Mean-field dynamo in relativistic disks accreting onto rotating black holes

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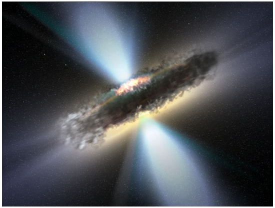
Necessity of amplifying magnetic fields



Accretion process:

- conversion of gravitational energy into thermal and radiative energy,
- geometrically thick and optically thin regime (SGR A*)

Large scale magnetic fields are fundamental in disks:

- viscosity generation by means of the magnetorotational instability (MRI) (Balbus and Hawley 1991);
- jets formation by means of Blandford-Znajek process (Blandford and Znajek 1977).

Mean-field dynamo is a good candidate in amplifying magnetic field.

INTRODUCTION TO THE MEAN-FIELD DYNAMO

Mean-field effect

Let us assume little turbulent fluttuations in a plasma

$$\boldsymbol{U} = \boldsymbol{U}_0 + \boldsymbol{u}, \qquad \boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b}.$$

The induction equation reads

$$\frac{\partial \boldsymbol{B}_0}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{U}_0 \times \boldsymbol{B}_0) + \eta \nabla^2 \boldsymbol{B}_0 + \boldsymbol{\nabla} \times \langle \boldsymbol{u} \times \boldsymbol{b} \rangle.$$

where

$$\langle \boldsymbol{u} \times \boldsymbol{b} \rangle = \alpha \boldsymbol{B}_0 - \beta \boldsymbol{\nabla} \times \boldsymbol{B}_0.$$

$$\downarrow$$

$$\frac{\partial \boldsymbol{B}_0}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{U}_0 \times \boldsymbol{B}_0) + (\eta + \beta) \nabla^2 \boldsymbol{B}_0 + \alpha \boldsymbol{\nabla} \times \boldsymbol{B}_0.$$

There are two effects:

- increase of the diffusion,
- generation of an electromotive force that produces a current parallel to the magnetic field.

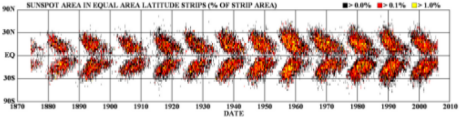
$\alpha\Omega$ dynamo cycle

In a plasma with differential rotation let us introduce the poloidal and toroidal components.

$$\boldsymbol{B} = \boldsymbol{B}_P + \boldsymbol{B}_T,$$

- Ω effect produced by the differential rotation ($B_P \rightarrow B_T$).
- α effect produced by the fluctuations ($B_T \rightarrow B_P$).

We can close the cycle in axisymmetric case.
An example of $\alpha\Omega$ dynamo:



Butterfly diagram which shows the time course of sunspot distribution. (`http://solarscience.msfc.nasa.gov/images/bfly.gif`).

Generalized Ohm's law

Covariant formulation of Ohm's equation (Bucciantini e Del Zanna 2013)

$$\boldsymbol{E}' = \eta \boldsymbol{J} + \xi \boldsymbol{B} \rightarrow \boxed{e^{\mu} = \eta j^{\mu} + \xi b^{\mu}}$$

where

- $\boldsymbol{E}' = \boldsymbol{E} + \boldsymbol{U} \times \boldsymbol{B}$ is the comoving electric field.
- η is the resistivity e $\xi := -\alpha$.

The spatial components:

$$\boxed{\begin{aligned}&\Gamma[\boldsymbol{E} + \boldsymbol{U} \times \boldsymbol{B} - (\boldsymbol{E} \cdot \boldsymbol{U})\boldsymbol{U}] = \\ &\eta(\boldsymbol{J} - Q\boldsymbol{U}) + \xi\Gamma[\boldsymbol{B} - \boldsymbol{U} \times \boldsymbol{E} - (\boldsymbol{B} \cdot \boldsymbol{U})\boldsymbol{U}],\end{aligned}}$$

$$\downarrow$$

Corrections in Ampere's law

ACCRETION DISKS

Polish doughnuts

Two ingredients:

- gravity (Kerr metric)
- perfect fluid ($p = K w^{\gamma_a}$)

HD equilibrium:

$$W - W_{in} + \frac{\gamma_a}{\gamma_a - 1}\frac{p}{w} = 0,$$

with $W(r,\theta)$ is a potential.
The disk is defined in that zones where $W - W_{in} < 0$ (Abramowicz 1978).
The fluid quantities are defined:

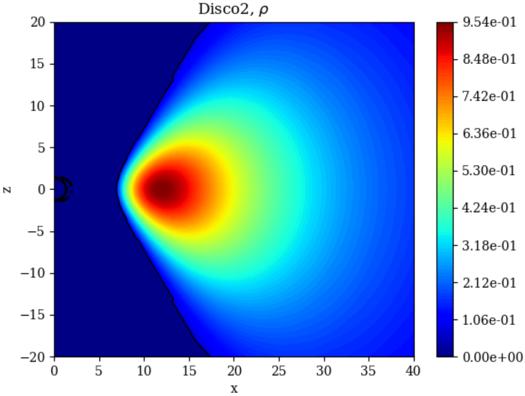
$$w = w_c \left(\frac{W_{in} - W}{W_{in} - W_c} \right)^{\frac{1}{\gamma_a - 1}}, \qquad \rho = w - \frac{\gamma_a}{\gamma_a - 1} p$$

Then a small magnetic field is introduced ($B^2 \ll w$). Two configurations:

- toroidal
- poloidal

Section of the Disk

$r_{in} = 6, \ r_c = 12, \ a_{BH} = 0.94$



Two-dimensional map for ρ.

NUMERICAL SIMULATIONS

ECHO code

Numerical simulations were performed with ECHO (Eulerian Conservaive High Order) (Del Zanna et al 2007).
Main features:

- Godunov type, shock capturing,
- finite differences,
- high-order reconstruction schemes,
- simplified Riemann solvers (HLL),
- generic metric
- stiff terms (IMEX)

Dynamo numbers

We define two numbers to quantify the dynamo action (Bugli et al. 2014):

$$C_\Omega = \frac{\Delta\Omega R^2}{\eta}, \qquad C_\xi = \frac{\xi R}{\eta}.$$

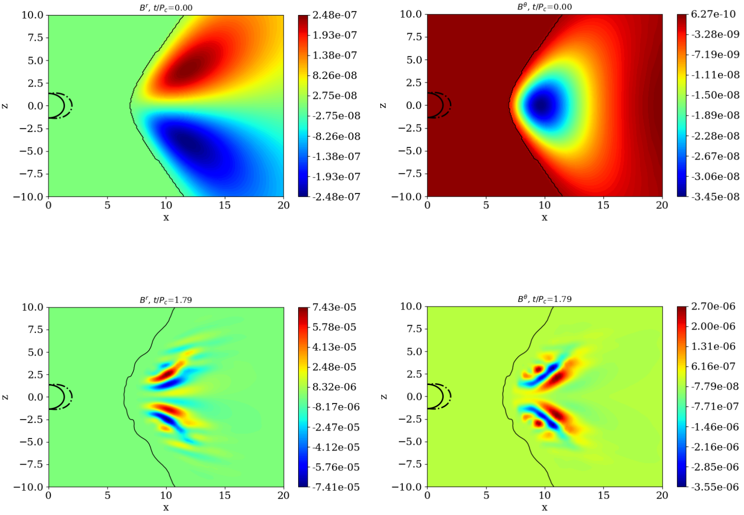
$\Delta\Omega$ is a typical angular velocity difference;
R is a typical lenght.
In our simulations we assumed:

$$C_\Omega \gg 1,$$

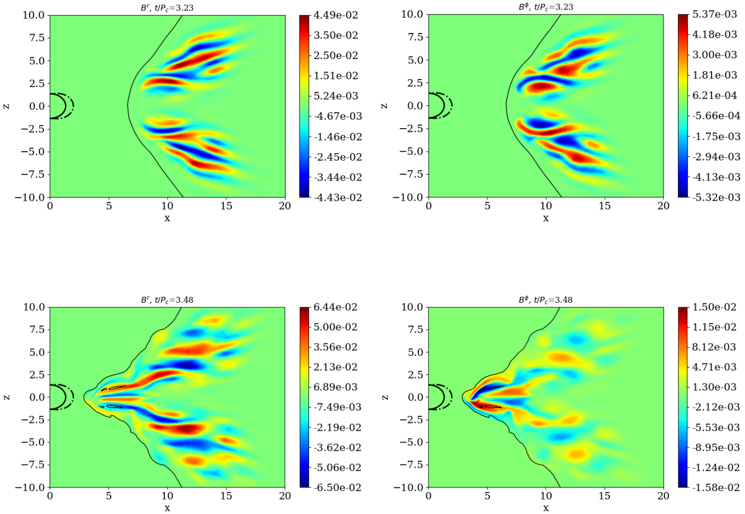
$$C_\xi \geq 1$$

- rotation stronger than diffusion;
- dynamo comparable with dissipation

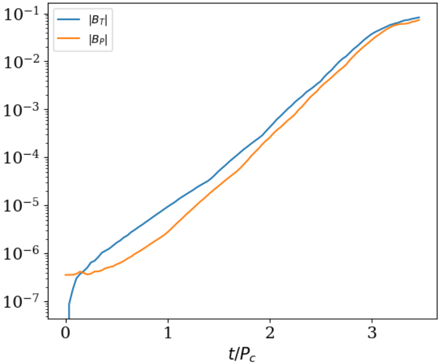
Initial conditions and evolution ($C_\Omega = 8200$ e $C_\xi = -120$)



Ω effect and evolution during mass loss



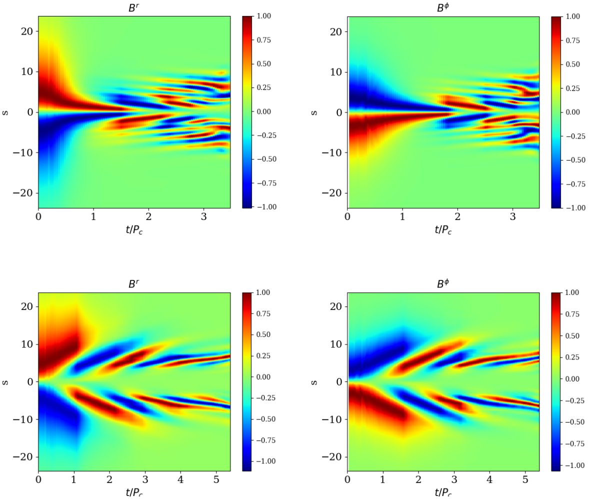
Exponential growth



Growth of the components of the field and saturation.

- growth rate: ~ 4.4,
- saturation value: 0.1,

Butterfly diagram



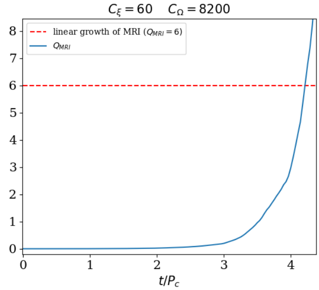
$(C_\Omega = 8200$ e $C_\xi = -120$ top$)$ $(C_\Omega = 8200$ e $C_\xi = 60$ bottom$)$.

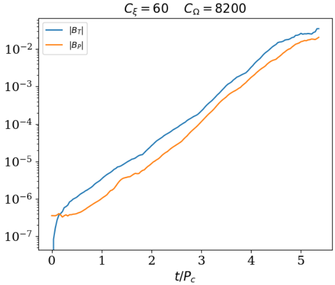
Saturation

Are we resolving MRI? → quality factor

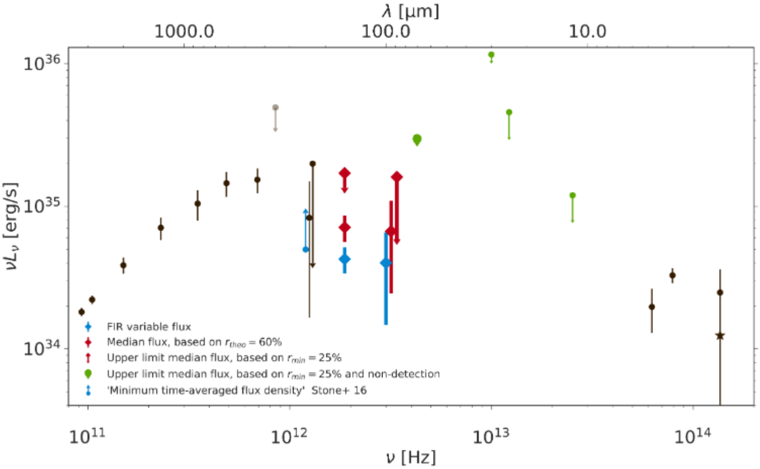
$$Q_{MRI} = \frac{\lambda_{MRI}}{ds} = \frac{2\pi V_A}{\Omega\sqrt{dr^2 + (rd\theta)^2}}.$$

If $Q_{MRI} > 6$ the code resolves MRI (Hogg e Reynolds 2018).





SED of SGR A*



SED of Sagittarius A* updated with the most recent measurements (von Fellenberg et al. 2018).

Magnetobremsstrahlung emission

This type of emission is synchrotron radiation produced by thermal electrons. The emissivity is given by:

$$j_\nu \simeq n_e \frac{\sqrt{2}\pi e^2 \nu_s}{6\Theta_e^2 c} X \exp\left(-X^{1/3}\right)$$

with $X = \nu/\nu_s$. The critical frequency ν_s is defined by:

$$\nu_s = (2/9)\nu_c \Theta_e^2 \sin\theta,$$

con

$$\nu_c = eB/(2\pi m_e c), \qquad \Theta_e = kT_e/(m_e c^2).$$

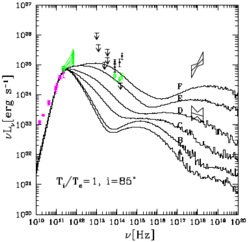
The total luminosity is defined (optically thin plasma):

$$L_\nu = \int_{r_{in}}^{r_{max}} r^2 dr \int_{\pi/3}^{2\pi/3} \sin\theta d\theta \int_0^{2\pi} d\phi \alpha \gamma^{1/2} j_\nu$$

Model of the disk around SGR A*

At the equilibrium we assume (Moscibrodzka et al. 2009):

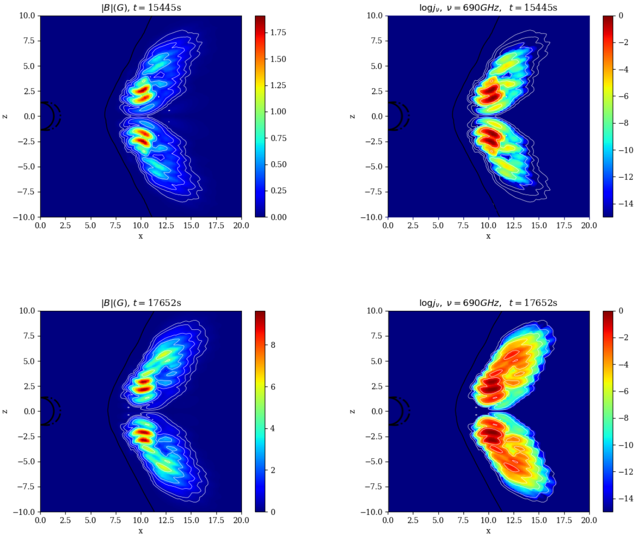
- geometrically thick and optically thin plasma ($r_{in} = 6$, $r_{out} = 12$),
- $T_e \sim T_i$,
- $n_e = 10^7$ cm^{-3}.



SED calculated with Monte Carlo scheme (Moscibrodzka et al. 2009).

- $B \sim 28$ G,
- $\Theta_e \sim 17 \rightarrow T_e \sim 10^{11}$ K

Maps of magnetic field and emissivity



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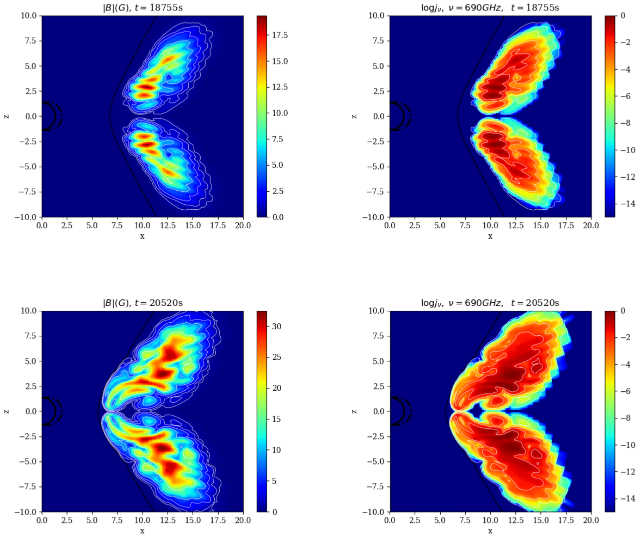
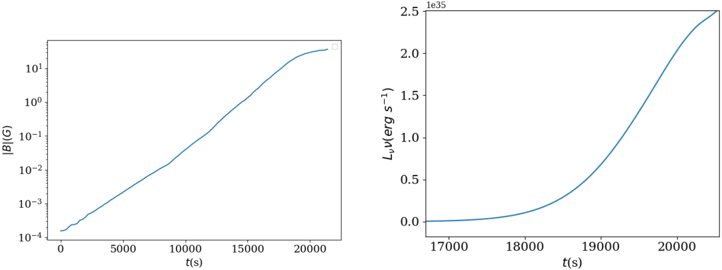


Consistency of the model

When the dynamo saturates, the field reaches the value of ∼ 30 G and the luminosity is consistent with the observed peak.



Evolution of the magnetic field B (sx) and $L_\nu \nu$ (dx).

$$\Theta_e \sim 4 \rightarrow T_e \sim 2 \cdot 10^{10} \text{ K}$$

CONCLUSIONS

Conclusions and future developments

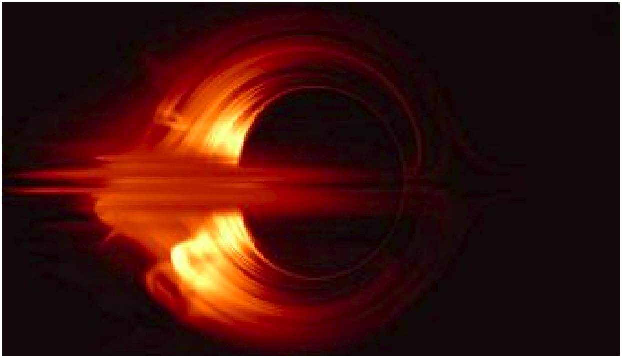
Conclusions:

- First dynamo simulation in an accretion disk in GRMHD regime with global evolution of all quantities.
- Extension of the kinematic case to a fully dynamic regime.
- Saturation possibly related to MRI.
- Magnetobremsstrahlung model: emissivity evolves according to the periodicity of the dynamo.
- Possible observational signature Event Horizon Telescope (EHT).

Future developments:

- More complete study of the parameter space.
- Better prescription for η e ξ.
- Implementation of α quenching.
- Interplay with MRI.

Conclusions and future developments



Numerical simulation of an EHT observation (Bronzwaer et. al 2018).

THE END