Certain Monomial Characters and Their Subnormal Constituents

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This is a joint work with G. Navarro.

Introduction

Let G be a group. A character $\chi \in \mathrm{Irr}(G)$ is said to be **monomial** if there exist a subgroup $U \subseteq G$ and a linear $\lambda \in \mathrm{Irr}(U)$, such that

$$\chi = \lambda^G.$$

A group G is said to be **monomial** if all its irreducible characters are monomial.

There are few results guaranteeing that a given character of a group is monomial. It is well-known the following

Theorem

Let G be a supersolvable group. Then all irreducible characters of G are monomial.

But this result depends more on the structure of the group than on characters themselves.

An interesting result.

Theorem (Gow)

Let G be a solvable group. Suppose that $\chi \in \mathrm{Irr}(G)$ takes real values and has odd degree. Then χ is rational-valued and monomial.

We give a monomiality criterium which also deals with fields of values and degrees of characters.

Notation: For n an integer, we write

$$\mathbb{Q}_n = \mathbb{Q}(\xi),$$

where ξ is a primitive nth root of unity.

Theorem A

Let G be a p-solvable group. Assume that $|\mathbf{N}_G(P) : P|$ is odd, where $P \in \mathrm{Syl}_p(G)$ for some prime p. If $\chi \in \mathrm{Irr}(G)$ has degree not divisible by p and the values of χ are contained in the cyclotomic extension $\mathbb{Q}_{|G|_p}$, then χ is monomial.

When $p = 2$, we can recover Gow's result from Theorem A.

The hypothesis about the index $|\mathbf{N}_G(P) : P|$ is necessary. Consider the group SL(2,3) and the prime p=3.

The solvability hypothesis is necessary in both Gow's and Theorem A. The alternating group A_6 is a counterexample in the two cases.

B_π Theory

We say that $\chi \in \mathrm{Irr}(G)$ is a π-**special character** of G, if

(a) $\chi(1)$ is a π-number.

(b) For every subnormal subgroup $N \lhd \lhd G$, the order of all the irreducible constituents of χ_N is π-number.

A $\mathbf{B}_\pi$ **character** of a group G may be thought as an irreducible character of G induced from a π-special character of some subgroup of G. (True in groups of odd order).

Main results

Now, I can state the main result.

Theorem B

Let G be a p-solvable group. Assume that $|\mathbf{N}_G(P) : P|$ is odd, where $P \in \mathrm{Syl}_p(G)$ for some prime p. If $\chi \in \mathrm{Irr}(G)$ has degree not divisible by p and its values are contained in the cyclotomic extension $\mathbb{Q}_{|G|_p}$, then χ is a B_p character of G.

We notice that B_p characters with degree not divisible by p are monomial.Thus Theorem B implies Theorem A.

Subnormal constituents of B_π characters are B_π characters. Then, as a Corollary of Theorem B we get.

Corollary C

Let G be a p-solvable group. Suppose that $|\mathbf{N}_G(P) : P|$ is odd, where $P \in \mathrm{Syl}_p(G)$ for some prime p. If $\chi \in \mathrm{Irr}(G)$ has degree not divisible by p and its field of values is contained in $\mathbb{Q}_{|G|_p}$, then every subnormal constituent of χ is monomial.

We also obtain the following consequence. The number of such characters can be computed locally.

Corollary D

Let G be a p-solvable group. Assume that $|\mathbf{N}_G(P) : P|$ is odd, where $P \in \mathrm{Syl}_p(G)$ for some prime p. The number of irreducible characters which have degree not divisible by p and field of values contained in $\mathbb{Q}_{|G|_p}$ equals the number of orbits under the natural action of $\mathbf{N}_G(P)$ on P/P'.

Thanks for your attention!