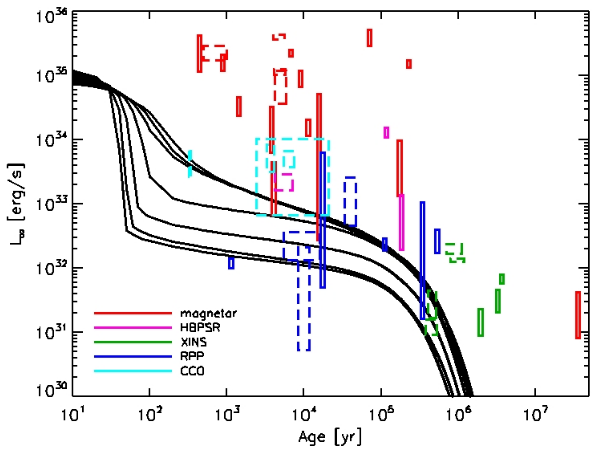
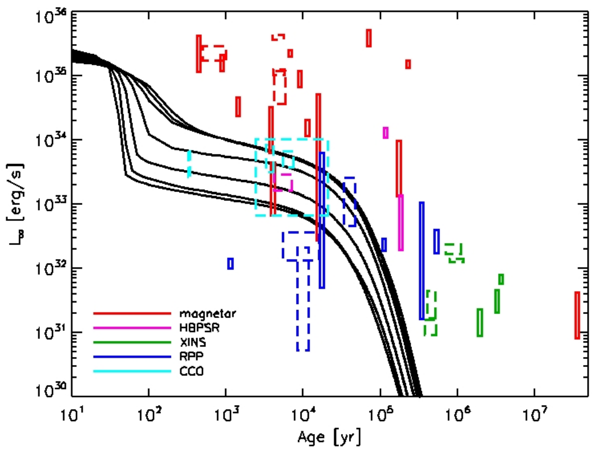
Magnetic field evolution in magnetars: the influence of the magnetosphere

Jose A. Pons
University of Alicante, Spain

Cooling curve, weak magnetic fields ($B \lesssim 10^{13}$ G)

Different lines: masses from 1.10 $M_\odot$ to 1.76 $M_\odot$. Iron (left)/accreted (right) envelope.
Caveat: solid boxes to be taken as AGE UPPER LIMITS ($\tau_c > t_{real}$)





Most magnetars, high-B PSRs, XINSs (and CCOs with *Fe* envelope) are hotter than expected.
$\Rightarrow$ **Extra energy needed: magnetic field decay explains energetics (luminosity),**
BUT T IS STILL TOO HIGH in some cases.

Magnetic AND thermal evolution are strongly coupled

How B field evolution is affected by temperature ?

1. Magnetic diffusivity is temperature (and composition) dependent.
2. Thermal evolution determines when phase transitions to superfluid/supercon states occur.
3. Thermoelectric field generation (low density).

How thermal evolution is affected by B field ?

1. Joule dissipation (source of heat Q_j, non-isothermal crust and envelope !)
2. anisotropic thermal conductivity $\hat{\kappa}$
3. neutrino synchrotron process
4. quantizing effects (only at low density)
5. **Vacuum vs. magnetosphere boundary conditions ??**

Magnetic field evolution in the CRUST

The crust be considered a simplified version of a Hall plasma: ions have very restricted mobility and only electrons can move freely through the lattice, carrying currents and heat: the proper equations are Hall MHD. If ions are strictly fixed in the lattice, the limit is known as EMHD (electron MHD).

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \left\{ \eta \vec{\nabla} \times (e^{\nu} \vec{B}) - \left[\frac{c e^{-\nu}}{4\pi e n_e} \vec{\nabla} \times (e^{\nu} \vec{B}) \right] \times (e^{\nu} \vec{B}) \right\}$$

$\eta = \frac{c^2}{4\pi\sigma}$ is the magnetic diffusivity, and the electron fluid velocity is

$$v_e = -\frac{\vec{J}}{e n_e} = -\frac{c e^{-\nu}}{4\pi e n_e} \vec{\nabla} \times (e^{\nu} \vec{B})$$

In the limit of small deformations, the metric is still spherically symmetric. Relativistic corrections included with the e^{ν} factors.

The Hall term

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \left\{ \eta \left[\vec{\nabla} \times \vec{B} - \omega_B \tau_e \left(\vec{\nabla} \times \vec{B} \right) \times \vec{b} \right] \right\}$$

Here $\omega_B \tau_e$ is the "magnetization parameter" (also $= \tau_{Ohm}/\tau_{Hall}$) , where ω_B is the gyro-frequency.
For high temperatures (large resistivity) or weak fields: diffusive regime $\omega_B \tau_e \ll 1$
For low T ($T \lesssim 10^8$ K) or strong fields: non-linear hyperbolic regime $\omega_B \tau_e \gg 1$, i.e. Hall activity.

Linear regime: wave modes *[Huba 2005]*

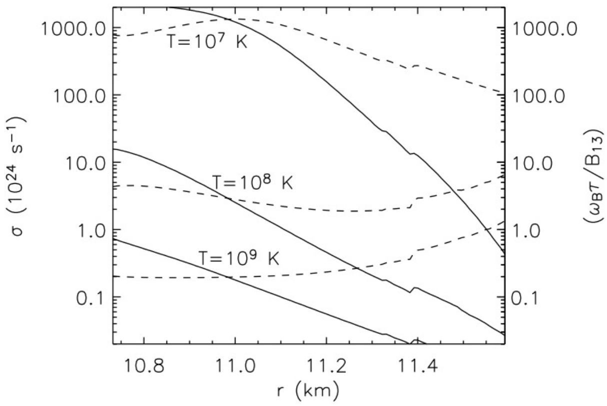
Backgroung field $\vec{B} = B_0 \hat{z}$

- constant n_e, **whistler (or helicon) waves** propagating along field lines
  dispersion relation $\omega = k^2 B / 4\pi e n_e$
  phase velocity $\propto k_z \Rightarrow$ restrictive Courant condition
- $n_e = n_e(x)$, **Hall drift waves** in the $\vec{B} \times \vec{\nabla} n_e$ direction
  dispersion relation $\omega = k_y B_0 / [4\pi e (dn_e/dx)]$
  phase velocity $B_0 / [4\pi e (dn_e/dx)]$

Hall timescale(s).

$$\tau_{Hall} = \frac{4\pi e n_e}{cB} L^2$$

But be careful with simple estimates.



PROBLEM: everything varies by several orders of magnitude. Estimates are not very useful, which region in this diagram should we look at ??
What is the real scale (L) of the B field ??

For example, consider the the Hall timescale

$$\tau_{Hall} = \frac{4\pi e n_e}{cB} L^2 \approx 6 \times 10^4 \frac{n_{e,10} L_{km}^2}{B_{13}} \mathrm{yr}$$

Consider a layer close to the neutron drip point, $n_{e,10} \approx 1$, a large scale field ($L = 10$ km) and ($B = 10^{14}$ G) seems to have a slow Hall timescale of about 0.6 Myr.

But a local structure (loop, twist) with $L = 1$ km and ($B = 10^{15}$ G) has a timescale of 600 years !

The Hall term: non-linearity

Consider the evolution equation for a purely toroidal MF, in a constant density medium, and neglecting resistivity, can be cast into the following form in cylindrical coordinates:

$$\frac{\partial B_\varphi}{\partial t} - \frac{c}{4\pi e n_e r}\frac{\partial B_\varphi^2}{\partial z} = 0 \; nonumber \tag{1}$$

Currents sheets

The evolution of the Burgers-like equation unavoidable leads to the formation of a shock (current sheet) in a finite time

$$\tau_{cs} = \frac{4\pi e n_e r}{c}\left(\frac{dB_{phi}}{dz}\right)^{-1}$$

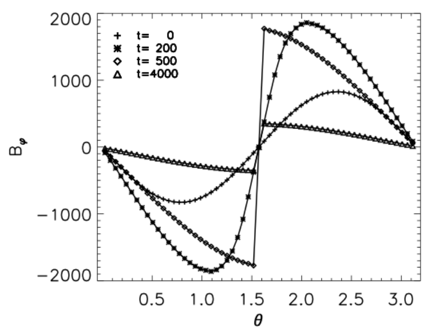
The Burgers part in toroidal B_ϕ is due to non-linear self-coupling! Very hard to capture with a spectral method, need for shock-capturing finite-difference methods.

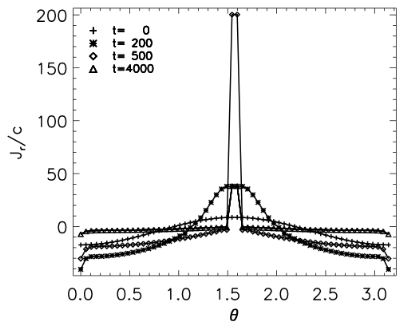
The numerical solutions: some technical details

[Viganò+ 2012, *Comput. Phys. Comm.* 183; Viganò+ 2018, *submitted*]

Numerical treatment

1. Finite-difference, divergence-preserving scheme: staggered grid with $\vec{E}$ and $\vec{B}$ components defined in displaced points.
2. Upwind scheme to follow the formation and dissipation of very localized current sheets
3. Cell reconstruction
4. "Semi-implicit" advance implying an alternated advance of B_ϕ and B_{pol}
5. Hyper-resistivity (well below the physical resistivity)
6. Conservation of total (magnetic+dissipated+Poynting flux) energy





Not clear what is the level of numerical noise in different codes with different approaches.

B field evolution in the core: Overview of driving forces.

Not clear how much flux penetrates into the core, and what is the evolution of a SC fluid (fluxoids drift and interact with vortices? magnetic buoyancy? Does ambipolar diffusion work with superfluid neutrons or SC protons?).

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1. Fluxoid buoyancy: bulk radial fluxtube drift (Baym & Pathick 1975, Muslimov & Tsygan 1985)
2. Viscous-like electron drag: electron-fluxtube scattering (Alpar et al. 1984)
3. Magnus force: redistribution to perpendicular flow (Jones 1987)
4. Fieldline tension (Harvey 1986, Konenkov & Geppert 2000)

The standard method is to estimate a bulk advection velocity ($\vec{v} \times \vec{B}$ term in the electric field) from force balance considerations

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \left\{ \eta \nabla \times \vec{B} - f_{Hall} \vec{j} \times \vec{B} + \vec{v} \times \vec{B} \right\}$$

But all recent studies (e.g. Graber et al 2015, Elfritz et al 2016) show a very slow drift velocity in all cases, of at most $v \approx 0.1 - 1$ km/Myr: NOT FAST ENOUGH to have a significant impact on magnetar/X-ray pulsars lifetimes.

Weak field

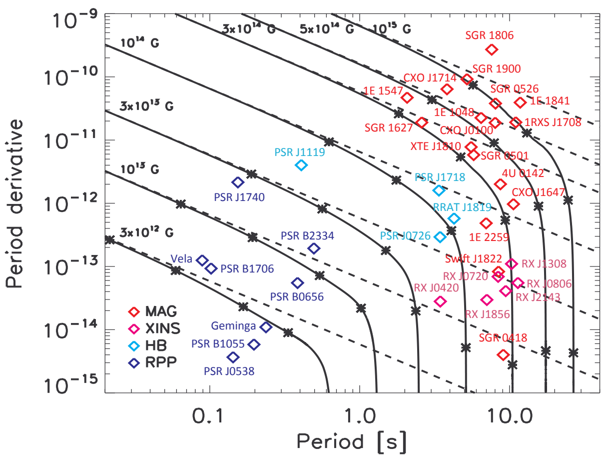
Strong field

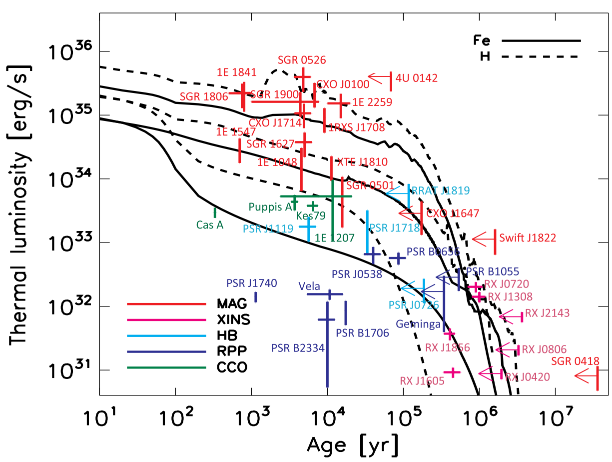
Core field

Toroidal field

Initial topology is quickly reorganized: the Hall term removes the freedom to chose arbitrarily large toroidal fields able to inject very large extra energy (Viganò + 2012, 2013)
The long term structure looks similar for all models (Pons & Geppert 2007, Vigano et al. 2013, Gourgouliatos+ 2013,2014) Models have more predictive power (lost memory of initial conditions), but only when they are sufficiently old. Only a few 3D simulations available (KG+ 2016,2018), interesting $m = 12 - 20$ mode appears.

Data vs. models



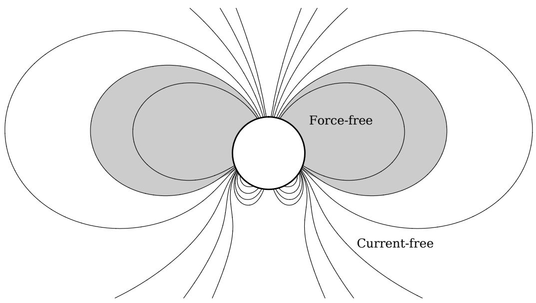


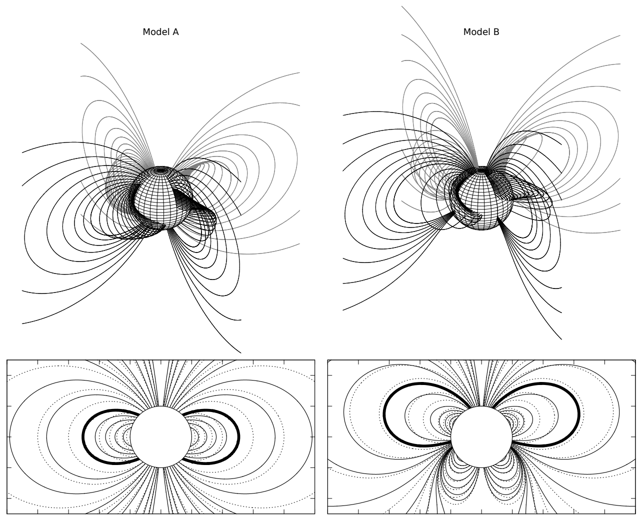
Constraining models

- Reasonable agreement of observations with models with CRUSTAL currents, not yet (?) possible with core fields (not enough heating, not enough field decay).
- BUT Modeled surface temperatures are systematically a factor $\sim 2-3$ lower than the fitted BB temperatures in quiescence.
- Neutrino cooling is very efficient in the interior, so few days/weeks after outbursts the surface temperature drops below $0.2-0.3$ keV: no way to keep a very high T all the time.

Coupling the interior evolution with the magnetosphere.

Up to now, simulations of the long term evolution only adopted vacuum boundary conditions.
Which is the effect of the magnetosphere ? Adapting recent force-free magnetosphere models (Akgün et al . 2016,2017,2018) as external BC to evolution codes.





Building axisymmetric force-free magnetospheres: Notation.

Magnetic field in terms of the poloidal and toroidal stream functions

$$\vec{B} = \vec{\nabla} P \times \frac{\hat{\phi}}{r \sin\theta} + T(P) \frac{\hat{\phi}}{r \sin\theta}$$

Poloidal field lines are lines of constant P

Grad-Shafranov equation

$$\Delta_{GS} P + TT'(P) = 0$$

$$\Delta_{GS} P \equiv \partial_r^2 + \frac{1-\mu^2}{r^2} \partial_\mu^2$$

here $\mu = \cos\theta$.

Current for a force-free field

$$\vec{J} = \frac{c}{4\pi} T'(P) \vec{B}$$

Matching conditions at the surface.

General procedure

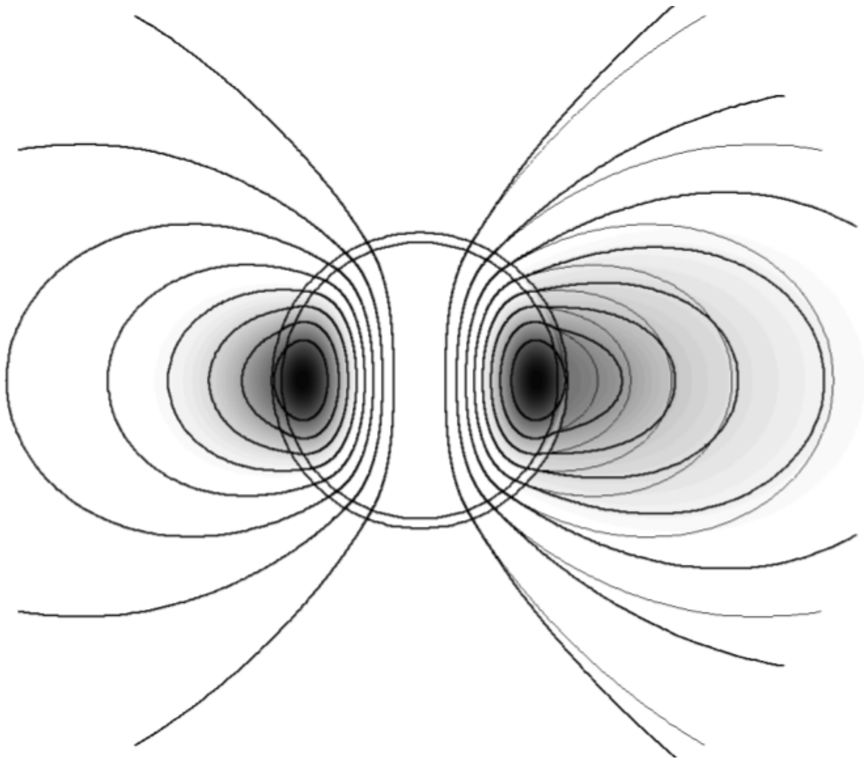
- Using the components B_r, B_ϕ given by the internal evolution we construct P and $T(P)$ at the star surface and then obtain the appropriate magnetospheric solution (Grad-Shafranov).
- The magnetosphere solution provides B_θ at the first cell above the surface, needed for the internal evolution.
- This is a generalization of the vacuum boundary conditions, which limit is easily recovered simply taking $T(P) = 0$. But now we allow for currents to flow through the surface.

Constructing $T(P)$: technical details

- The force-free magnetosphere requires a single-valued function $T(P)$: same value at both surface footprints of a magnetic field line.
- The internal evolution gives rise to a general function $T(P)$, generally not satisfying the condition, but it can be separated into a symmetric part and an antisymmetric part with respect to the footprints.
- The antisymmetric part cannot propagate into the magnetosphere and must be reflected into the interior, or very quickly damped. (Similar to the results of Gabler et al. 2014 for ideal MHD simulations of internal torsional oscillations.)

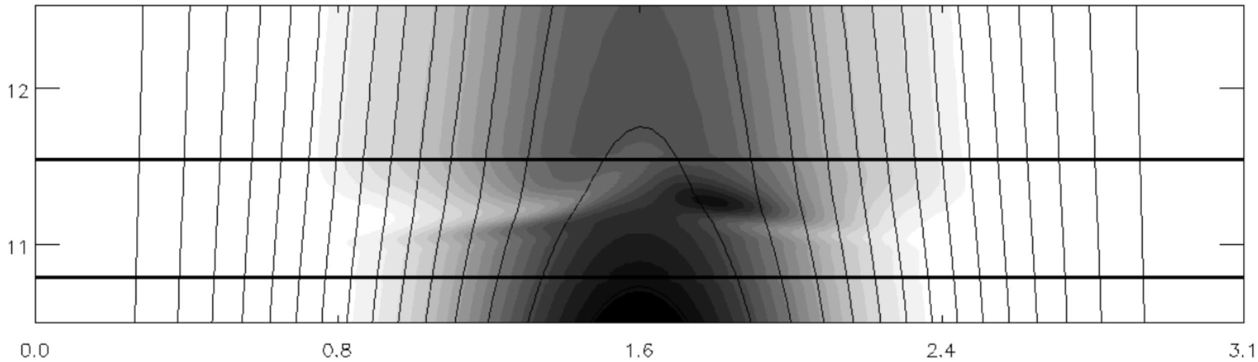
Sample evolution

- Left: initial configuration.
- Right: final configuration at $t = 2150$ yr.
- The star surface and the core-crust boundary shown by circles.



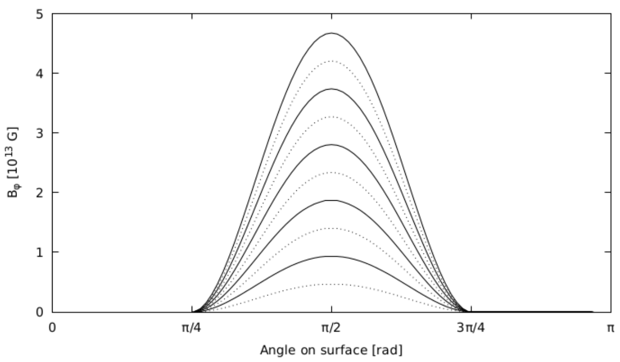
Sample evolution: detail

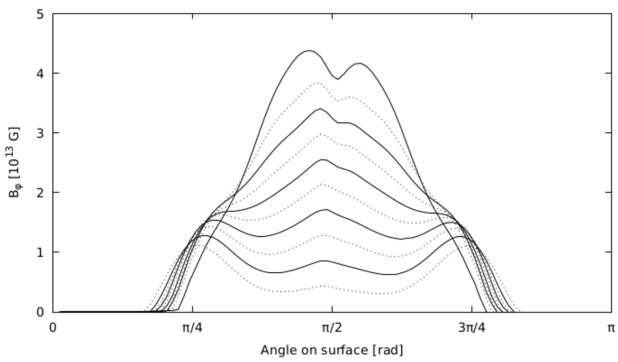
- Cartesian projection as a function of the polar angle (in radians, horizontal axis) and radial distance (in km, vertical axis) at the end (at 2150 yr).
- The horizontal black lines indicate the crust-core boundary (at $r = 10.8$ km) and the surface ($R = 11.6$ km).
- The grayscale represents the intensity of the toroidal function T.



Evolution of toroidal field profiles

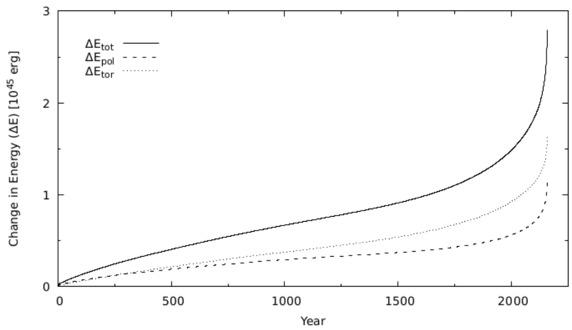
- B_ϕ profiles for different simulations (varying strength). Left: initial model. Right: at the end of simulations (instability threshold), when a critical point is reached, beyond which connected force-free solutions do not exist.
- Note that currents pile up near the toroidal border (more vertical profiles in the plots), and may change sign.

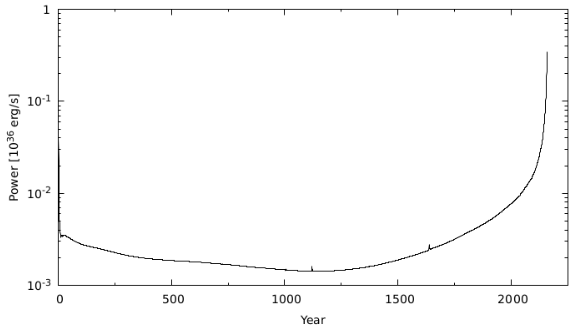




Energy evolution

- Top: evolution of the (relative change, wrt initial model) energy stored in the magnetosphere (total, poloidal and toroidal energies).
- Bottom: total power input into the magnetosphere (dE_{tot}/dt).
- For most of the time, the power is relatively constant, and is of the order of the quiescence luminosity of magnetars (10^{33} to 10^{34} erg/s).
- Near the critical point, the power input increases substantially, consistent with peak luminosities of transient energetic events (outbursts).

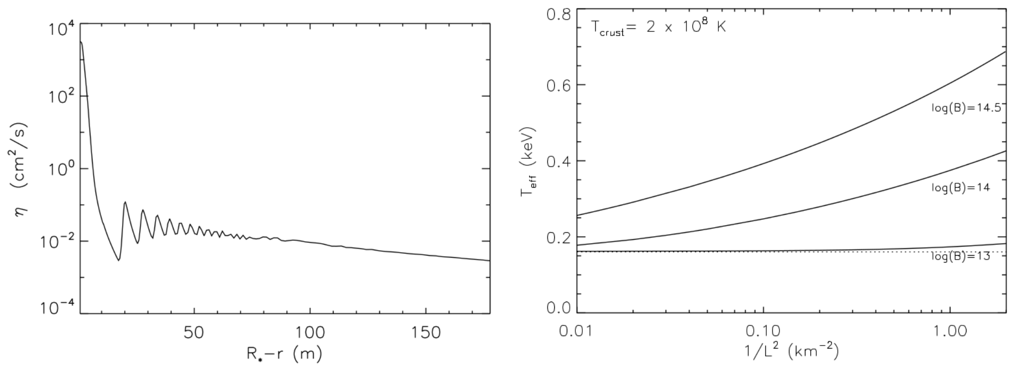




Implications: surface temperature

- Ohmic dissipation can be more effective in the outermost 100 m of the star (the envelope), normally not included in simulations (boundary condition)
- The magnetic diffusivity (η) is very large close to the surface (see also quantizing effects), and the dissipation timescale $\tau_{Ohm} = L^2/\eta$ can become very short ($\approx$ weeks-months).
- A simple estimate (Akgün et al 2018) gives

$$T_{eff} \approx 0.3keV \left[\frac{\Delta r}{1\mathrm{m}}\right]^{1/4} \left[\frac{\eta}{10^3\mathrm{cm}^2\mathrm{s}^{-1}}\right]^{1/4} \left[\frac{B}{10^{14}\mathrm{G}}\right]^{1/2} \left[\frac{1\mathrm{km}}{L}\right]^{1/2}$$



Coupling the interior evolution with the magnetosphere.

Main effects:

- The dipole content and the energy increases up to 10-20 % (compared to vacuum solutions). This would be the energy available for flares.
- There is a maximum value of the toroidal/poloidal field ratio (s) above which no magnetosphere models can be found (no convergence of the iterative procedure). This ratio is of order unity.
- For initial models with s above a critical value, the evolution leads to the magnetosphere expansion as s increases: will reach the limit and trigger a flare ?
- This limit value is reached when the maximum twist is $\approx 2 - 3$ rad, rather independently of the particular model.
- **Extending vacuum boundary conditions and including consistent magnetospheres matching the internal evolution may be the key to explain high T. It implies that currents can flow in/out through the outermost few meters, where the resistivity is largest. Not much impact on global energetics (small volume involved), but locally hot spots easy to create.**