR are used to model the higher moments quantiles of the asset returns. Considering daily returns of 12 equities of the S&P500 a rank of them is made to form portfolios, obtaining a clear outperform by the assets selected with the SP quantile ratios, especially those that do not go far from the center of the distribution.

Keywords: CAViaR, Asymmetric GARCH, Higher moments, Sharpe Ratios, Quantile Ratios.

1. Introduction

This work studies how different the Performance measures (PMs) sort assets and the behaviour of the portfolios constructed using this criteria. Using the PMs as a screening rule we can choose the assets to be invested in.

In this environment the Sharpe Ratio (SR) (Sharpe 1996) is the benchmark strategy, which requires the returns to follow a Gaussian distribution[1] or quadratic preferences. It has been well documented that this assumption is far from reality, as the assets deviate from normality, causing an underestimation of the risk. Following León et al (2015), the debate about the significance in investment applications of PMs regarding the SR is still open.

In this work we show a class of PMs that can account for higher moments and two ways to measure them. Using a parametric (P) approach based in an asymmetric GARCH style, and a semiparametric (SP) approach based on the CAViaR proposed by Engle and Manganelli (2004) the higher moments and some selected quantiles of the returns distributions are modelled.

A set of six PMs that pertain to two different groups are used, the SR and its extension, the Adjusted Sharpe Ratio with skewness and kurtosis, and a group of PMs based on quantiles, two Value at Risk (VaR) and two Expected Shortfall (ES) ratios, what makes 12 different portfolios. These PMs are computed and ranked for daily returns of a subset of 12 stocks pertaining to the S&P 500, and the four stocks with the best performance are selected for the next day to form an equally-weighted portfolio. This analysis is made for both the in sample (IS) and the out of sample period (OOS). The resulting cumulative returns for each portfolio and the differences in composition made by every PM are studied.

For both sample periods differences in ranks are found for the different groups of PMs, and for the different ways to measure them. The group of ratios based on quantiles, and especially the SP-VaR ratios outperform every other measure during the in sample period. According to these results we pay more attention to the SP PMs during the OOS, and we estimate again the models every year of the period.

The outline of this work is as follows. In section 2 the PMs used are presented and section 3 shows the way in which higher moments and quantiles are modelled in every framework. Section 4 shows the data used and comment the results obtained, while section 5 discuses the portfolios from the previous

[1] Owen et al.1983

estimations. Sections 6 and 7 show the conclusions and discusses figures and tables that contain the results.

2. Performance Measures

Two main groups have been used here, based on the moments of the returns distribution (Sharpe Ratio and the Adjusted Sharpe Ratio), and based on the quantiles (Value at Risk Ratio and Expected Shortfall Ratio).

The standard PM is the Sharpe Ratio (SR) (Sharpe, 1966 and 1994), which is defined as

$$SR(\theta) = \frac{\mu-\theta}{\sigma},$$

where μ and σ are the expected return and volatility of return distributions respectively. θ is the threshold, usually the risk free rate. To account for higher moments of the returns distribution an extension of SR is used, the Amplified Sharpe Ratio (ASR), suggested by Pézier and White (2008)

$$ASR(\theta) = SR(\theta)\left[1 + \frac{sk}{6}SR(\theta) - \frac{ku-3}{24}SR^2(\theta)\right],$$

Where sk and ku are the skewness and kurtosis of returns distributions respectively.

The other group of measures are based on quantiles, so first are introduced the downside risk measures used.

The VaR at the α confidence level of a distribution $F(r)$ is the α-quantile of $F(r)$ is:

$$VaR(\alpha) = inf\{r|F(r) \geq \alpha\},$$

The other measure is the Expected Shortfall or Conditional VaR, which measures the expected value of the returns given that the VaR level has been exceeded.

$$ES(\alpha) = \mathbb{E}\{r|r \leq -VaR(\alpha)\},$$

VaR Ratio, introduced in Caporin and Lisi (2011) uses symmetric quantiles of the returns distributions:

$$VaRR(\alpha) = \left|\frac{VaR(-r;\alpha)}{VaR(r;\alpha)}\right|,$$

where $|\cdot|$ is the absolute value function. The confidence levels used are 80 and 20 (VaRR8020) and 90 10 (VaRR9010). Following this scheme we have the Expected Shortfall Ratio (ESR), at the same levels than VaRR.

$$ESR(\alpha) = \left|\frac{ES(-r;\alpha)}{ES(r;\alpha)}\right|,$$

where the numerator is the expected gain (the right tail) and the denominator is the expected loss (the left tail).

3. Modelling time-varying skewness and kurtosis

In this section we show the quantiles and the higher moments to construct the PMs.

In previous literature two main approaches have been used to modelling higher moments. The *direct approach,* where the evolution of skewness and kurtosis are defined in an explicit equation. This approach was proposed by Harvey & Siddique (1999), Brooks et al. (2005) and León et al. (2005). Regarding the last model, which captures the dynamics in both skewness and kurtosis and extend the other ones, shows a modification of the Gram-Charlier density for standardized returns. This model is the one used by White et al. (2010) to compare with their multi-quantile CAViaR.

As addressed by Anatolyev & Petukhov (2016), this approach would be very attractive but it shows some drawbacks. There are a few distributions that have skewness and kurtosis as parameters, and there exists a theoretical bound in which all possible values for skewness-kurtosis must lie (see Jondeau & Rockinger, 2003), while the dynamics they propose is not restricted to this bound. In León et al. (2005) to overcome the boundedness problem, they modify the density to be defined by any pair of skewness-kurtosis, but those parameters are no longer the skewness and kurtosis desired with respect to the modified density.

The *indirect approach* to modelling conditional higher moments consists of implementing some distribution with parameters that reflect asymmetry and heavy tailedness. Jondeau & Rockinger (2003) is one of the first to study the effect of conditional higher moments, and they selected the Hansen (1994) Skewed-t distribution (ST), which has two parameters that drive skewness and kurtosis. This approach has also been used by Fenou et al. (2014), which models conditional skewness with the Binormal distribution, ST, the Skewed Generalized Error Distribution (SGED) of Theodossiou (2000), Lalancette and Simonato (2017) which used the Johnson Su distribution to model conditional skewness and kurtosis of the VIX index, and finally, Bali et al. (2007) who implemented used the Skewed Generalized t distribution (SGT) of Theodossiou (1998) to estimate the conditional value at risk (VaR).

In this work two kind of approaches have been used, a semiparametric approach based on the quantiles of the returns distribution estimated via CAViaR (Engle and Manganelli, 2004), and a parametric approach based on the modellization of the conditional moments of the Skewed t of Hansen (Jondeau & Rockinger, 2003).

3.1. Parametric measures of conditional skewness and kurtosis

3.1.1. The Hansen skewed t distribution

The next model builds on an asymmetric GARCH model, with errors following the skewed t distribution of Hansen (1994) with time varying moments.

This density function, introduced by Hansen (1994) permits the residuals to have asymmetries and fat tails:

$$ST(z|\nu,\lambda) = \begin{cases} bc\left(1+\frac{1}{\nu-2}\left(\frac{bz+a}{1-\lambda}\right)^2\right)^{-\frac{(\nu+1)}{2}} & if\ z < -\frac{a}{b} \\ bc\left(1+\frac{1}{\nu-2}\left(\frac{bz+a}{1+\lambda}\right)^2\right)^{-\frac{(\nu+1)}{2}} & if\ z \geq -\frac{a}{b} \end{cases}$$

where

$$a \equiv 4\lambda c \frac{\nu - 2}{\nu - 1}, \quad b^2 \equiv 1 + 3\lambda^2 - a^2, \quad c \equiv \frac{\Gamma\big((\nu+1)/2\big)}{\sqrt{\pi(\nu-2)}\Gamma(\nu/2)}.$$

This distribution is defined for $2 < \nu < \infty$ and $-1 < \lambda < 1$, and includes other known distributions such as the Student's t as λ goes to 0, and the Normal distribution when ν tends to infinity. This distribution, like the traditional Student-t distribution with ν degrees of freedom, allows the existence of the moments up to the νth. Then, it exists if $\nu > 2$, but the kurtosis does exists for the restriction of $\nu > 4$ is imposed.

The theoretical formulae for skewness and kurtosis are extracted from Jondeau and Rockinger (2003), given the notations

$$m_2 = 1 + 3\lambda^2,$$

$$m_3 = 16c\lambda(1 + \lambda^2)\frac{(\nu-2)^2}{(\nu-2)(\nu-3)} \, if \; \nu > 3\,,$$

$$m_4 = 3\frac{(\nu-2)}{(\nu-4)}(1 + 10\lambda^2 + 5\lambda^4) \, if \; \nu > 4\,,$$

Both skewness and kurtosis are defined as

$$Sk = [m_3 - 3am_2 + 2a^3]/b^3\,,$$

$$K = [m_4 - 4am_3 + 6a^2m_2 - 3a^4]/b^4\,,$$

For more details, see Jondeau and Rockinger (2003).

3.1.2 GARCH model

Let $r_{i,t}$ denote the daily log-return of asset i at time t, then.

$$r_{i,t} = \sqrt{h_{i,t}}\varepsilon_{i,t},$$

$$h_{i,t} = \beta_{0,i} + \beta_{1,i}h_{i,t-1} + \beta_{2,i}^{+}z_{i,t-1}^2\mathbf{1}_{\{z_{i,t-1\geq 0}\}} + \beta_{2,i}^{-}z_{i,t-1}^2\mathbf{1}_{\{z_{i,t-1<0}\}},$$

$$z_{i,t} = \sqrt{h_{i,t}}\varepsilon_{i,t},$$

$$\varepsilon_{i,t}|\mathcal{F}_{i,t-1} \sim ST(z_{i,t}; \upsilon_{i,t}, \lambda_{i,t}),$$

where $h_{i,t}$ is the expected return and variance conditioned on $\mathcal{F}_{i,t-1}$ the information set, and $\varepsilon_{i,t}$ is the corresponding residual.

Asymmetric GARCH, (the GJR-GARCH (Glosten et al. 1993)) is employed for the dynamic of the conditional variance. This specification accounts for volatility clustering and leverage effects. Innovations

$\varepsilon_{i,t}$ are driven by a skewed Student's t distribution, which capture heavy tails and skewness through the degrees of freedom υ_i and the asymmetry parameter λ_i (Hansen 1994).

The specification of the innovation's distributions allows for time-varying higher moments as follows:

$$\tilde{\upsilon}_{i,t} = \alpha_{0,i} + \alpha_{1,i}\tilde{\upsilon}_{i,t-1} + \alpha_{2,i}^{+} z_{i,t-1}^{+} + \alpha_{2,i}^{-} z_{i,t-1}^{-},$$

$$\tilde{\lambda}_{i,t} = \gamma_{0,i} + \gamma_{1,i}\tilde{\lambda}_{i,t-1} + \gamma_{2,i}^{+} z_{i,t-1}^{+} + \gamma_{2,i}^{-} z_{i,t-1}^{-},$$

$$\upsilon_{i,t} = \wedge_{[4,30]}\left(\tilde{\upsilon}_{i,t}\right), \qquad \lambda_{i,t} = \wedge_{(-1,1)}\left(\tilde{\lambda}_{i,t}\right),$$

Where $z_{i,t}^{+} = \max\,(z_{i,t}, 0)$, $z_{i,t}^{-} = \min\,(z_{i,t}, 0)$, $y = \wedge_{[l,u]}\,(x) = l + \frac{u-l}{1+\exp(-x)}$ denotes the logistic map in order to keep the transformed variable y in the domain (l, u) for all $x \in \mathbb{R}$. This kind of specification $\tilde{\upsilon}_{i,t}$, and $\tilde{\lambda}_{i,t}$ may depend on their lagged values and react differently to positive and negative shocks. For more details, see Jondeau and Rockinger (2003).

This model is solved using maximum likelihood estimation (MLE), but the optimality conditions make us wonder if the optimum is reached. The steps followed in the final estimation of the model consist of estimating the model firstly accounting only for the dynamic of the volatility. Then model both volatility and degree of freedom parameter with the asymmetry constant, and finally the whole model, taking into account the previous estimations. For the finally results, the MLE parameters are used as initial values for the Bayesian estimation. As explained below we use Markov Chain Monte Carlo (MCMC) with the Metropolis-Hastings algorithm to overcome the parameter uncertainty. See appendix for more details.

Once the return distribution is modelled it is straightforward to obtain the different quantiles that would describe the VaR and ES

$$q_{\theta,i,t} = \sigma_{i,t-1} ST^{-1}_{\upsilon_{i,t-1},\lambda_{i,t-1}}(\theta)$$

where $q_{\theta,i,t}$ is the desired quantile at the θ confidence level, for the asset i and the moment t, $\sigma_{i,t-1}$ is the estimated volatility of the previous day for that asset ($\sqrt{h_{i,t}}$), and $ST^{-1}_{\upsilon_{i,t-1},\lambda_{i,t-1}}(\theta)$ is the inverse of the Hansen t distribution given the estimated parameters and for the confidence level.

3.2 Semiparametric measures of conditional skewness and kurtosis.

Kim and White (2004) estimate by using more robust measures both the third and fourth moments of standardized random variables. For instance, is Bowley's (1920) coefficient of skewness is given by

$$SK_2 = \frac{q_3^* + q_1^* - 2q_2^*}{q_3^* - q_1^*},$$

where $q_1^* = F^{-1}(0.25), q_2^* = F^{-1}(0.5), q_3^* = F^{-1}(0.75)$, where $F(y) \equiv P_0[Y_t < y]$ is the unconditional cumulative density function (CDF) of Y_t. The kurtosis coefficients of Crow & Siddiqui's (1967) is given by

$$KR_4 = \frac{q_4^* - q_0^*}{q_3^* - q_1^*} - 2.91,$$

where $q_0^* = F^{-1}(0.025), q_4^* = F^{-1}(0.975)$. The choice of these measures can be seen in White, et al. (2010)., and Kim and White (2004).

These measures are based on unconditional quantiles, so they can't incorporate the dynamic evolution of quantiles over time. To avoid these limitations White et al. (2010), aim to build conditional skewness and kurtosis measures by using conditional quantiles $q_{j,t}^*$ instead of the unconditional ones, q_j^*. These conditional measures are given by

$$CSK_2 = \frac{q_{3,t}^* + q_{1,t}^* - 2q_{2,t}^*}{q_{3,t}^* - q_{1,t}^*}$$

$$CKR_4 = \frac{q_{4,t}^* + q_{0,t}^*}{q_{3,t}^* - q_{1,t}^*} - 2.91.$$

For more details, see White et al. (2010).

Finally the methodology by Taylor (2005) is used to estimate volatility based on symmetric quantiles

$$CV = \frac{q_{4,t}^* - q_{0,t}^*}{3.92}$$

Note that the denominator is based on the central distances between the quantiles under the Pearson curves (see Pearson and Tukey 1965).

3.2.1 CAViaR model

Let $r_{i,t}$ denote the daily log-return of an asset i at time t.

Let $\boldsymbol{\beta}$ be a vector of unknown parameters, then a generic CAViaR specification (Engle and Manganelli 2004) for a symmetric absolute value[2] on the news impact curve might be:

$$q_t(\boldsymbol{\beta}) = \beta_0 + \beta_1 q_{t-1}(\boldsymbol{\beta}) + \beta_2 |r_{t-1}|$$

The parameters of CAViaR models are estimated by quantile regression, introduced by Koenker and Basset (1978), Engle and Manganelli (2004).

The θth regression quantile is defined as:

$$\min_{\beta} \frac{1}{T} \sum_{t=1}^{T} [\theta - I_{\{r_t < q_t(\boldsymbol{\beta})\}}] |r_t - q_t(\boldsymbol{\beta})|$$

where $I_{\{\cdot\}}$ is the indicator function.

The previous CAViaR model is used to estimate the five quantiles needed for the conditional skewness and kurtosis: $\theta_j = 0.025, 0.25, 0.5, 0.75, 0.975,$ and the ones used in VaR and ES measures: $\theta_j = 0.01, 0.02, \ldots 0.19, 0.2, 0.8, 0.81, \ldots 0.98{,}0.99,$ what makes a total of 45 quantiles.

[2] Kuester et al. (2006) shows the good performance of the Symmetric Absolute Value in relation to more sophisticated CAViaR specifications.

$$q^*_{0.01,i,t} = \beta^*_0 + \beta^*_1 |r_{i,t-1}| + \beta^*_2 q^*_{0.01,i,t-1}$$

$$q^*_{0.02,i,t} = \beta^*_0 + \beta^*_1 |r_{i,t-1}| + \beta^*_2 q^*_{0.02,i,t-1}$$

$$\vdots$$

$$q^*_{0.99,i,t} = \beta^*_0 + \beta^*_1 |r_{i,t-1}| + \beta^*_2 q^*_{0.99,i,t-1}$$

Trying to estimate this model as a VAR is difficult from the computational point of view as estimated in White et al. (2010) as they had to estimate 35 parameters for their model of 5 quantiles. In this case, the model is much more simple, with a different equation for every quantile and without interaction across them, so the problems gets simplified to estimate only 3 parameters for each quantile[3].

This conditional quantiles $q^*_{0.025,i,t}, \dots q^*_{0.975,i,t}$ are then used to estimate CSK2 and CSK4, the quantiles $q^*_{0.1,i,t}, q^*_{0.2,i,t}, q^*_{0.8,i,t}, q^*_{0.9,i,t}$ are used to get the VaR measures, and to compute the ES an arithmetic mean of the quantiles beyond the given VaR are used (for the ES10 for example the mean of $q^*_{0.01,i,t}, q^*_{0.02,i,t}, \dots q^*_{0.09,i,t}, q^*_{0.1,i,t}$ is computed).

4. Data

In this study 12 stock return series from different sectors of the S&P500 have been chosen to ensure different statistic types, obtained from Datastream. The next symbols are used for the equities: APPL, JNJ, BRK.A, DIS, GE, KO, XOM, SO, SPG, DOW, T, JPM respectively Apple, Johnson & Johnson, Berkshire Hathaway, Walt Disney, General Electric, Coca Cola, Exxon Mobil, Southern, Simon Property Group, Dow Jones, AT&T and JP Morgan. It covers 16 years of daily data, from March 18, 1997 to May 29, 2013, what makes 4217 return observations used for the in sample, and 1000 observations to March 28, 2017 for out of sample purposes.

Table 1 displays the summary statistics for the in sample market data. It shows the annualized average mean that ranges between 0.85% and 27,24% , and annualized volatility from 21,45% to 49,77%. Skewness and Kurtosis are different from 0 (ranged between -2,81 to 0,25) and higher than 3 in every stock (from 7,96 to 77,49) along with the Jarque-Bera (JB) test that rejects the normality assumption in the data.

Finally, there is no evidence for serial correlation across all returns series, in contrast with the case of squared returns that are all strongly correlated but for the case of Apple.

[INSERT TABLE 1 AROUND HERE]

4.1 Model estimation

4.1.1 Parametric approach

Table 2 and Table 3 exhibit Bayesian parameter estimates of the GARCH and Skew-t dynamics are reported. In every case except for T, we can see how bad news have a greater effect in the volatility, what

[3] The reason to do this is to simplify calculations, and as showed in their paper of MQ-CAViaR the estimations do not vary much from those in the univariate problem.

suggests asymmetric returns. The volatility persistence $(\beta_1 + (\beta_2^+ + \beta_2^+)/2)$ is high in very case, close to 1 in the case of XOM and the lowest 0.954 of T.

[INSERT TABLE 2 AROUND HERE]

[INSERT TABLE 3 AROUND HERE]

Table 3 exhibit the parameters for the higher-moment dynamics. Regarding the degree of freedom parameter we can see that the autoregressive parameter is far from 1 in every case, what might suggest that there is not a high persistence in this parameter. It is not easy to decide if either positive or negative returns have a greater effect on this parameter as in every equation have different sign and magnitude, but all the constant terms seem to be negative under this dynamic specification.

Regarding the asymmetry parameter there are similar results, but in this case the constant term seem to be lower and close to 0 in every case. There is no evidence of strong persistence in this parameter, except BRK.A and DOW seem to be higher, and hence a less erratic shape of the dynamics.

4.1.2 Semiparametric approach

Only the estimates for the 5 quantiles driving the skewness and kurtosis are exhibited, instead of all the quantile estimation, what would make 1620 parameters.

Regarding the autoregressive parameters is high in every case and with low deviation except for the case of the median that we will comment below. The parameters that drive the impact news (β_2) get higher as we try to estimate the quantiles in the extremes of the distribution, what makes sense as there are more pikes in this part of the distribution. The case with the median (q2) is quite different. Since these series tend to be around 0 they become difficult to estimate the parameters underlying this equation, as shown in higher standard deviation of these parameters.

4.2 Dynamics of the higher moments

4.2.1 Parametric approach

In figures 1 to 5 the dynamics of the moments are displayed as well as Both the asymmetry and degree of freedom parameters that drive skewness and kurtosis for four selected equities: APPL, KO, GE and JPM, what let us have an idea of the rest of time-series.

[INSERT FIGURE 1 AROUND HERE]

[INSERT FIGURE 2 AROUND HERE]

[INSERT FIGURE 3 AROUND HERE]

[INSERT FIGURE 4 AROUND HERE]

[INSERT FIGURE 5 AROUND HERE]

In general a low degree of freedom leads a higher kurtosis and vice versa, and an asymmetry parameter close to 0 leads to a lower skewness. An example of this can be seen with the dynamics of the degrees of freedom estimated for KO, around 29, and a kurtosis close to 3.

In figure 2 we can see the high volatility in APPL in 2001, corresponding to the dot-com bubble, as well as with JPM in year 2009 with the financial crisis, in this case also with spikes of higher kurtosis.

4.2.2 Semiparametric approach

Figures 6 to 9 show the evolution of the quantiles driving the volatility, skewness and kurtosis. This way of modelling the moments of the distribution is less sensitive to those based on moments as we can see with the corresponding rise in volatility of APPL modelled with a GARCH.

[INSERT FIGURE 6 AROUND HERE]

[INSERT FIGURE 7 AROUND HERE]

[INSERT FIGURE 8 AROUND HERE]

[INSERT FIGURE 9 AROUND HERE]

4.3 Risk measures estimates

Figures 10 to 13 show the returns, the estimates of VaR measures and ES at the 10% and 90% confidence level, under both the P and SP ways for the selected equities, and a selection of measures, as the results are similar for the 20% and 80% estimates. As expected, the ES measures are always over the VaR estimates.

[INSERT FIGURE 10 AROUND HERE]

[INSERT FIGURE 11 AROUND HERE]

[INSERT FIGURE 12 AROUND HERE]

[INSERT FIGURE 13 AROUND HERE]

The SP approach by Engle and Manganelli (2004) catches the quantile really well in the for sample period. Meanwhile, that is based on P approach based on the Hansen's t distribution predicts higher risk. Anyway both measures have almost similar number of violations.

5. PM and portfolio analysis

We study the differences in sorting stocks according to the different strategies, as well as a performance measure to compare the different portfolio strategies.

5.1. In sample period

In order to study the alternative compositions that produce the PMs, we analyze the rankings for stocks. The portfolios are constructed using an equally weighted portfolio of the four stocks that best perform, so those are the ones that are more relevant to us. Table 5 exhibits the equities that appear a higher number of times from each strategy, under either P or SP approaches. If a number is repeated it means that takes simultaneously those positions, so the portfolio has more weight in that equity. The first difference is that according to the measures based on the CAViaR, the first asset (APPL), achieves the first position in the six selected measures. Regarding the P measures based on ES there are not many differences, against the results showed in the SP measures. Note that the P measure is a compound of the product of dynamic volatility and a given quantile of the distribution studied, while in the SP measures the quantiles are directly estimated for the return distribution. Under P, the volatility seems to have the biggest relative importance, so the quantile may not vary enough to produce different sorting. The equities that perform the best, as we will see below are the ones chosen with the SP VaR8020, which selects the stocks 1, 9 and 8 (APPL, SPG and SO) as the best in this period.

We analyze how different are the sorts made by these PMs, by using a measure based on the Euclidean norm. Thus, we divide the norm by the max of these values, and as a result we obtain a measure ranging between 0, (no difference), to 1, (completely different). Table 6 shows the matrix that produces these measures. As this measure is symmetric, the results are only displayed over the diagonal. There are two zeros in this matrix, corresponding to the ES9010 and the ES8020 with the parametric approach, and to the SR and ASR measured via CAViaR, which produce exactly the same results. The northeast matrix displays the differences between both approaches to obtain PMs. As expected, we find the biggest differences between both methods. The measures that sort completely different are those based in the ES8020 and ES9010 (which are the same), against the ES9010 based on the SP approach. Also these measures are the most different against the SP ones.

[INSERT TABLE 6 AROUND HERE]

Table 7 shows the number of days that the portfolio remain without rebalance. The measures that need less rebalance are the ones measured with ES and VaR ratios in the SP case, and the portfolio that best performs, based on the SP VaR8020, is also the one that needs less trades, and therefore has less transaction costs.

[INSERT TABLE 7 AROUND HERE]

5.1.1. Performance fee measure

The selected tool to evaluate the economic value to accounting for higher moments is the *performance fee* measure proposed by West et al. (1993) and Fleming, et al. (2001), following Jondeau and Rockinger (2012). It measures the management fee that an investor would pay to switch from a given strategy to a different one. This performance fee or opportunity cost, denoted by ϑ, is defined as the average return that has to be subtracted from the return of the strategy based in SR against the other strategies, such that the investor becomes indifferent to both strategies

$$E_t\left[U\left(1+\hat{r}_{p,t+1}\right)\right] = E_t\left[U\left(1+{r^*}_{p,t+1}-\vartheta\right)\right]$$

Where $\hat{r}_{p,t+1}$ is the portfolio return of the SR based strategy and ${r^*}_{p,t+1}$ is the portfolio return under the alternative strategy. The performance fee ϑ is obtained by solving this equation numerically.

To measure the utility, it can be expressed as infinite-order Taylor series expansion around the wealth at day t

$$U(W_{t+1}) = \sum_{k=0}^{\infty} \frac{U^{(k)}(W_t)}{k!}(W_{t+1}-W_t)^k$$

where $W_{t+1} - W_t = r_{p,t+1}$ is the portfolio return at day t+1, and $U^{(k)}$ is the kth derivative of the utility function. Then, the expected utility is given by

$$E_t[U(W_{t+1})] = E_t\left[\sum_{k=0}^{\infty} \frac{U^{(k)}(W_t) r_{p,t+1}^k}{k!}\right] = \sum_{k=0}^{\infty} \frac{U^{(k)}(W_t)}{k!} m_{p,t+1}^{(k)}$$

where $m_{p,t+1}^{(k)} = E_t\left[r_{p,t+1}^k\right]$ denotes the noncentered moments of order k. Following this approach, the expected utility depends on all the moments of the distribution of the portfolio return.

We aim to compare some measures that account for higher moments against SR, which only accounts explicitly for the first two moments. A Taylor series expansion up to the fourth order is given by

$$\hat{E}_t\left[U_{[4]}(W_{t+1})\right] = \varphi_0 + \varphi_1 m_{p,t+1}^{(1)} + \varphi_2 m_{p,t+1}^{(2)} + \varphi_3 m_{p,t+1}^{(3)} + \varphi_4 m_{p,t+1}^{(4)}$$

where $\varphi_k = \frac{1}{k!} U^{(k)}(W_t)$. Following Jondeau and Rockinger (2012), the parameters φ_k are calibrated using the power utility function $U(W_{t+1}) = W_{t+1}^{1-\gamma}/(1-\gamma)$, where $\gamma > 0$ $(\gamma \neq 1)$ is the relative risk aversion coefficient. In this case, the parameters are $\varphi_0 = 1/(1-\gamma)$, $\varphi_1 = 1$, $\varphi_2 = -\gamma/2$, $\varphi_3 = -\gamma(\gamma+1)/3!$, $\varphi_4 = \gamma(\gamma+1)(\gamma+2)/4!$.

The alternative strategies would be compared against the benchmark SR based strategy, which only accounts for the first two moments. So the Taylor series expansion becomes.

$$\hat{E}_t\left[U_{[2]}(W_{t+1})\right] = \varphi_0 + \varphi_1 m_{p,t+1}^{(1)} + \varphi_2 m_{p,t+1}^{(2)}$$

Table 8 exhibits the payment that an investor is willing to pay yearly to switch from a strategy based on the SR measure to the alternative. If the SR strategy is better than the alternative the payment is negative, hence, and hence, the investor would not change. It is shown that the strategy which deserves the highest pay is the one based on the VaR8020, of a 1,04% every year. Under SP this result is higher as the investor gets more risk averse.

[INSERT TABLE 8 AROUND HERE]

5.1.2. Dynamic Quantile test

As a measure of accuracy a Dynamic Quantile Test (DQ test) proposed by Engle and Manganelli (2004) is carried out. This test consists of using a linear regression model to link current to past violations, testing whether the null hypothesis of the Hit_t is uncorrelated with any variable that belongs to the information set $\mathcal{F}_{t-1}$ available when the VaR was calculated and have a mean value of zero, implying the absence of autocorrelation in the hits. If

$$Hit_t(\theta) = \begin{cases} 1-\theta & if\ r_t < VaR_{t|t-1}(\theta) \\ \theta & if\ r_t \geq VaR_{t|t-1}(\theta) \end{cases}$$

Consider the following linear regression model,

$$Hit_t(\theta) = \beta_0 + \sum_{i=1}^{p} \beta_i\, Hit_{t-i}(\theta) + \sum_{j=p+1}^{q} \beta_j\, X_j + e_t$$

where X_j is the vector of explanatory variables contained in $\mathcal{F}_{t-1}$. Engle and Manganelli (2004) suggest $X_1 = VaR(\theta)$. This means testing whether the probability of an exception depends on the level of the VaR. They derive the following Wald statistic for the DQ test:

$$\frac{\widehat{\beta}' X' X \hat{\beta}}{\theta(1-\theta)} \overset{a}{\sim} \mathcal{X}^2(p+q+1)$$

Under the null hypothesis $E[Hit_t(\theta)] = E(e_t) = 0,$ which implies $E[I_t(\theta)] = \theta,$ the hits are unbiased and uncorrelated.

As the core of this work is not related to perfectly estimate risk measures, this test is only made to account for the in sample behaviour. The instruments used in the regression are a constant, one lag of the hits and the corresponding quantile, and the results show that the SP passes the test 93% of the times, while the Pr models get an 83%.

5.1.3. Cumulative Returns behaviour

Figure 14 exhibits the cumulative portfolio returns under different measures and methods. There is a clear result, the PMs measured with the SP models outperform the best, with the PMs based on the VaR and ES ratios. The best portfolio is the one based on the VaR8020. This suggests that for an investor the information that has to care about is not in the extreme percentiles of the distribution (90 vs 10, or a mix of quantiles), but in the quantiles which are close to three fourths and one fourth of the return distribution. In this case, measuring the skewness and kurtosis of the individual assets does not seem to be very effective to select assets.

[INSERT FIGURE 14 AROUND HERE]

5.2. Out of sample period

The position of assets (Table 9) are very similar for the parametric measures except for the VaR ratios. Indeed the VaR9010 produces similar results to the ES ratios, while the VaR8020 changes the positions for the assets 2 to 4 that most appear. The SR and ASR remain unchanged under the SP framework. It is held that the first asset for every measure is again APPL, but there are differences in the other assets chosen for the VaR and ES ratios. The VaR8020, which makes the best performance during the in sample period, contains most of the time the two stocks APPL and T.

[INSERT TABLE 9 AROUND HERE]

Figure 15 displays in separated graphs the cumulative returns for different PMs under P and SP without reestimation. It is exhibited that during the first two years of the OOS the SP outperform the P models. Specifically, the best performance is according to ES8020. After this period, the same behaviour can be seen. The strategies of the SR, (according both P and SP methods) outperform the other measures, and at the end of the period the results change again. For the P models very similar results are obtained, while in the SP model the VaR8020 takes the best performance, corresponding to the same strategy leading in the IS.

[INSERT FIGURE 15 AROUND HERE]

Figure 16 only considers the SP models, which have been reestimated every 250 days. We would expect a different result due to the reestimation, but there are only slight differences. Depending on the investor's horizon the PM to select assets is different. Specifically, for 1 or 2 years the optimal strategy would be the ES8020, for 3 years it is not very clear which PM to be selected, and the VaR8020 would be chosen if the horizon were 4 years. Figure 17 displays the previous data from the point of view of an investor entering that year to the strategy, what let us know the benefits obtained in each strategy if we entered in a different year.

[INSERT FIGURE 16 AROUND HERE]

[INSERT FIGURE 17 AROUND HERE]

6. Conclusions

We show two methods, P and SP, to model different PMs to select assets.

We show that not only the PMs selected matter, but also the approach to estimate them is important as well. The family of VaR ratios provide a better performance, especially when measured for quantiles closer to the center of the distribution than the usual ones studied in risk measures, like the 5 or 1 percent confidence levels. For an investor, the relevant information relies on the quantiles that do not go far from the center of the returns distribution.

The SP measures seem to be more appropriate in this framework, not only for the higher returns these portfolios provide, but also because under this method the PMs exhibit less noise. Note that this leads to a less rebalancing strategy and therefore less transaction costs.

In the OOS the results are quite different. It is not so clear the strategy that should be used to select assets, the optimal PM seems to be here time dependent. We let the study of this behaviour for further research.

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8. Appendix

Bayesian estimation

Following Jondeau and Rockinger (2012), Bayesian estimation is used to obtain the parameter estimates, using Markov chain Monte Carlo (MCMC), and obtaining the chain using the Metropolis-Hastings algorithm. A description of this methodology is given below.

Lets denote $\theta^{(t)}$ the vector of parameters obtained at step t. In each step, a new guess X is generated for a vector of parameters. For the first step, $\theta^{(1)}$ is drawn from the asymptotically normal distribution of the ML estimated parameters, and next steps for $t > 1$ follow the next scheme:

1. Generate $X \sim q(x|\theta^{(t)})$.
2. Take

$$\theta^{(t+1)} = \begin{cases} X & with\, probability\; \rho(\theta^{(t)}, X), \\ \theta^{(t)} & with\, probability\; 1 - \rho(\theta^{(t)}, X) \end{cases}$$

Where

$$\rho(\theta^{(t)}, X) = min\left\{ \frac{p(X|y)q(\theta^{(t)}|X)}{p(\theta^{(t)}|y)q(X|\theta^{(t)})}, 1 \right\}.$$

$q(x|\theta^{(t)})$ is the proposal density, in this case the asymptotic multivariate normal distribution resulting from the ML estimation. $p(\theta|y)$ is the objective or tarjet density, with $y = \{r_t\}_{t=1}^{T}$. The target density is obtained or posterior distribution:

$$p(\theta|y) = L(y|\theta)f(\theta),$$

Where $L(y|\theta)$ is the data density or likelihood of the model (in this case of the Hansen's t), and $f(\theta)$ is the prior density of the parameter set.

In this case for each equity the chain samples 50.000 estimations, and after the burning period of 40.000 the mean and the standard deviation of the chain are taken as the estimated parameters and their standard deviation respectively. Comparing the new likelihood of the model against the previous with the parameters estimated via Nelder-Mead and Quasi-Newton let us know the improvement on the parameter estimation.

The prior distributions are chosen as in Jondeau and Rockinger (2009) to ensure that the model is stationary. Parameters of the GARCH processes are drawn from a $Beta(p, q)$, distributions to ensure positivity

$$f(\beta_0) = B(p_0, q_0), \quad f(\beta_1) = B(p_1, q_1),$$
$$f(\beta_2^+) = B(p_2^+, q_2^+), \quad f(\beta_2^-) = B(p_2^-, q_2^-),$$

Parameters p_i and q_i are chosen to ensure that their values are in the usual range for daily returns[4].

With the following prior distribution for the degrees of freedom, $\upsilon_{i,t}$ and the asymmetry parameter $\lambda_{i,t}$.

[4] The selected values are $p_0 = 2, q_0 = 50, p_1 = 150, q_1 = 15, p_2^+ = 2, q_2^+ = 20, p_2^- = 2.5, q_2^- = 20$.

$$f(\alpha_0) \propto N\left(\mu_{\alpha_0}, \sigma_{\alpha_0}^2\right) \qquad f(\gamma_0) \propto N\left(\mu_{\gamma_0}, \sigma_{\gamma_0}^2\right)$$
$$f(\alpha_1) \propto N\left(\mu_{\alpha_1}, \sigma_{\alpha_1}^2\right)\mathcal{I}_{\alpha_1 \in [-1,1]} \qquad f(\gamma_1) \propto N\left(\mu_{\gamma_1}, \sigma_{\gamma_1}^2\right)\mathcal{I}_{\gamma_1 \in [-1,1]}$$
$$f(\alpha_2^+) \propto N\left(\mu_{\alpha_2^+}, \sigma_{\alpha_2^+}^2\right) \qquad f(\gamma_2^+) \propto N\left(\mu_{\gamma_2^+}, \sigma_{\gamma_2^+}^2\right)$$
$$f(\alpha_2^-) \propto N\left(\mu_{\alpha_2^-}, \sigma_{\alpha_2^-}^2\right) \qquad f(\gamma_2^-) \propto N\left(\mu_{\gamma_2^-}, \sigma_{\gamma_2^-}^2\right)$$

These distributions are chosen as in Jondeau and Rockinger (2012), consistent with the null hypothesis that there are no dynamics in the conditional distribution, with a mean value of 0 for the parameters that drive the residuals, and a variance of 2. The mean value for the autoregressive parameters are also 0, but with a lower variance of 0.3, with a truncation to ensure that the range of this parameter is between -1 and 1. And for the constant terms the mean value is chosen to be similar to the previous estimations, and a large variance of 5.

Tables and Figures

This section shows the tables and figures containing the results.

Table 1 Summary statistics for market returns.

	APPL	JNJ	BRK.A	DIS	GE	KO	XOM	SO	SPG	DOW	T	JPM
Mean	27,24	3,42	6,15	2,85	0,85	1,01	3,69	7,04	5,84	0,67	0,85	1,39
SD	49,77	21,45	44,50	33,08	32,00	23,93	25,93	33,84	35,44	36,25	29,05	42,93
Skewness	-2,81	-0,36	0,08	-0,09	0,01	0,05	0,06	-0,21	0,25	-0,30	0,08	0,24
Kurtosis	77,49	14,51	10,70	10,74	10,28	9,73	11,88	11,67	20,98	10,18	7,96	13,68
JB	980.609,27	23.366,05	10.435,58	10.532,36	9.300,65	7.955,72	13.862,91	13.245,48	56.826,24	9.117,53	4.328,89	20.069,60
LB(1)	4,31	0,00	51,65	6,77	0,94	0,19	51,54	11,85	133,75	8,51	5,96	24,40
Engle(1)	3,02	59,93	138,90	62,09	256,41	194,87	346,12	210,23	525,38	115,96	84,02	436,25
$\rho(r)$	-0,03	0,00	-0,11	-0,04	-0,01	-0,01	-0,11	-0,05	-0,18	-0,04	-0,04	-0,08
$\rho(r^2)$	0,03	0,12	0,18	0,12	0,25	0,21	0,29	0,22	0,35	0,17	0,14	0,32

This table reports summary statistics for the market returns on the in sample period: The annualized average return and standard deviation, the skewness and kurtosis, the Jarque-Bera test (JB), the Ljung-Box test statistic for no serial correlation (LB(1)), the Engle test (Engle(1)) for no serial correlation in squared returns, the first-order serial correlation of returns $\rho(r)$ and of the squared returns $\rho(r^2)$. The critical values at 5% are 5,99 and 3,84 respectively for the JB test and the Ljung-Box and Engle

Table 2 Parameter estimation of the GARCH model.

		Mean	SD	5%	Median	95%
APPL	β_0	0,0821	0,0072	0,0722	0,0803	0,0959
	β_1	0,9524	0,0018	0,9499	0,9523	0,9551
	β_2^+	0,0288	0,0045	0,0234	0,0273	0,0375
	β_2^-	0,0650	0,0024	0,0603	0,0650	0,0691
JNJ	β_0	0,0110	0,0048	0,0041	0,0108	0,0198
	β_1	0,9060	0,0086	0,8924	0,9071	0,9189
	β_2^+	0,0374	0,0019	0,0343	0,0376	0,0403
	β_2^-	0,1628	0,0181	0,1339	0,1626	0,1926
BRK.A	β_0	0,0422	0,0058	0,0325	0,0421	0,0503
	β_1	0,9297	0,0026	0,9254	0,9298	0,9347
	β_2^+	0,0173	0,0036	0,0098	0,0184	0,0217
	β_2^-	0,0491	0,0063	0,0387	0,0488	0,0609
DIS	β_0	0,0117	0,0018	0,0085	0,0122	0,0138
	β_1	0,9495	0,0041	0,9446	0,9480	0,9565
	β_2^+	0,0316	0,0030	0,0274	0,0311	0,0360
	β_2^-	0,0743	0,0049	0,0653	0,0759	0,0810
GE	β_0	0,0251	0,0072	0,0145	0,0249	0,0357
	β_1	0,9070	0,0142	0,8844	0,9064	0,9268
	β_2^+	0,0476	0,0113	0,0307	0,0495	0,0656
	β_2^-	0,1335	0,0169	0,1102	0,1336	0,1615
KO	β_0	0,0087	0,0036	0,0028	0,0088	0,0146
	β_1	0,9116	0,0132	0,8904	0,9111	0,9339
	β_2^+	0,0808	0,0150	0,0551	0,0813	0,1054

	β_2^-	0,1311	0,0275	0,0857	0,1314	0,1760
XOM	β_0	0,0127	0,0072	0,0029	0,0114	0,0260
	β_1	0,9275	0,0051	0,9178	0,9285	0,9340
	β_2^+	0,0443	0,0041	0,0365	0,0455	0,0494
	β_2^-	0,1421	0,0089	0,1277	0,1421	0,1576
SO	β_0	0,0688	0,0109	0,0527	0,0677	0,0838
	β_1	0,9375	0,0061	0,9258	0,9389	0,9457
	β_2^+	0,0056	0,0040	0,0010	0,0047	0,0157
	β_2^-	0,0708	0,0062	0,0635	0,0692	0,0822
SPG	β_0	0,0148	0,0038	0,0081	0,0154	0,0210
	β_1	0,8780	0,0186	0,8466	0,8782	0,9087
	β_2^+	0,1163	0,0105	0,0984	0,1174	0,1328
	β_2^-	0,1878	0,0209	0,1547	0,1873	0,2241
DOW	β_0	0,0423	0,0058	0,0325	0,0421	0,0503
	β_1	0,9297	0,0026	0,9254	0,9298	0,9347
	β_2^+	0,0173	0,0036	0,0098	0,0184	0,0217
	β_2^-	0,0491	0,0063	0,0387	0,0488	0,0610
T	β_0	0,0293	0,0064	0,0169	0,0299	0,0389
	β_1	0,9173	0,0111	0,9002	0,9164	0,9381
	β_2^+	0,0770	0,0058	0,0683	0,0765	0,0865
	β_2^-	0,0684	0,0214	0,0302	0,0706	0,1026
JPM	β_0	0,0076	0,0022	0,0040	0,0074	0,0109
	β_1	0,9371	0,0061	0,9269	0,9370	0,9472
	β_2^+	0,0195	0,0090	0,0103	0,0159	0,0376
	β_2^-	0,1140	0,0102	0,0949	0,1152	0,1288

This table reports the Bayesian parameter estimates for the asymmetric GARCH process. The Mean and SD columns make reference to the mean and standard deviation of the posterior distribution of the parameters, and the last two columns contain the 5%, median and 95% quantiles of the distribution.

Table 3. Estimates of the Skew-t dynamic parameters

		Mean	SD	0,0500	Median	0,9500
APPL	α_0	-3,1813	0,1262	-3,3395	-3,2142	-2,9669
	α_1	0,0443	0,0085	0,0309	0,0455	0,0557
	α_2^+	0,1340	0,0106	0,1108	0,1378	0,1459
	α_2^-	-0,6504	0,0444	-0,7238	-0,6477	-0,5958
	γ_0	-0,0197	0,0099	-0,0339	-0,0198	-0,0043
	γ_1	-0,0262	0,0158	-0,0510	-0,0278	-0,0039
	γ_2^+	-0,0901	0,0197	-0,1147	-0,0970	-0,0490
	γ_2^-	0,1782	0,0090	0,1652	0,1796	0,1919
JNJ	α_0	-1,7066	0,0102	-1,7214	-1,7072	-1,6884
	α_1	-0,0026	0,0047	-0,0106	-0,0029	0,0059
	α_2^+	0,2081	0,0110	0,1887	0,2084	0,2231
	α_2^-	0,0094	0,0264	-0,0337	0,0052	0,0500
	γ_0	-0,0078	0,0037	-0,0115	-0,0087	0,0009
	γ_1	0,0212	0,0134	0,0006	0,0215	0,0434
	γ_2^+	0,1323	0,0069	0,1208	0,1319	0,1439
	γ_2^-	-0,0120	0,0074	-0,0222	-0,0138	0,0024
BRK.A	α_0	-1,3087	0,2019	-1,5538	-1,3421	-0,8604
	α_1	0,4577	0,0813	0,3449	0,4462	0,6326
	α_2^+	-0,4820	0,1329	-0,7779	-0,4590	-0,2677
	α_2^-	-0,6710	0,1292	-0,8836	-0,6381	-0,4963
	γ_0	0,1274	0,0120	0,1104	0,1256	0,1459
	γ_1	0,8322	0,1467	0,5737	0,8207	1,1194
	γ_2^+	-0,4836	0,0689	-0,6095	-0,4801	-0,3604
	γ_2^-	0,5489	0,0404	0,4854	0,5483	0,6292
DIS	α_0	-1,9974	0,0076	-2,0080	-1,9990	-1,9837
	α_1	0,1144	0,0247	0,0781	0,1152	0,1603
	α_2^+	0,0732	0,0070	0,0630	0,0720	0,0854
	α_2^-	0,0017	0,0145	-0,0263	0,0028	0,0221
	γ_0	0,0852	0,0007	0,0839	0,0854	0,0861
	γ_1	-0,1158	0,0039	-0,1220	-0,1160	-0,1099
	γ_2^+	-0,1400	0,0076	-0,1504	-0,1426	-0,1256
	γ_2^-	0,1529	0,0138	0,1324	0,1542	0,1803
GE	α_0	-2,2557	0,0486	-2,3440	-2,2441	-2,1948
	α_1	0,0792	0,0161	0,0564	0,0786	0,1060
	α_2^+	0,0258	0,0127	0,0064	0,0236	0,0456
	α_2^-	-0,6455	0,0487	-0,7118	-0,6462	-0,5683
	γ_0	-0,0119	0,0139	-0,0306	-0,0134	0,0092
	γ_1	-0,0539	0,0227	-0,0869	-0,0519	-0,0206
	γ_2^+	0,0653	0,0284	0,0186	0,0700	0,1048
	γ_2^-	0,0800	0,0063	0,0674	0,0796	0,0892
KO	α_0	-2,1516	0,0110	-2,1693	-2,1527	-2,1342
	α_1	0,1992	0,0470	0,1248	0,1998	0,2757
	α_2^+	-0,2432	0,0769	-0,3792	-0,2417	-0,1236

	α_2^-	-0,4825	0,0798	-0,6216	-0,4816	-0,3595
	γ_0	0,0351	0,0580	-0,0626	0,0360	0,1292
	γ_1	-0,0034	0,0158	-0,0319	-0,0003	0,0205
	γ_2^+	-0,3054	0,1885	-0,6280	-0,3009	-0,0053
	γ_2^-	0,0174	0,0121	-0,0049	0,0195	0,0342
XOM	α_0	-2,4302	0,0841	-2,5779	-2,4331	-2,3067
	α_1	-0,1000	0,0110	-0,1173	-0,1004	-0,0828
	α_2^+	0,1847	0,0611	0,0880	0,1895	0,2846
	α_2^-	-1,6288	0,0955	-1,7720	-1,6234	-1,4642
	γ_0	-0,2780	0,0294	-0,3294	-0,2797	-0,2333
	γ_1	0,1213	0,0205	0,0871	0,1210	0,1533
	γ_2^+	-0,1451	0,0178	-0,1743	-0,1454	-0,1195
	γ_2^-	0,1537	0,0425	0,0859	0,1580	0,2230
SO	α_0	-1,8936	0,2621	-2,2374	-1,8616	-1,4602
	α_1	0,0327	0,0115	0,0140	0,0318	0,0492
	α_2^+	0,4471	0,0834	0,3128	0,4319	0,5810
	α_2^-	-0,1053	0,1744	-0,3425	-0,0934	0,2021
	γ_0	0,0119	0,0181	-0,0140	0,0097	0,0417
	γ_1	0,1811	0,0499	0,1088	0,1767	0,2658
	γ_2^+	-0,2182	0,0432	-0,2990	-0,2070	-0,1621
	γ_2^-	0,2665	0,0763	0,1607	0,2706	0,4058
SPG	α_0	-0,7008	0,8546	-2,0754	-0,7175	0,7346
	α_1	-0,3201	0,3115	-0,8452	-0,3133	0,1784
	α_2^+	0,3964	0,1148	0,2026	0,3980	0,5822
	α_2^-	0,0245	0,1213	-0,1659	0,0311	0,2220
	γ_0	-0,0709	0,1258	-0,2794	-0,0704	0,1342
	γ_1	0,0981	0,1536	-0,1547	0,0999	0,3492
	γ_2^+	0,1262	0,1404	-0,1093	0,1287	0,3537
	γ_2^-	0,3467	0,0795	0,2231	0,3439	0,4833
DOW	α_0	-1,3087	0,2019	-1,5539	-1,3416	-0,8605
	α_1	0,4577	0,0813	0,3449	0,4463	0,6326
	α_2^+	-0,4820	0,1329	-0,7780	-0,4592	-0,2677
	α_2^-	-0,6710	0,1292	-0,8835	-0,6381	-0,4963
	γ_0	0,1274	0,0120	0,1104	0,1256	0,1459
	γ_1	0,8322	0,1467	0,5737	0,8206	1,1192
	γ_2^+	-0,4836	0,0689	-0,6095	-0,4801	-0,3603
	γ_2^-	0,5489	0,0404	0,4854	0,5483	0,6293
T	α_0	-2,1167	0,0901	-2,2676	-2,1159	-1,9699
	α_1	0,0533	0,0483	-0,0261	0,0516	0,1352
	α_2^+	0,4884	0,0586	0,3942	0,4870	0,5904
	α_2^-	-0,0680	0,0718	-0,2016	-0,0681	0,0629
	γ_0	-0,0976	0,0238	-0,1362	-0,0971	-0,0590
	γ_1	-0,0815	0,0351	-0,1373	-0,0768	-0,0255
	γ_2^+	0,2885	0,0568	0,2002	0,2841	0,3921
	γ_2^-	0,0954	0,0216	0,0573	0,0961	0,1325
JPM	α_0	-1,3636	0,5390	-2,2914	-1,3432	-0,5148

α_1	-0,1985	0,3609	-0,8274	-0,1910	0,3832
α_2^+	0,3135	0,1879	-0,0047	0,3388	0,5914
α_2^-	-0,8658	0,4371	-1,6297	-0,8468	-0,1943
γ_0	0,0391	0,1031	-0,1530	0,0768	0,1729
γ_1	-0,1517	0,2320	-0,4158	-0,2310	0,2699
γ_2^+	0,2396	0,5020	-0,5664	0,2273	1,0992
γ_2^-	-0,0452	0,6100	-1,0190	-0,0674	1,0182

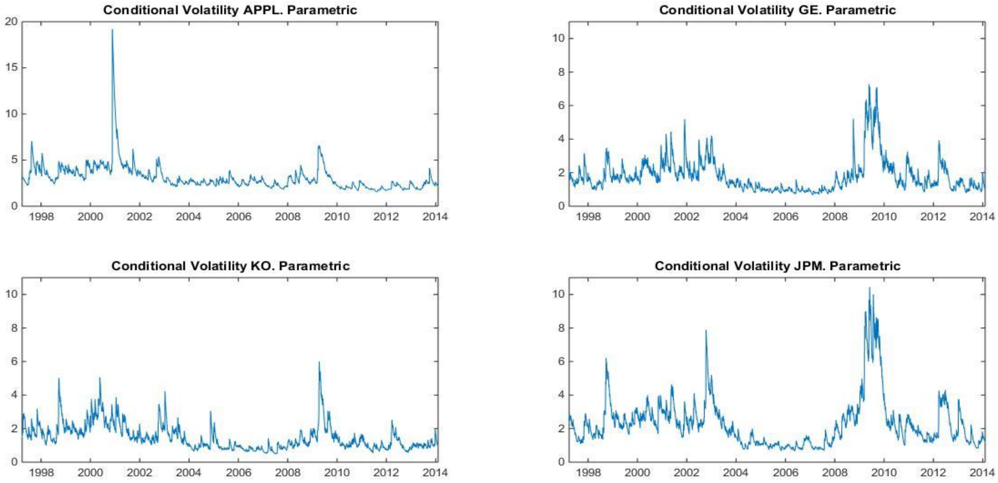
This table reports the Bayesian parameter estimates for the time-varying parameters of Skew-t distribution. The Mean and SD columns make reference to the mean and standard deviation of the posterior distribution of the parameters, and the last two columns contain the 5%, median and 95% quantiles of the distribution.

Table 4. Estimates of the parameters of the semiparametric model

		q0		q1		q2		q3		q4	
		parameter	SD	parameter	SD	parameter	SD	parameter	SD	parameter	SD
APPL	β_0	0,0529	0,0333	-0,0118	0,0333	-0,0090	0,0147	-0,0046	0,0029	-0,0393	0,0220
	β_1	0,9465	0,0089	0,9427	0,0089	0,9420	0,0567	0,9852	0,0042	0,9591	0,0123
	β_2	0,1072	0,0112	0,0458	0,0112	0,0036	0,0065	-0,0101	0,0030	-0,0951	0,0283
JNJ	β_0	0,0204	0,0076	0,0048	0,0076	0,0105	0,0080	-0,0052	0,0114	-0,0587	0,0203
	β_1	0,9419	0,0056	0,9397	0,0056	0,0975	0,4645	0,8742	0,0487	0,8999	0,0177
	β_2	0,1298	0,0120	0,0396	0,0120	-0,0271	0,0092	-0,0974	0,0393	-0,2102	0,0277
BRK.A	β_0	0,0963	0,0546	0,0173	0,0546	0,0000	0,0000	-0,0307	0,0088	-0,0100	0,0197
	β_1	0,9214	0,0236	0,8860	0,0236	-0,8618	0,0307	0,9174	0,0227	0,9502	0,0117
	β_2	0,1610	0,0476	0,0756	0,0476	0,0000	0,0000	-0,0454	0,0156	-0,1426	0,0309
DIS	β_0	0,1302	0,0531	0,0002	0,0531	0,0000	0,0001	-0,0080	0,0054	-0,0189	0,0145
	β_1	0,8728	0,0234	0,9298	0,0234	0,9921	0,4666	0,9429	0,0154	0,9562	0,0116
	β_2	0,2420	0,0351	0,0544	0,0351	0,0000	0,0001	-0,0396	0,0110	-0,1050	0,0289
GE	β_0	0,0204	0,0177	0,0025	0,0177	0,0000	0,0124	-0,0219	0,0116	-0,0656	0,0375
	β_1	0,9250	0,0164	0,9547	0,0164	-0,9680	0,0000	0,8853	0,0302	0,8924	0,0267
	β_2	0,1813	0,0458	0,0327	0,0458	0,0000	0,0035	-0,0748	0,0256	-0,2432	0,0484
KO	β_0	0,0586	0,0330	-0,0003	0,0330	0,0000	0,0000	-0,0048	0,0046	-0,0348	0,0133
	β_1	0,8931	0,0377	0,9589	0,0377	-0,5063	0,8280	0,9243	0,0206	0,9298	0,0151
	β_2	0,2218	0,0844	0,0327	0,0844	0,0000	0,0000	-0,0545	0,0154	-0,1571	0,0365
XOM	β_0	0,0892	0,0569	0,0047	0,0569	0,0000	0,0000	-0,0183	0,0129	-0,0621	0,0128
	β_1	0,8886	0,0436	0,9528	0,0436	-0,4734	0,6900	0,8969	0,0306	0,9025	0,0104
	β_2	0,2259	0,1167	0,0324	0,1167	0,0000	0,0000	-0,0730	0,0236	-0,1965	0,0177
SO	β_0	0,0909	0,0230	-0,0030	0,0230	-0,0025	0,0031	-0,0160	0,0264	-0,0333	0,0348
	β_1	0,9193	0,0189	0,9368	0,0189	-0,7769	0,1902	0,9215	0,0313	0,9455	0,0180
	β_2	0,1560	0,0417	0,0499	0,0417	-0,0027	0,0027	-0,0565	0,0214	-0,1207	0,0410
SPG	β_0	0,0438	0,0080	-0,0086	0,0080	-0,0020	0,0039	-0,0156	0,0217	-0,0696	0,0266
	β_1	0,9445	0,0043	0,9237	0,0043	0,9622	0,0800	0,9072	0,0430	0,8642	0,0347
	β_2	0,1110	0,0100	0,0648	0,0100	0,0012	0,0023	-0,0652	0,0264	-0,3280	0,1028
DOW	β_0	0,0154	0,0314	0,0089	0,0314	0,0000	0,0000	-0,0149	0,0101	-0,0214	0,0128
	β_1	0,9541	0,0199	0,9404	0,0199	-0,6433	0,5644	0,9109	0,0277	0,9506	0,0104
	β_2	0,1128	0,0395	0,0408	0,0395	0,0000	0,0000	-0,0622	0,0189	-0,1158	0,0177
T	β_0	0,0224	0,0131	-0,0004	0,0131	0,0000	0,0000	-0,0169	0,0072	-0,0105	0,0348
	β_1	0,9567	0,0065	0,9582	0,0065	-0,5761	0,6645	0,9159	0,0166	0,9529	0,0180
	β_2	0,1008	0,0125	0,0332	0,0125	0,0000	0,0000	-0,0540	0,0149	-0,1160	0,0410
JPM	β_0	0,0453	0,0232	0,0005	0,0232	0,0000	0,0001	-0,0224	0,0152	-0,0207	0,0266
	β_1	0,9208	0,0103	0,9373	0,0103	0,9928	0,5678	0,8670	0,0191	0,9143	0,0347
	β_2	0,1782	0,0120	0,0486	0,0120	0,0000	0,0002	-0,0916	0,0136	-0,2190	0,1028

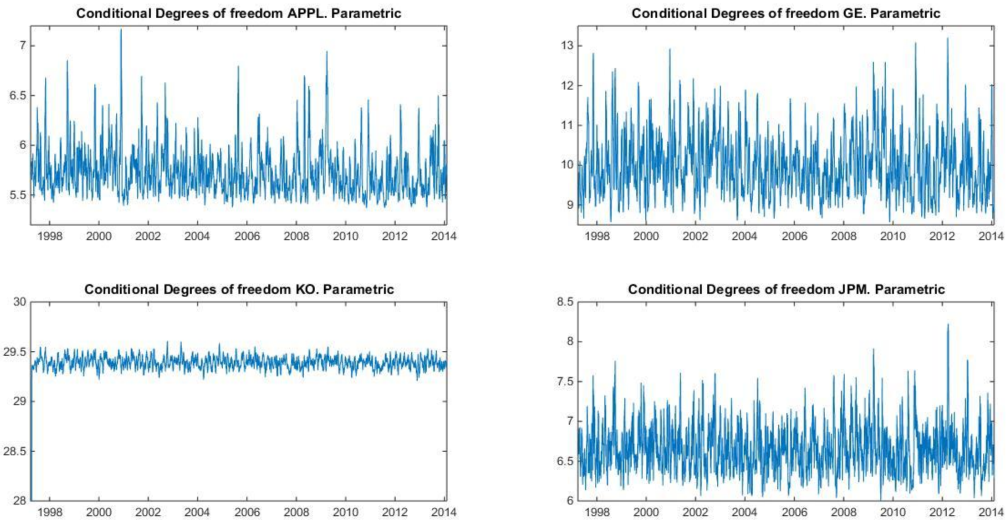
This table reports the parameters (parameter) and their standard deviation (SD) estimated with Nelder-Mead algorithm (following Engle and Manganelli 2004) for the quantiles 0,025 (q0) , 0,25 (q1), 0,5 (q2), 0,75 (q3), 0,975 (q4).

Figure 1. Conditional Volatility parametric approach



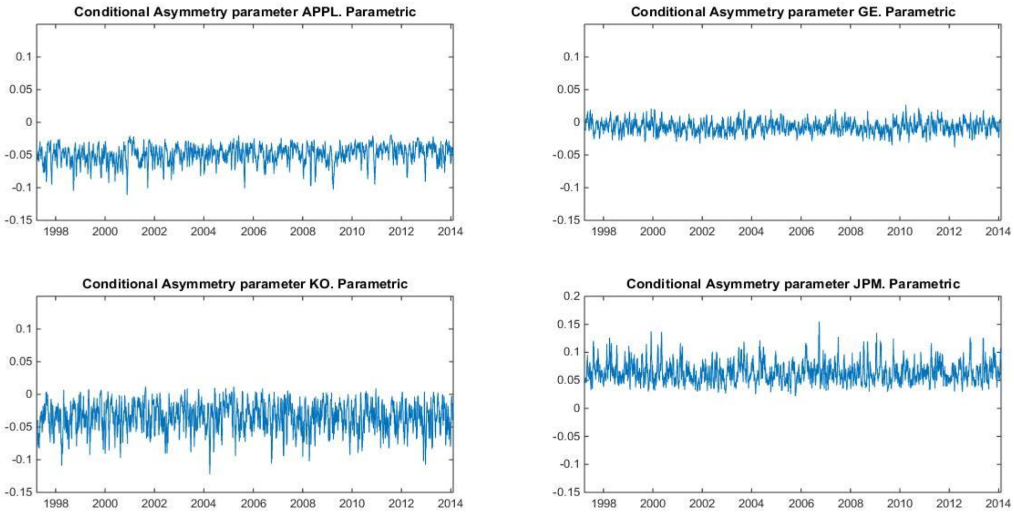
This figure displays the conditional Volatility resulting from the parameter estimates reported in table 2 for 4 selected stocks of the sample.

Figure 2. Degree of Freedom parameter parametric approach



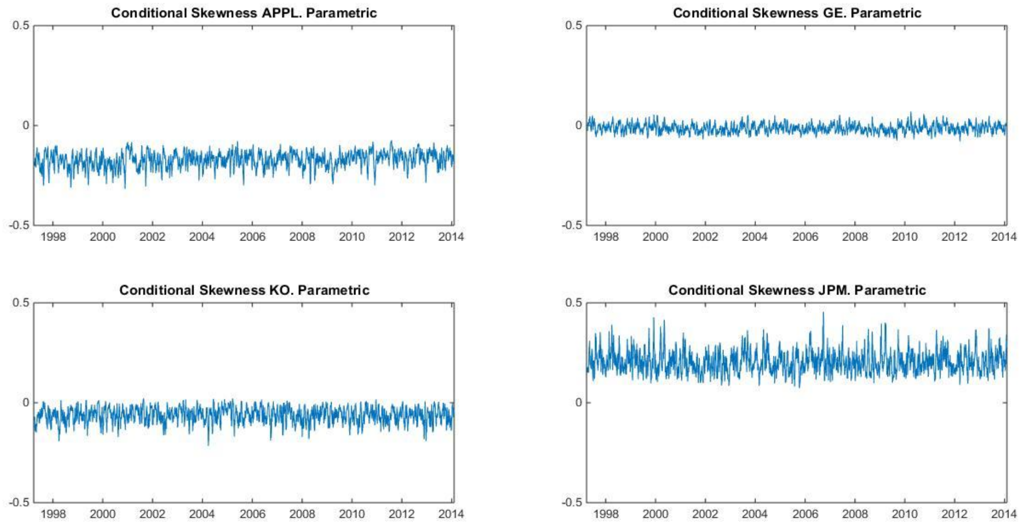
This figure displays the conditional degrees of freedom resulting from the parameter estimates reported in table 3 for 4 selected stocks of the sample. The series are smoothed using a 2 week simple moving average following Jondeau and Rockinger 2012.

Figure 3. Asymmetry parameter parametric approach



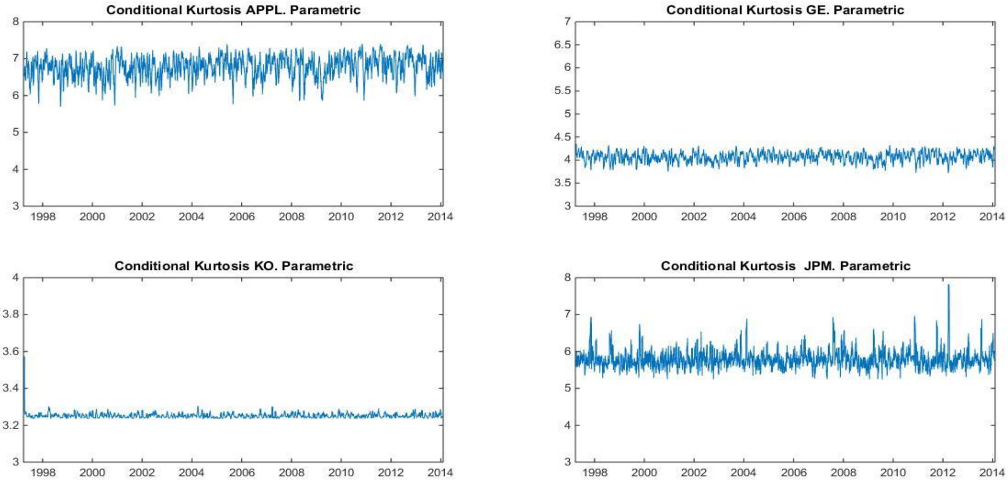
This figure displays the conditional asymmetry parameter resulting from the parameter estimates reported in table 3 for 4 selected stocks of the sample. The series are smoothed using a 2 week simple moving average.

Figure 4. Conditional Skewness parametric approach



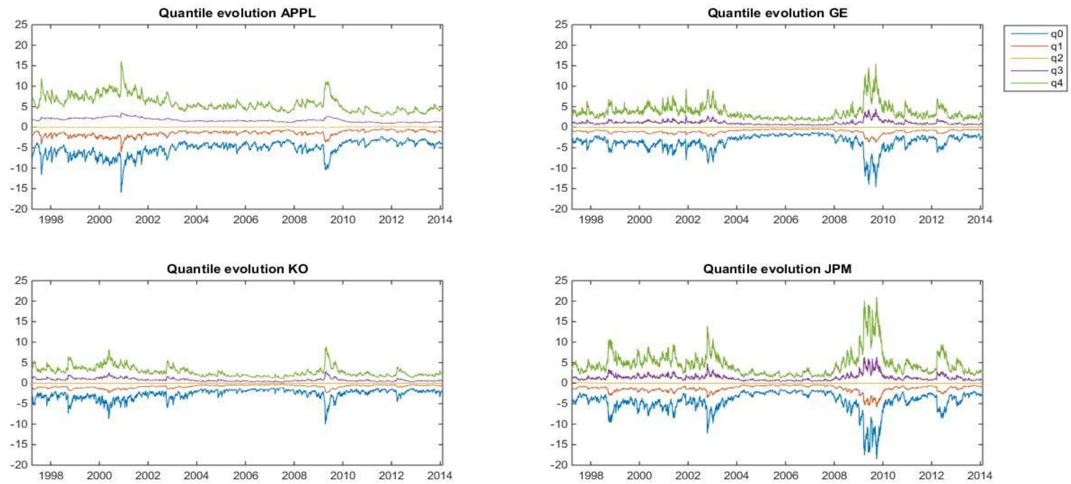
This figure displays the conditional Skewness resulting from the parameter estimates reported in table 3 for 4 selected stocks of the sample. The series are smoothed using a 2 week simple moving average.

Figure 5. Conditional Kurtosis parametric approach



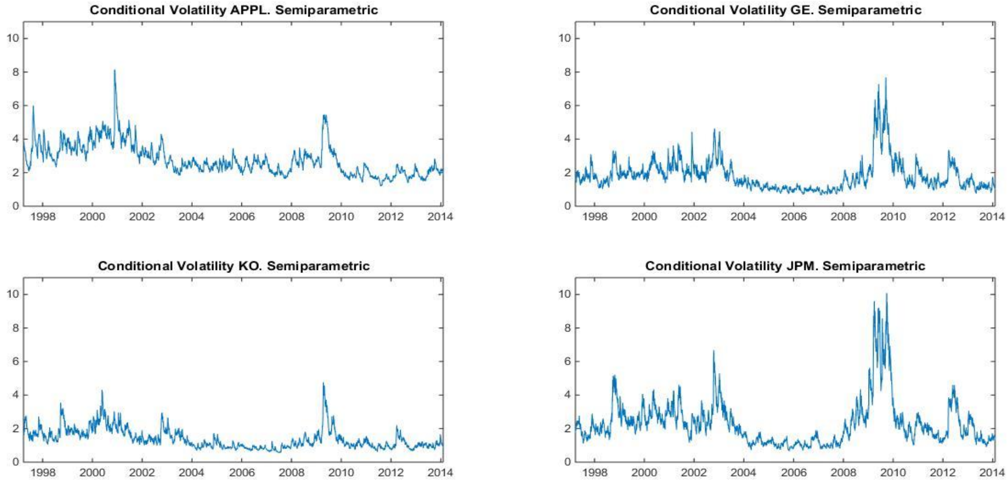
This figure displays the conditional Kurtosis resulting from the parameter estimates reported in table 3 for 4 selected stocks of the sample. The series are smoothed using a 2 week simple moving average.

Figure 6. Quantile evolution semiparametric approach



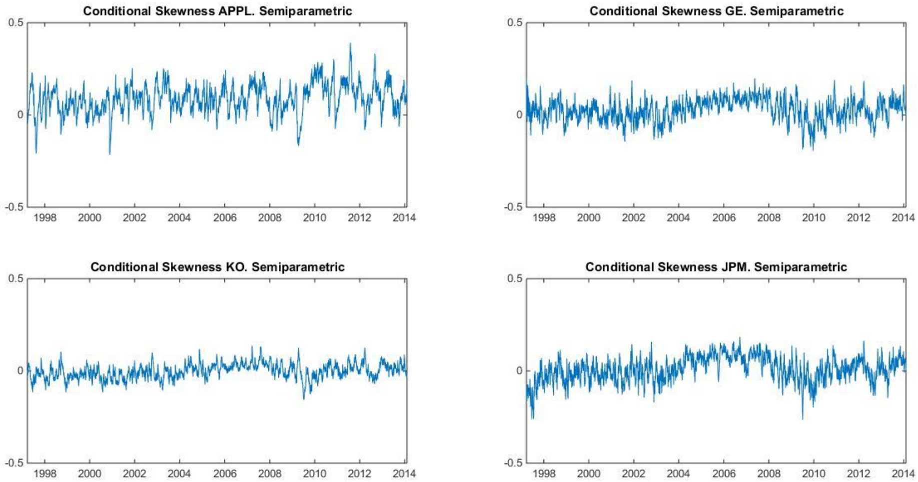
This figure displays the conditional quantiles at the confidence level of 0,025, 0,25, 0,5, 0,75, 0,975 resulting from the parameter estimates reported in table 4 for 4 selected stocks of the sample.

Figure 7. Conditional Volatility semiparametric approach



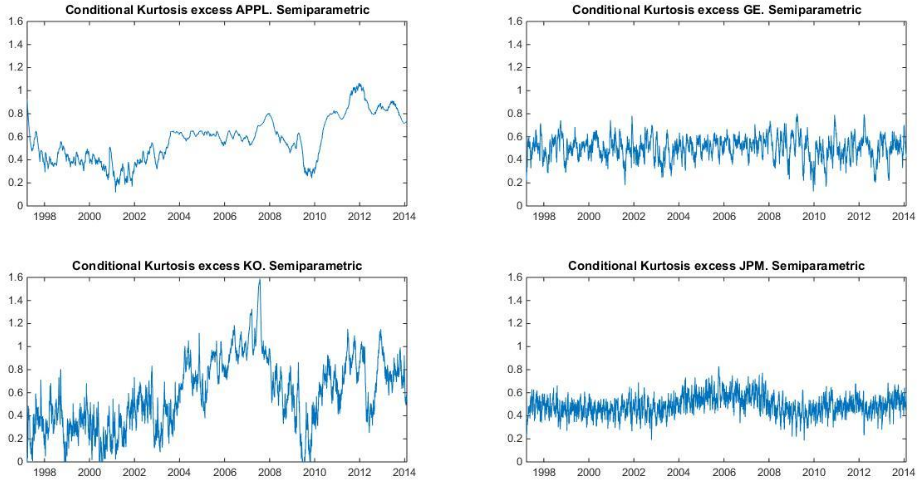
This figure displays the conditional Volatility based on quantiles resulting from the parameter estimates reported in table 4 for 4 selected stocks of the sample.

Figure 8. Conditional Skewness semiparametric approach



This figure displays the conditional skewness based on quantiles resulting from the parameter estimates reported in table 4 for 4 selected stocks of the sample.

Figure 9. Conditional excess of Kurtosis semiparametric approach

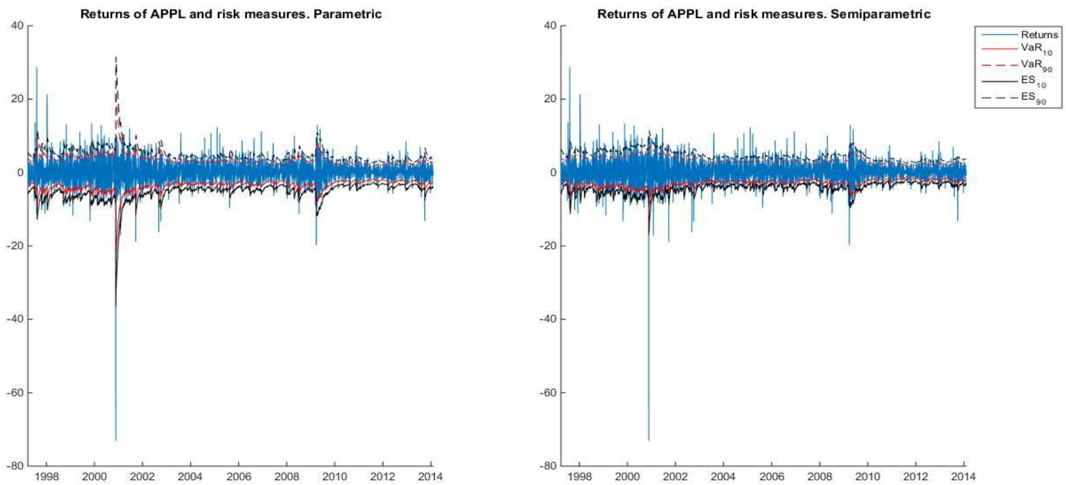


This figure displays the conditional excess of kurtosis based on quantiles resulting from the parameter estimates reported in table 4 for 4 selected stocks of the sample.

7.2. Risk measures

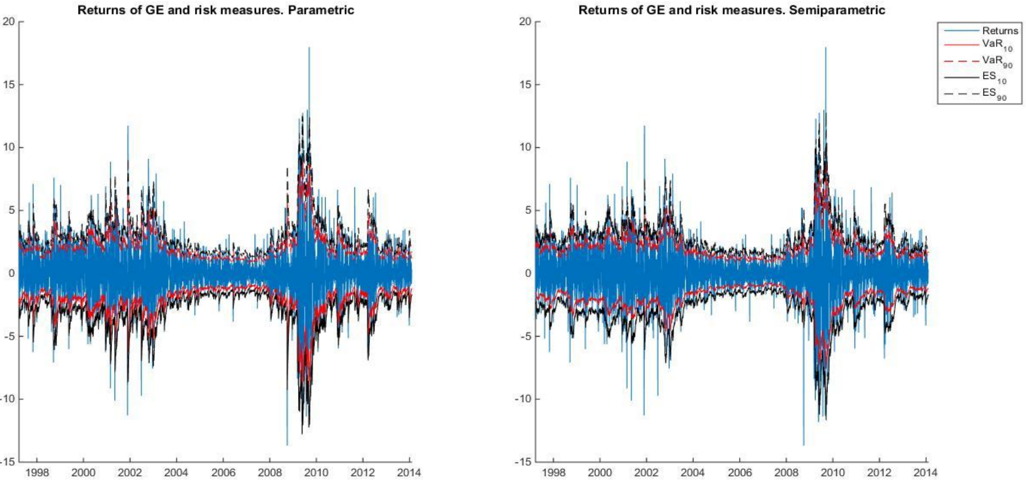
Next in this part of the appendix figures of the VaR and ES risk measures for selected confidence levels are displayed.

Figure 10. VaR and ES estimates of APPL



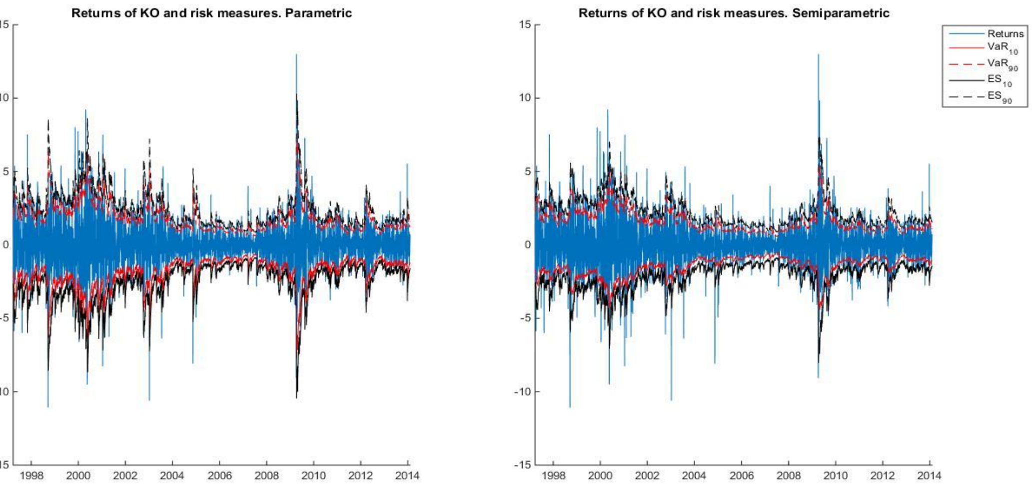
This figure displays the returns of APPL and the estimated VaR in the upper (dashed line) and lower tails (solid lines) (at 90% and 10% confidence level respectively) in red, and the corresponding ES measures in black, estimated with the P approach (left figure) and with the SP approach (right figure) .

Figure 11. VaR and ES estimates of GE



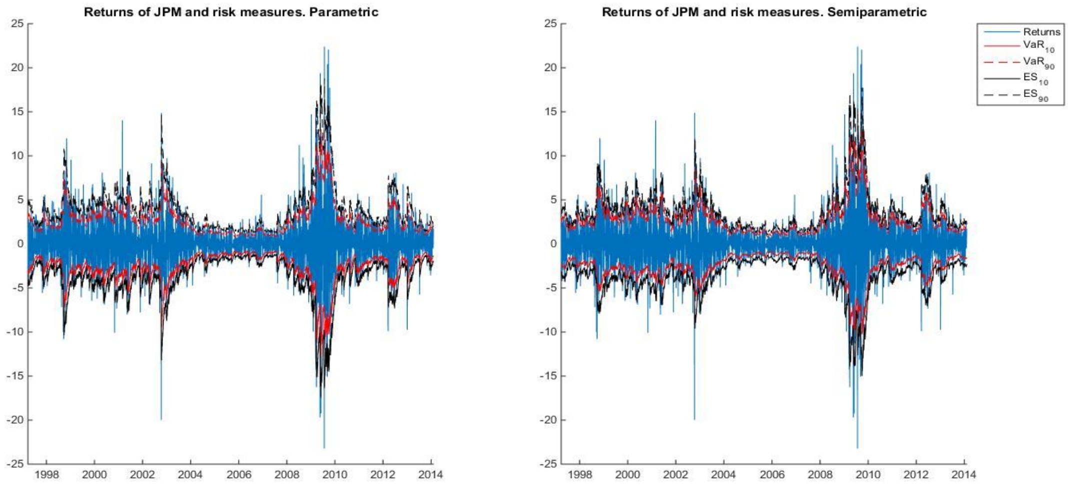
This figure displays the returns of GE and the estimated VaR in the upper (dashed line) and lower tails (solid lines) (at 90% and 10% confidence level respectively) in red, and the corresponding ES measures in black, estimated with the P approach (left figure) and with the SP approach (right figure).

Figure 12. VaR and ES estimates of KO



This figure displays the returns of KO and the estimated VaR in the upper (dashed line) and lower tails (solid lines) (at 90% and 10% confidence level respectively) in red, and the corresponding ES measures in black, estimated with the P approach (left figure) and with the SP approach (right figure).

Figure 13. VaR and ES estimates of JPM



This figure displays the returns of JPM and the estimated VaR in the upper (dashed line) and lower tails (solid lines) (at 90% and 10% confidence level respectively) in red, and the corresponding ES measures in black, estimated with the P approach (left figure) and with the SP approach (right figure).

Table 5. Equity positioning, in sample period

	Parametric						Semiparametric					
Position	SR	ASR	ES8020	ES9010	VaR8020	VaR9010	SR	ASR	ES8020	ES9010	VaR8020	VaR9010
1	3	1	12	12	7	6	1	1	1	1	1	1
2	2	2	12	12	1	4	2	2	4	1	9	1
3	3	3	2	2	9	4	3	3	4	6	8	4
4	4	4	2	2	12	5	4	4	6	6	8	2
5	5	5	5	5	8	5	5	5	6	6	8	2
6	6	6	1	1	2	2	6	6	6	10	8	2
7	7	7	1	1	2	8	7	7	6	10	10	2
8	8	8	1	1	5	8	8	8	10	11	10	6
9	9	9	8	8	5	8	9	9	10	11	10	10
10	10	10	7	7	4	1	10	10	10	11	6	10
11	11	11	7	7	4	1	11	11	7	7	6	10
12	3	3	3	3	6	7	12	12	3	7	3	3

Numeration	1	2	3	4	5	6	7	8	9	10	11	12
	APPL	JNJ	BRK.A	DIS	GE	KO	XOM	SO	SPG	DOW	T	JPM

This table displays the position, in descending order, that each equity has occupied for the longest time during the in sample period for both P and SP approaches of measuring the PMs.

Table 6. Differences in sorting, in sample period

		Parametric						Semiparametric					
		1	2	3	4	5	6	7	8	9	10	11	12
Paramatric	1	0	0,083	0,746	0,746	0,703	0,631	0,383	0,383	0,250	0,395	0,529	0,338
	2	0	0	0,791	0,791	0,728	0,652	0,374	0,374	0,235	0,386	0,523	0,328
	3	0	0	0	0	0,787	0,720	0,875	0,875	0,778	1,000	0,861	0,725
	4	0	0	0	0	0,787	0,720	0,875	0,875	0,778	1,000	0,861	0,725
	5	0	0	0	0	0	0,536	0,759	0,759	0,652	0,771	0,701	0,725
	6	0	0	0	0	0	0	0,664	0,664	0,570	0,673	0,585	0,689
Semiparametric	7	0	0	0	0	0	0	0	0	0,442	0,405	0,643	0,497
	8	0	0	0	0	0	0	0	0	0,442	0,405	0,643	0,497
	9	0	0	0	0	0	0	0	0	0	0,333	0,386	0,412
	10	0	0	0	0	0	0	0	0	0	0	0,463	0,611
	11	0	0	0	0	0	0	0	0	0	0	0	0,720
	12	0	0	0	0	0	0	0	0	0	0	0	0

Numeration	1	2	3	4	5	6	7	8	9	10	11	12
	SR	ASR	ES8020	ES9010	VaR8020	VaR9010	SR	ASR	ES8020	ES9010	VaR8020	VaR9010

This table displays the differences in sorting with the different PMs, measured as the Euclidean norm divided by the maximum value, so it ranges from 0 (minimum difference) to 1 (maximum difference).

Table 7. Number of days (%), in which the portfolio remains unchanged during the in sample

Parametric						Semiparametric					
SR	ASR	ES8020	ES9010	VaR8020	VaR9010	SR	ASR	ES8020	ES9010	VaR8020	VaR9010
9,43	9,25	22,53	22,59	35,35	21,85	9,12	9,11	53,36	53,68	55,41	52,85

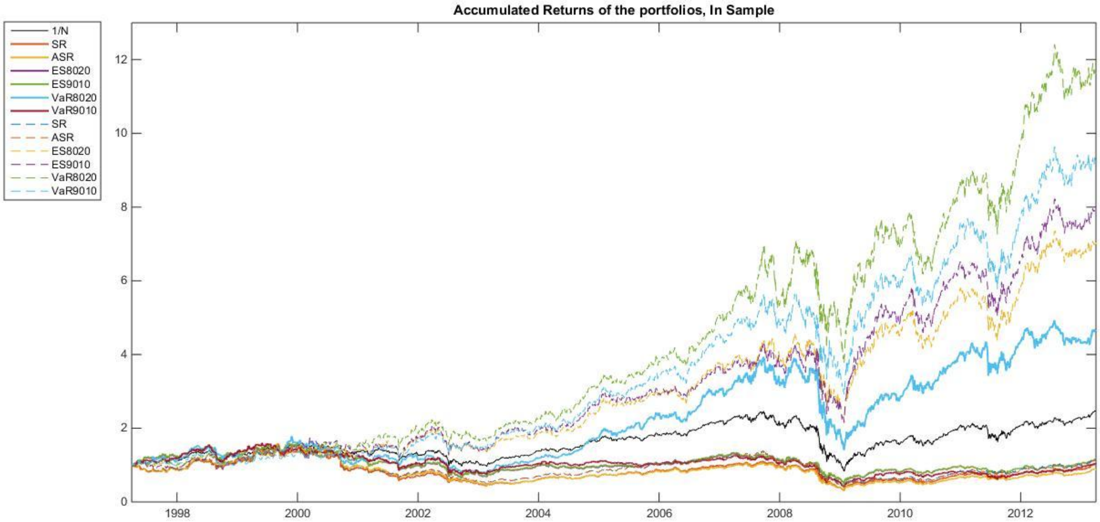
This table displays the number of days as a percentage that the assets remain the same, so the portfolios does not need a rebalance.

Table 8. Perfomance fee measure (%), In sample period

		ASR	ES8020	ES9010	VaR8020	VaR9010
Parametric	γ = 5	-0,0005	0,0006	0,0006	0,0065	-0,0001
	γ = 10	-0,0004	0,0014	0,0014	0,0067	0,0007
	γ = 15	0,0007	-0,0007	-0,0007	0,0061	0,0009
Semiparametric	γ = 5	-0,0001	0,0072	0,0078	0,0090	0,0083
	γ = 10	0,0000	0,0080	0,0086	0,0098	0,0090
	γ = 15	0,0011	0,0080	0,0082	0,0104	0,0098

This table displays the annualized Performance Fee for the in sample period measure of changing from the SR based strategy to the other proposed strategies. The results are percentage and computed for 3 different values of the risk aversion parameter ranging from 5 (less risk averse) to 15 (more risk averse).

Figure 14. Cumulative returns for the in sample portfolios



This figure displays the cumulative returns for the In Sample period, the dashed lines represent the portfolios based on the SP measures and the black line represents the equally weighted portfolio.

7.3.2. Out of sample period

In this section the results for the out of sample period are displayed

Table 9. Equity positioning, out of sample period

	Parametric						Semiparametric					
Position	SR	ASR	ES8020	ES9010	VaR8020	VaR9010	SR	ASR	ES8020	ES9010	VaR8020	VaR9010
1	3	1	12	12	7	12	1	1	1	1	1	1
2	2	2	12	12	7	12	2	2	5	5	11	4
3	3	3	2	2	3	2	3	3	9	2	11	4
4	4	4	2	2	8	2	4	4	9	9	11	11
5	5	5	5	5	1	5	5	5	6	10	8	11
6	6	6	1	1	1	1	6	6	6	10	8	6
7	7	7	1	1	5	1	7	7	11	10	8	6
8	8	8	1	1	5	1	8	8	6	8	8	9
9	7	9	8	8	2	8	9	9	10	11	10	2
10	10	10	7	7	2	7	10	10	10	11	10	10
11	11	11	7	7	2	7	11	11	12	11	2	10
12	3	3	7	7	12	7	12	12	3	7	3	3

Numeration	1	2	3	4	5	6	7	8	9	10	11	12
	APPL	JNJ	BRK.A	DIS	GE	KO	XOM	SO	SPG	DOW	T	JPM

This table displays the position, in descending order, that each equity has occupied for the longest time during the out of sample period for both parametric and SP ways of measuring the PMs.

Table 10. Differences in sorting, out of sample period

		Parametric						Semiparametric					
		1	2	3	4	5	6	7	8	9	10	11	12
Paramatric	1	0	0,126	0,817	0,817	0,846	0,817	0,419	0,419	0,456	0,491	0,773	0,491
	2	0	0	0,864	0,864	0,896	0,864	0,400	0,400	0,429	0,458	0,757	0,530
	3	0	0	0	0	0,697	0,000	0,875	0,875	0,967	0,990	0,988	0,984
	4	0	0	0	0	0,697	0,000	0,875	0,875	0,967	0,990	0,988	0,984
	5	0	0	0	0	0	0,697	0,801	0,801	0,967	1,000	0,943	0,896
	6	0	0	0	0	0	0	0,875	0,875	0,967	0,990	0,988	0,984
Semiparametric	7	0	0	0	0	0	0	0	0	0,586	0,477	0,856	0,664
	8	0	0	0	0	0	0	0	0	0,586	0,477	0,856	0,664
	9	0	0	0	0	0	0	0	0	0	0,456	0,571	0,557
	10	0	0	0	0	0	0	0	0	0	0	0,677	0,530
	11	0	0	0	0	0	0	0	0	0	0	0	0,694
	12	0	0	0	0	0	0	0	0	0	0	0	0

Numeration	1	2	3	4	5	6	7	8	9	10	11	12
	SR	ASR	ES8020	ES9010	VaR8020	VaR9010	SR	ASR	ES8020	ES9010	VaR8020	VaR9010

This table displays the differences in sorting with the different PMs for the out of sample period, measured as the Euclidean norm divided by the maximum value, so it ranges from 0 (minimum difference) to 1 (maximum difference).

Table 11. Number of days (%), in which the portfolio remains unchanged

Parametric						Semiparametric					
SR	ASR	ES8020	ES9010	VaR8020	VaR9010	SR	ASR	ES8020	ES9010	VaR8020	VaR9010
8,125	8,4	22,4	22,6	24,9	22,1	7,875	8	54,125	53,05	61,375	56

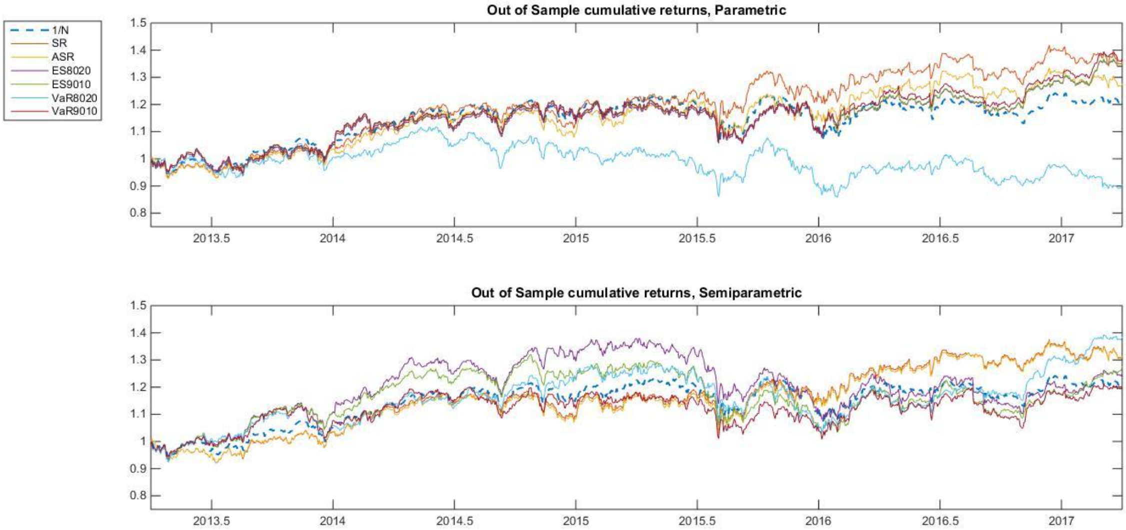
This table displays the number of days as a percentage that the assets remain the same, so the portfolios does not need a rebalance.

Table 12. Perfomance fee measure (%), out of sample period

		ASR	ES8020	ES9010	VaR8020	VaR9010
Parametric	γ = 5	-0,0011	-0,0002	-0,0001	-0,0066	0,0001
	γ = 10	-0,0010	-0,0001	0,0000	-0,0065	0,0002
	γ = 15	-0,0007	0,0002	0,0002	-0,0062	0,0004
Semiparametric	γ = 5	-0,0011	-0,0002	-0,0001	-0,0066	0,0001
	γ = 10	-0,0010	-0,0001	0,0000	-0,0065	0,0002
	γ = 15	-0,0007	0,0002	0,0002	-0,0062	0,0004

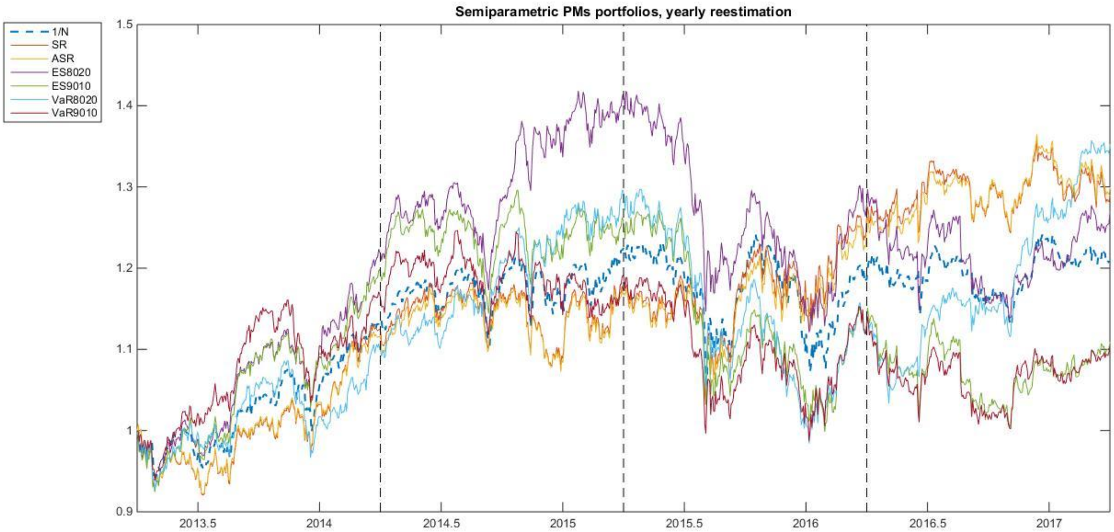
This table displays the annualized Performance Fee for the out of sample period measure of changing from the SR based strategy to the other proposed strategies. The results are as a percentage and computed for 3 different values of the risk aversion parameter ranging from 5 (less risk averse) to 15 (more risk averse).

Figure 15. Cumulative returns for the out of sample portfolios



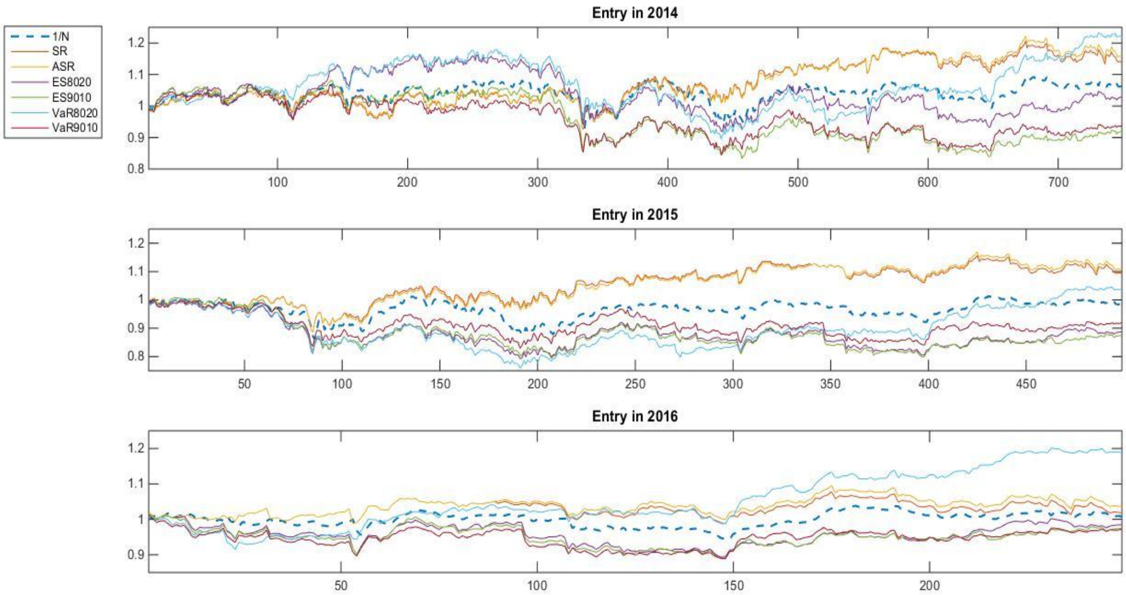
This figure displays the cumulative returns of the portfolios for the Out of Sample period, the dashed lines represent the equally weighted portfolio of the 12 equities. Parametric and SP portfolios are respectively in the upper and lower figures.

Figure 16. Cumulative returns for the out of sample SP portfolios yearly reestimated



This figure displays the cumulative returns of the SP PM based portfolios. The dashed line represents the equally weighted portfolio of the 12 equities. The Vertical lines represent the day where the reestimation is made.

Figure 17. Cumulative returns for the out of sample SP portfolios depending on the entrance.



This figure displays the cumulative returns of the SP PM based portfolios. The dashed line represents the equally weighted portfolio of the 12 equities. Each year the cumulative returns are restarted as if we entered in the portfolio and stayed in that position until the end in 2017.