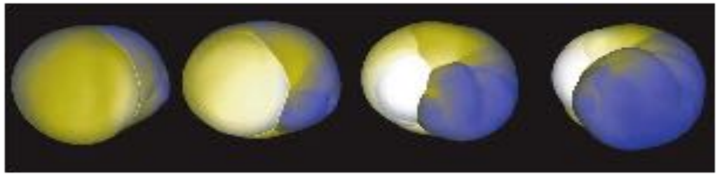
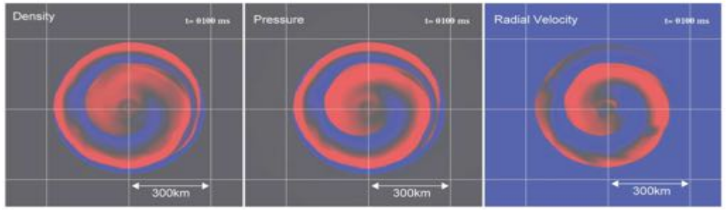
New insights on the low T/W instability in shocked accretion flows



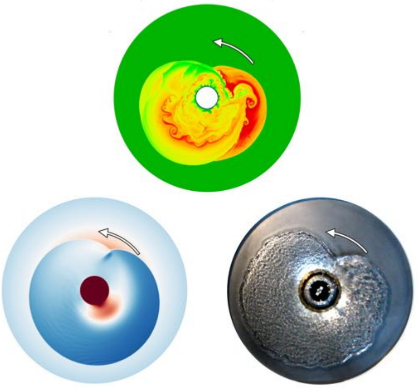
Blondin & Mezzacappa 07

How much rotation can make a difference on the explosion threshold and NS birth?



Takiwaki+16

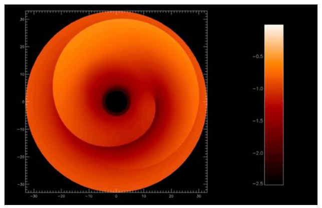
Why is the prograde mode of SASI destabilized by rotation?

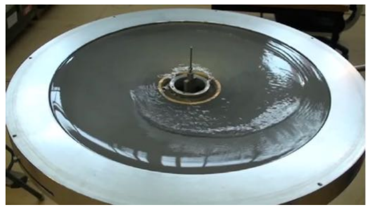


What is the interplay between SASI and the corotation 'low T/|W|' instability?

Is the corotation instability similar to isolated NS with differential rotation ?

Should we expect different GW signatures from these two instabilities?

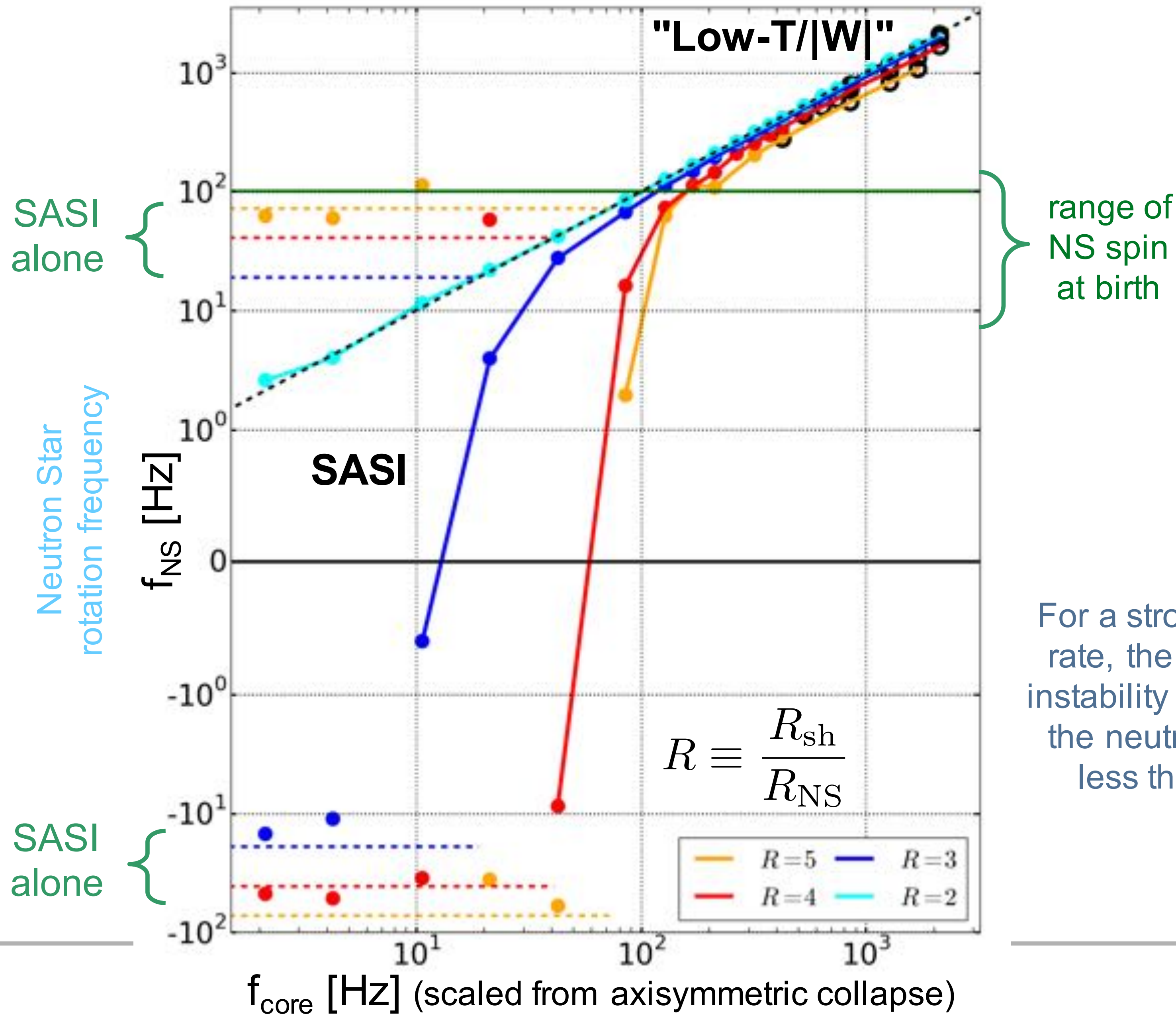




Thierry Foglizzo, Rémi Kazeroni*, Jérôme Guilet, Matthias Gonzalez, Gilles Durand, Bruno Pagani
CEA Saclay, *MPA Garching

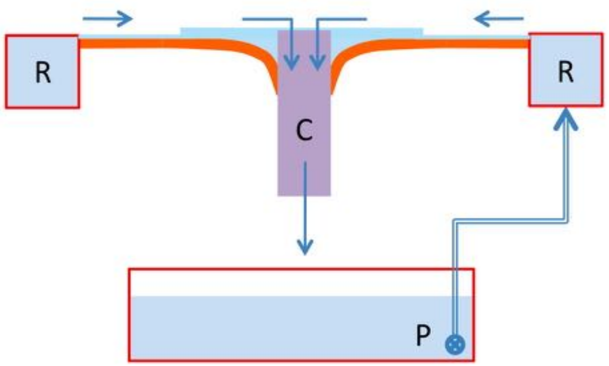
Spin-up or spin-down of the neutron star?

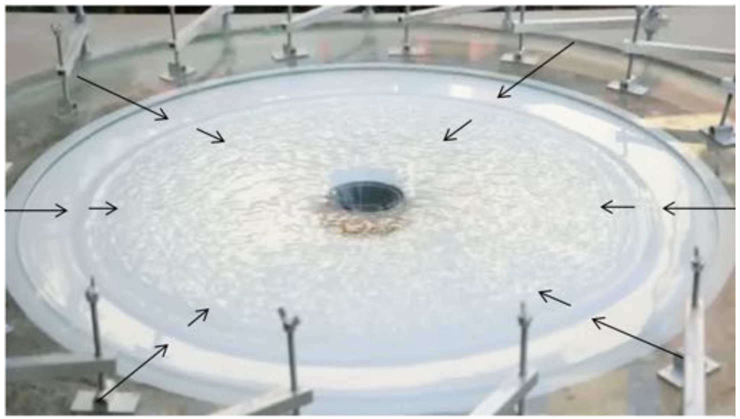
(Kazeroni+17)



For a strong rotation rate, the corotation instability decelerates the neutron star by less than 30%.

Physical insight from an experimental analogue of SASI





adiabatic gas

$$c_s^2 \equiv \frac{\gamma P}{\rho}$$

$$\Phi \equiv -\frac{GM_{ns}}{r}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

$$\frac{\partial v}{\partial t} + (\nabla \times v) \times v + \nabla \left(\frac{v^2}{2} + \frac{c_s^2}{\gamma - 1} + \Phi \right) = \frac{c_s^2}{\gamma} \nabla S$$

Inviscid shallow water is analogue to an isentropic gas γ=2

St Venant

$$c_{sw}^2 \equiv gH$$

$$\Phi \equiv gH_\Phi$$

$$\frac{\partial H}{\partial t} + \nabla \cdot (Hv) = 0$$

$$\frac{\partial v}{\partial t} + (\nabla \times v) \times v + \nabla \left(\frac{v^2}{2} + c_{sw}^2 + \Phi \right) = 0$$

acoustic waves shock wave density ρ	$\longleftrightarrow$	surface waves hydraulic jump depth H

expected scaling
$$\frac{t_{ff}^{sh}}{t_{ff}^{jp}} \equiv \left(\frac{r_{sh}}{r_{jp}}\right) \left(\frac{r_{sh} g H_\Phi^{jp}}{GM_{ns}}\right)^{\frac{1}{2}} \sim 10^{-2}$$

shock radius $\times 10^{-6}$ 200 km $\rightarrow$ 20 cm

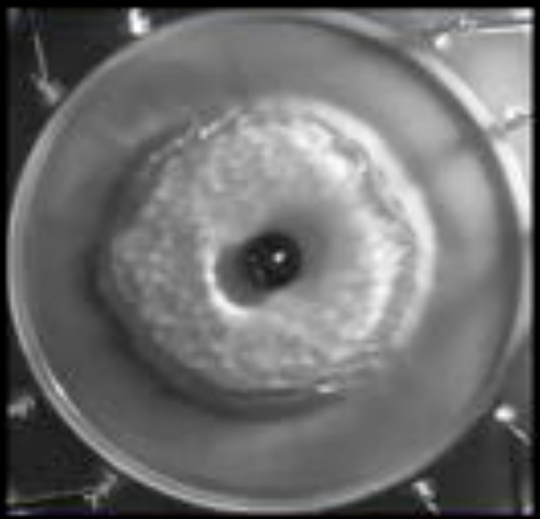
oscillation period $\times 10^{2}$ 30 ms $\rightarrow$ 3 s

Dynamics of water
in the fountain

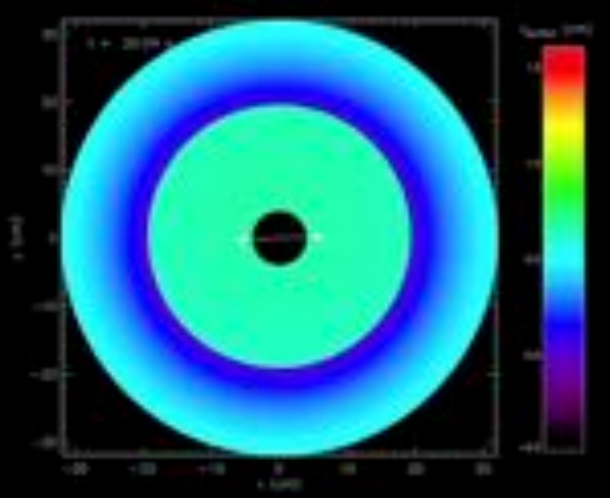
Dynamics of the gas
in the supernova core

diameter 40cm ← 1 000 000 x bigger → diameter 400km

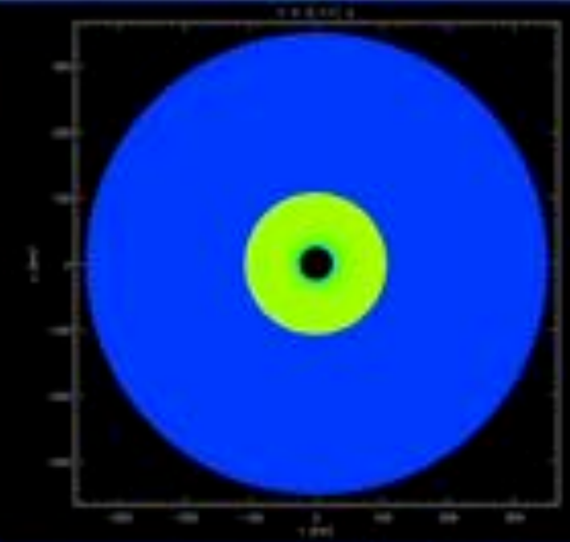
3s/oscillation ← 100 x faster → 0.03s/oscillation



Expérience hydraulique

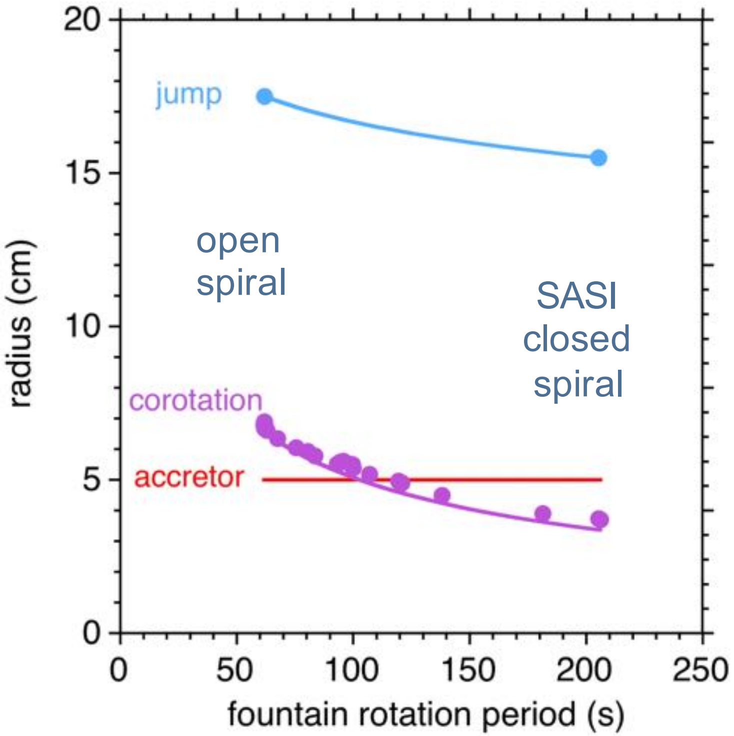


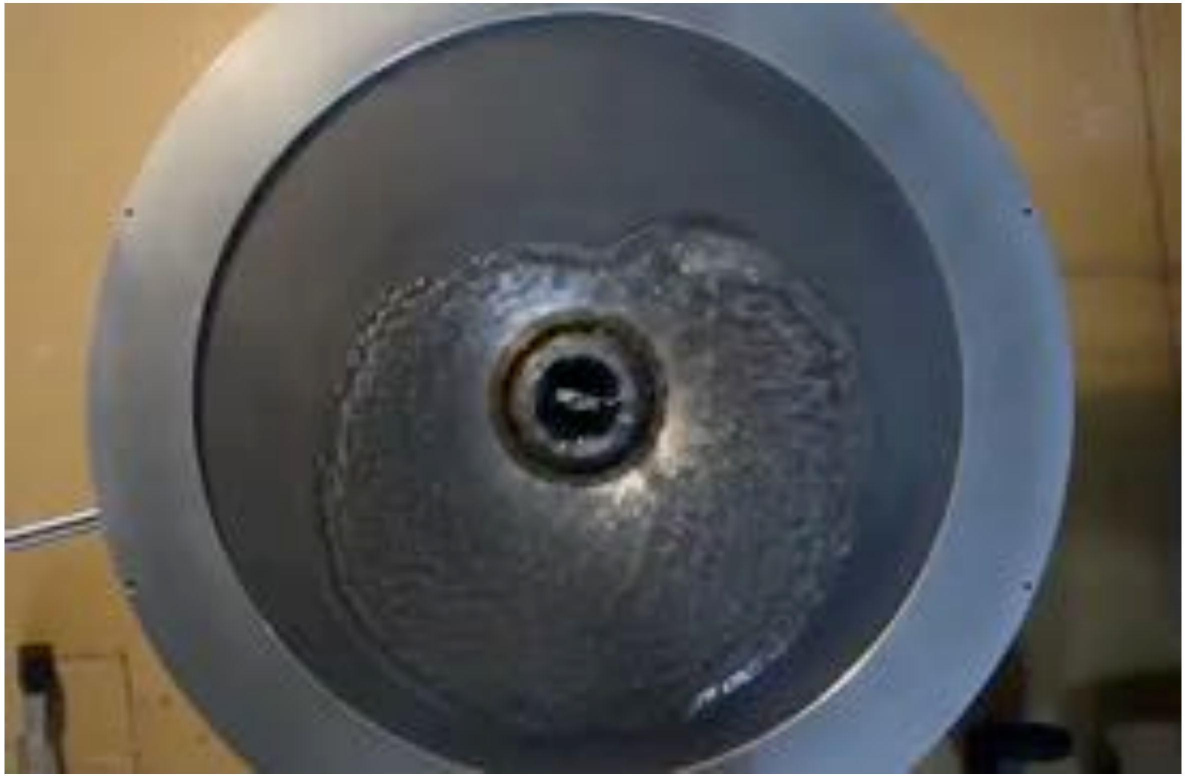
Simulation numérique de l'expérience hydraulique



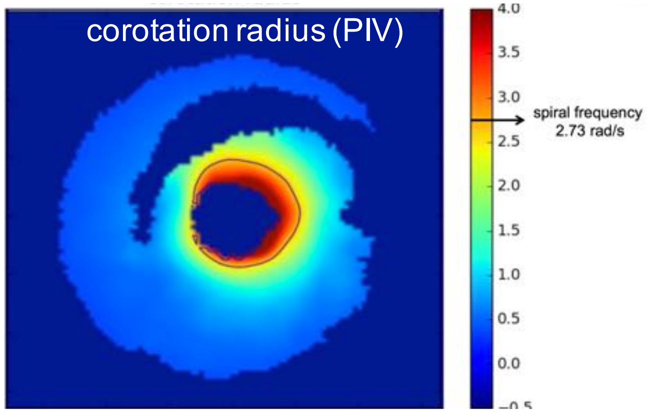
Simulation numérique de l'onde de choc dans le coeur de la supernova

Increasing the rotation rate:
continuous transition from SASI to the corotation instability

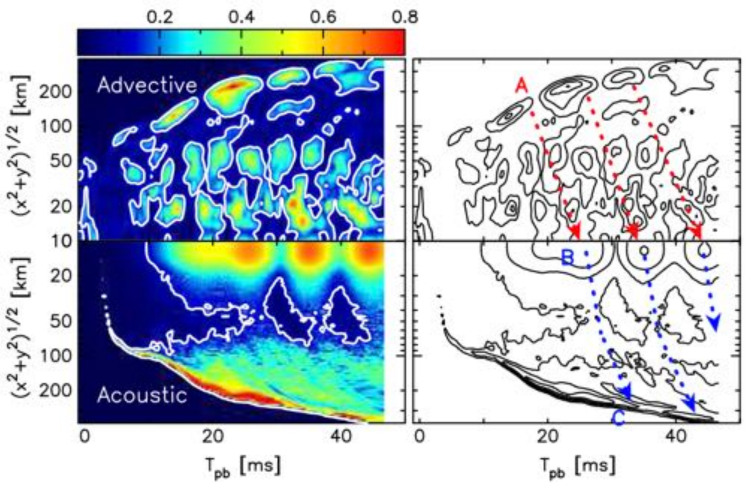




the rotation period is gradually decreased (205s → 62s)
the flow rate is gradually decreased (1.1 L/s → 0.59 L/s)



→the gravitational wave signature of the low T/|W| instability may be hard to disentangle from the SASI oscillations (Kuroda+14)

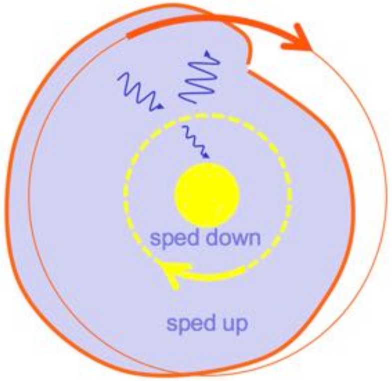


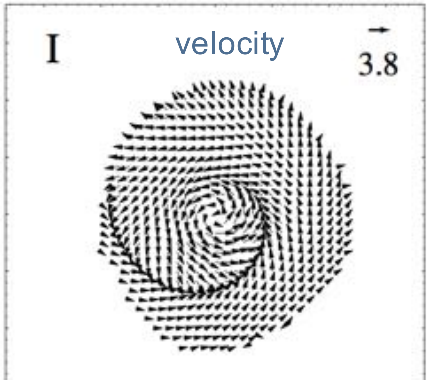
Robust spiral mode driven at the corotation radius (~20% Kepler)
Which mechanism? acoustic over-reflection or vortical-acoustic coupling ?

$$\frac{\Omega}{\Omega_{\rm NS}} \propto \left(\frac{R_{\rm NS}}{R}\right)^2$$ Radial accretion enforces differential rotation

corotation-enhanced SASI:
-weak jump = weak outer acoustic reflection
-the growth time scales like the advection time

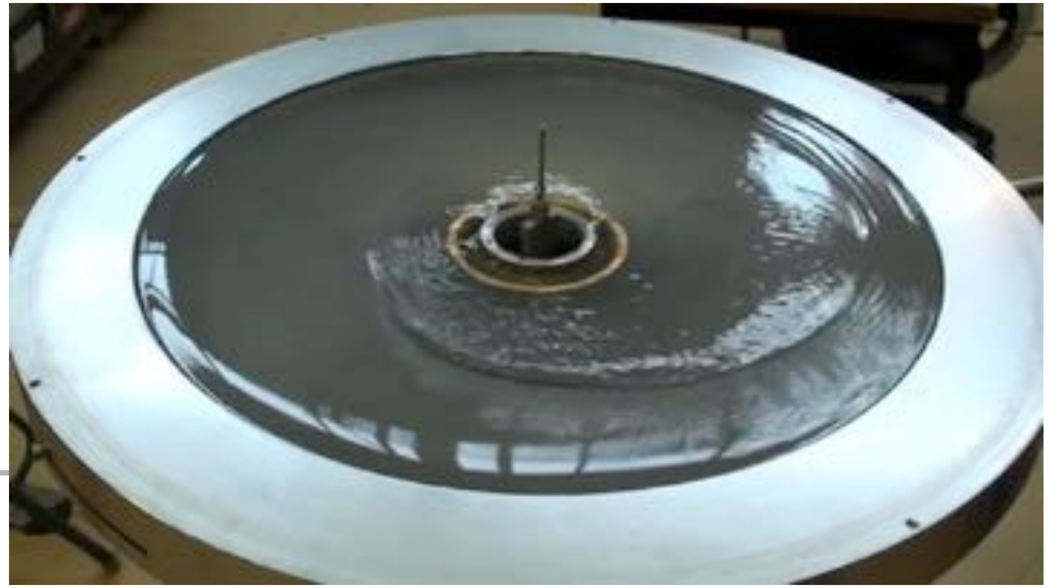
classical corotation instability in astrophysics:
- -neutron star rotating differentially ("low T/|W|") (Shibata+02, Passamonti & Andersson 15)
- -keplerian torus with a reflecting edge (Papaloizou & Pringle 84, Goldreich & Narayan 85)



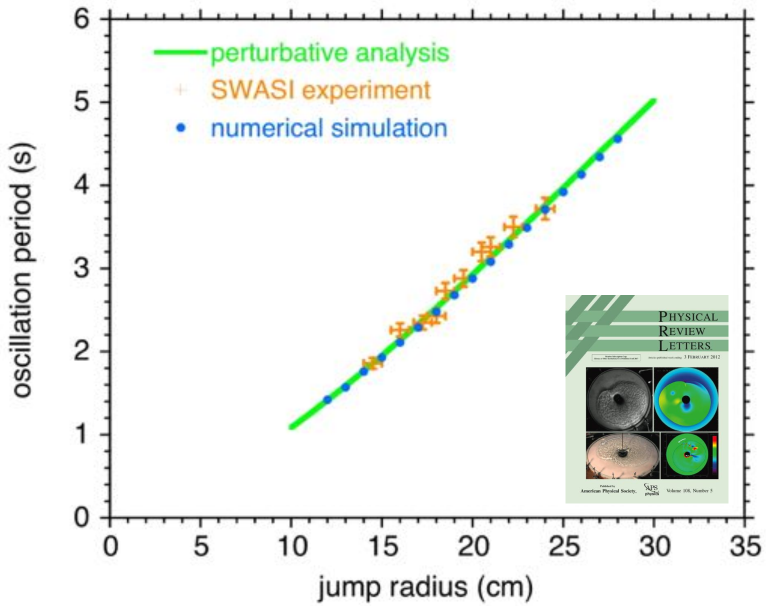


Saijo & Yoshida 06

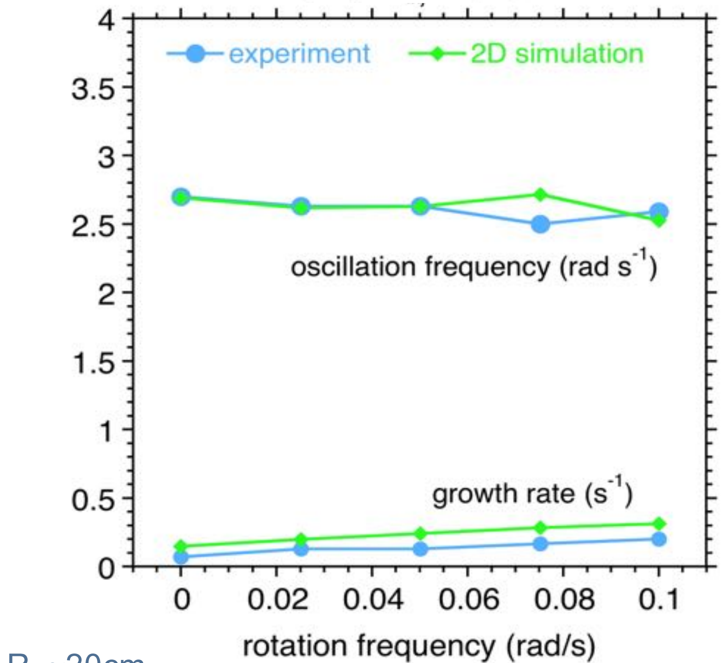
Corotation instability with subsonic accretion



Experimental growth rate and oscillation period compared to shallow water modelling: a hint for an advective mechanism



Foglizzo+12



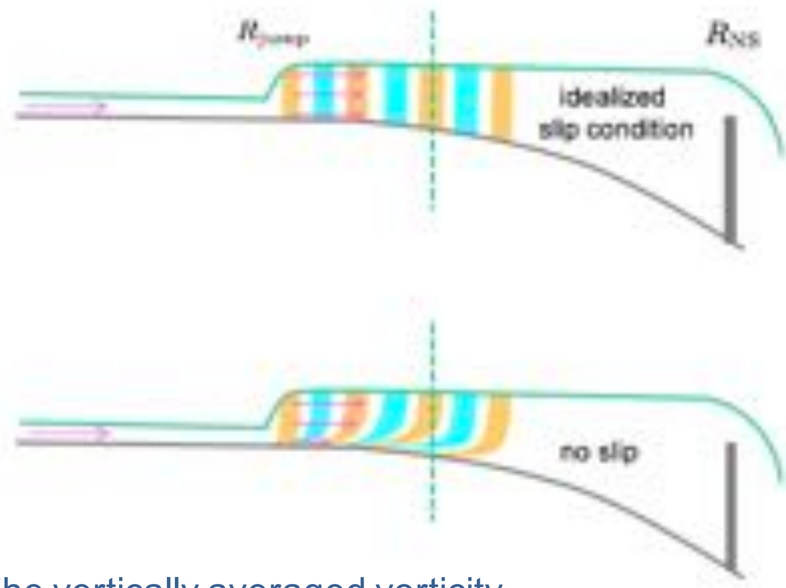
R_{jp}~20cm

-excellent modelling of the oscillation frequency

limited by the measured radial width of the hydraulic jump

-systematic offset of the experimental growth rate

expected phase mixing of the dragged vorticity



The vertically averaged vorticity is damped by a factor Q

$$Q \sim \int_0^H \frac{\mathrm{d}z}{H} \cos\left[\frac{\omega_{\rm SASI}\Delta R}{v(z)}\right] \sim 0.27\ (\text{laminar})$$

$$\sim 0.52\ (\text{turbulent})$$

→ at odds with the idea of a transition to an acoustic corotation instability?

Compact formulation of the perturbative problem with rotation

uniform specific angular momentum

$$L \equiv r^2\Omega(r) = \mathrm{cte}$$

Differential system for the linearized perturbations

$$\left\{\begin{aligned} \frac{\partial H}{\partial t} + \nabla\cdot(H\boldsymbol{v}) &= 0, \\ \frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{w}\times\boldsymbol{v} + \nabla\left(\frac{|\boldsymbol{v}|^2}{2} + gH + \Delta\Phi\right) &= 0. \end{aligned}\right.$$

mass conservation

Euler equation

Doppler shifted frequency

$$\omega' \equiv \omega - \frac{mL}{r^2},$$

same as in a cylindrical flow (Yamasaki & Foglizzo 08)

$$\delta w_z = \frac{r_{\rm sh} v_{\rm sh}}{r v_r} (\delta w_z)_{\rm sh} \mathrm{e}^{\int_{\rm sh} i\omega' \frac{\mathrm{d}r}{v_r}},$$

conserved specific vorticity

same changes of variables as in a plane parallel flow (Foglizzo 09)

$$\left\{\begin{aligned} \frac{\mathrm{d}X}{\mathrm{d}r} &\equiv \frac{v_r}{1-\mathrm{Fr}^2}, \\ \delta\tilde{v}_\theta &\equiv \mathrm{e}^{\int_{\rm sh} i\omega' \frac{\mathrm{d}X}{c^2}} \delta v_\theta, \end{aligned}\right.$$

new variable X

phase shifted velocity

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}X^2} + \frac{\omega'^2 - \frac{m^2}{r^2}(c^2 - v_r^2)}{v_r^2 c^2}\right\} r\delta\tilde{v}_\theta = \mathrm{e}^{\int_{\rm sh} i\omega' \frac{\mathrm{d}X}{c^2}} \frac{\mathrm{d}}{\mathrm{d}X}\frac{r\delta w_z}{v_r}.$$

acoustic equation with a source term similar to Foglizzo 09 without rotation

\+ boundary conditions from conservation equations

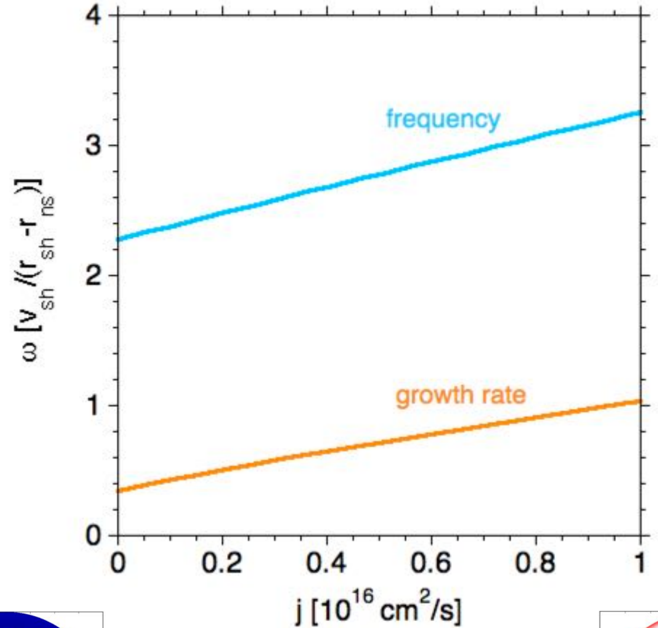
-at the shock/jump

$$\left\{\begin{aligned} (\delta w_z)_{\rm sh} &= -\frac{im}{r_{\rm sh} v_{\rm sh}}\left(1 - \frac{v_{\rm sh}}{v_1}\right)\Delta\zeta\left[-i\omega' v_1\left(1-\frac{v_{\rm sh}}{v_1}\right) + \frac{\partial\Phi}{\partial r} - \frac{v_1 v_{\rm sh}}{r_{\rm sh}} - \frac{L^2}{r_{\rm sh}^3}\right], \\ (r\delta v_\theta)_{\rm sh} &= im\left(1-\frac{v_{\rm sh}}{v_1}\right) v_1 \Delta\zeta, \\ \frac{\mathrm{d}}{\mathrm{d}X}(r\delta\tilde{v}_\theta)_{\rm sh} &= -\frac{im}{v_{\rm sh}^2}\left(1-\frac{v_{\rm sh}}{v_1}\right)\Delta\zeta\left[-i\omega' v_1\left(1 - 2\frac{v_{\rm sh}}{v_1}\right) + \frac{\partial\Phi}{\partial r} - \frac{v_1 v_{\rm sh}}{r_{\rm sh}} - \frac{L^2}{r_{\rm sh}^3}\right], \end{aligned}\right.$$

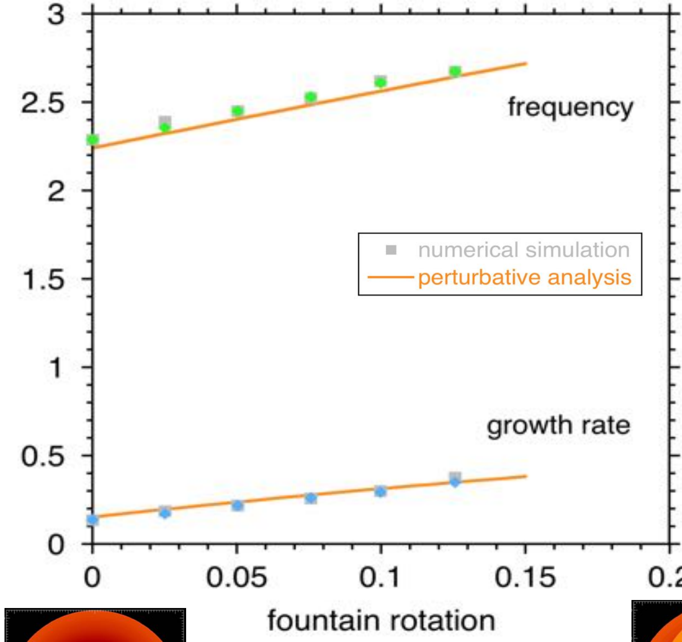
-at the inner boundary

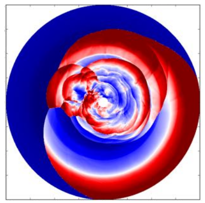
$$v_*^2 \frac{\mathrm{d}}{\mathrm{d}X}\delta(r\delta\tilde{v}_\theta) = i\omega' \mathrm{Fr}_*^{\frac{4}{3}} (r\delta\tilde{v}_\theta)_* + \left(1 - \mathrm{Fr}_*^{\frac{4}{3}}\right)(r v_r \delta\tilde{w})_*$$

Rotation effect in shallow water equations is similar to gas dynamics

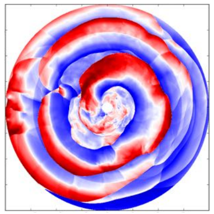


Yamazaki & Foglizzo 08

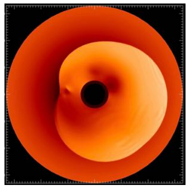




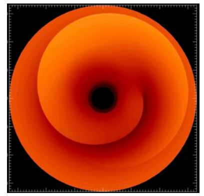
shocked gas dynamics

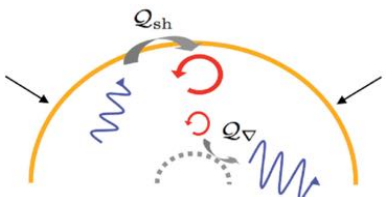


Kazeroni+17

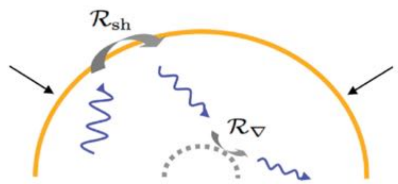


shallow water dynamics





Why is the prograde mode of SASI destabilized by rotation?



$$\omega' \equiv \omega - \frac{mL}{r^2}$$

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}X^2} + \frac{\omega'^2 - \frac{m^2}{r^2}(c^2 - v_r^2)}{v_r^2 c^2}\right\} r\delta\tilde{v}_\theta = \mathrm{e}^{\int_{\rm sh} i\omega' \frac{\mathrm{d}X}{c^2}} \frac{\mathrm{d}}{\mathrm{d}X}\frac{r\delta w_z}{v_r}.$$

The vortical-acoustic coupling $\mathcal{Q}_\nabla$ depends on - the stationary flow gradients

- the relative phase of advected and acoustic perturbations

Wronskien resolution: convolution of the acoustic solution with the source term

acoustic solution satisfying the inner boundary condition

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}X^2}+\frac{\omega'^2-\frac{m^2}{r^2}(c^2-v_r^2)}{v_r^2c^2}\right\}r\delta\tilde{v}_\theta^0=0,$$

inner boundary condition

$$v_*^2\frac{\mathrm{d}}{\mathrm{d}X}\delta(r\delta\tilde{v}_\theta^0)=i\omega'\mathrm{Fr}_*^{\frac{4}{3}}(r\delta\tilde{v}_\theta^0)_*,$$

definition of the perturbed mass flux

$$h^0\equiv\frac{\delta v_r}{v_r}+\frac{\delta H}{H}=\frac{1-\mathrm{Fr}^2}{imv_r}\mathrm{e}^{\int_{\mathrm{sh}}-i\omega'\frac{v_r}{1-\mathrm{Fr}^2}\frac{\mathrm{d}r}{c^2}}\frac{\mathrm{d}}{\mathrm{d}r}\left(\mathrm{e}^{\int_{\mathrm{sh}}i\omega'\frac{v_r}{1-\mathrm{Fr}^2}\frac{\mathrm{d}r}{c^2}}r\delta v_\theta^0\right).$$

acoustic solution

advected phase of the source term

$$\int_*^{\mathrm{sh}}\left(h^0+\frac{\omega'}{mc^2}r\delta v_\theta^0\right)\frac{\mathrm{e}^{\int_* i\omega'\frac{1+\mathrm{Fr}^2}{1-\mathrm{Fr}^2}\frac{\mathrm{d}r}{v_r}}}{1-\mathrm{Fr}^2}\frac{\mathrm{d}r}{v_r}=-\frac{\frac{\omega'}{mv_{\mathrm{sh}}}(r\delta v_\theta)_{\mathrm{sh}}^0+v_1h_{\mathrm{sh}}^0}{\frac{\partial\Phi}{\partial r}-\frac{v_1v_{\mathrm{sh}}}{r_{\mathrm{sh}}}-\frac{L^2}{r_{\mathrm{sh}}^3}-i\omega'v_1\left(1-\frac{H_1}{H_{\mathrm{sh}}}\right)}\mathrm{e}^{\int_*^{\mathrm{sh}}i\omega'\frac{1+\mathrm{Fr}^2}{1-\mathrm{Fr}^2}\frac{\mathrm{d}r}{v_r}}-\frac{h_*^0}{i\omega_*'}.$$

advective-acoustic coupling

shock boundary condition

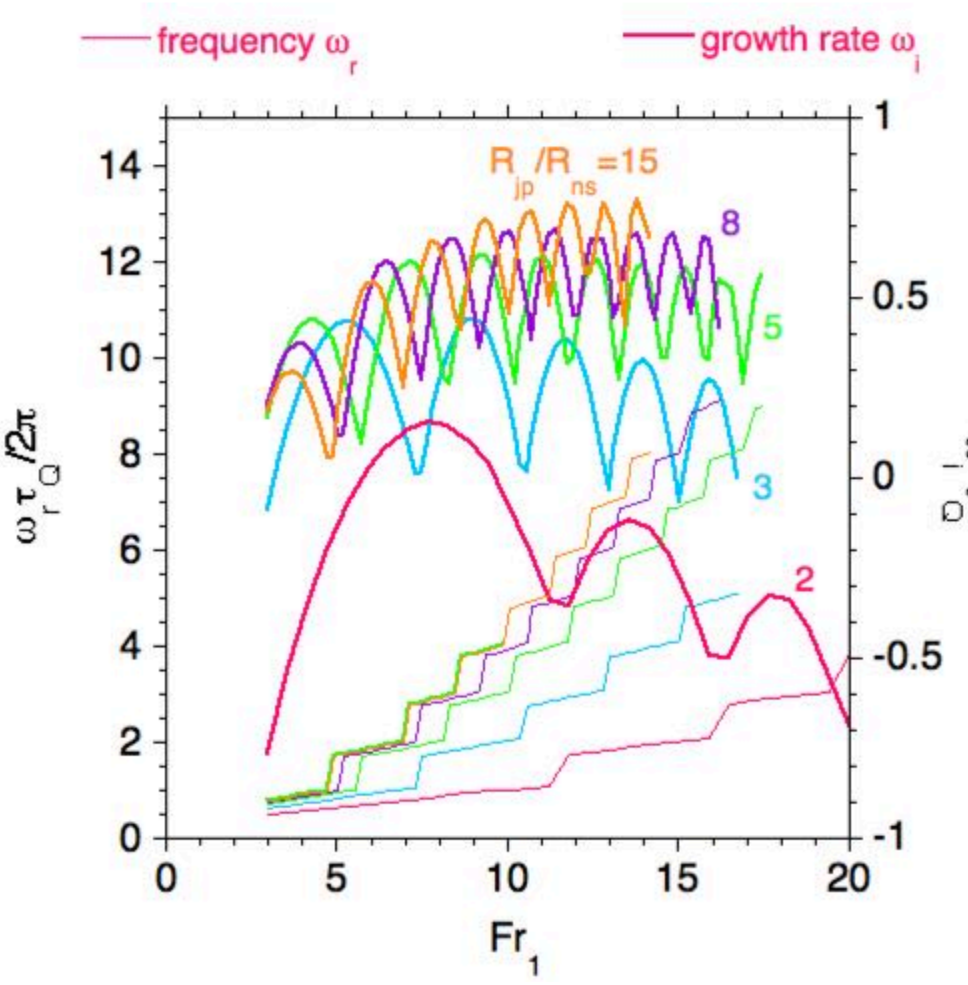
inner boundary condition

The Doppler shifted frequency $\omega'\equiv\omega-\frac{mL}{r^2}$ affects the phase mixing between the source and the acoustic wave

The frequency of the prograde mode is locally decreased by the doppler shift: the decrease of $\omega'\tau_\nabla$ is favourable to the advective-acoustic coupling as in Sheck+08 and Foglizzo 09 without rotation.

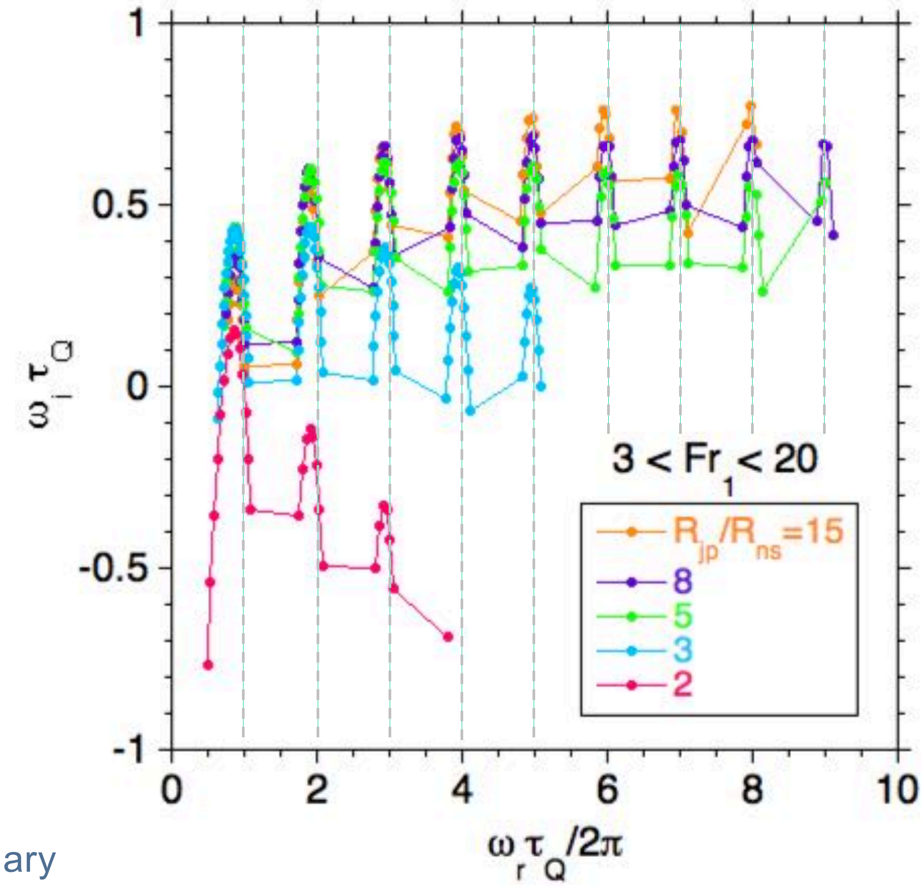
The corotation condition $\omega'=0$ favours the advective-acoustic coupling: the stationary phase prevents phase mixing

Analogue of SASI modes without rotation: Fr_1, R_{jp}/R_{ns}



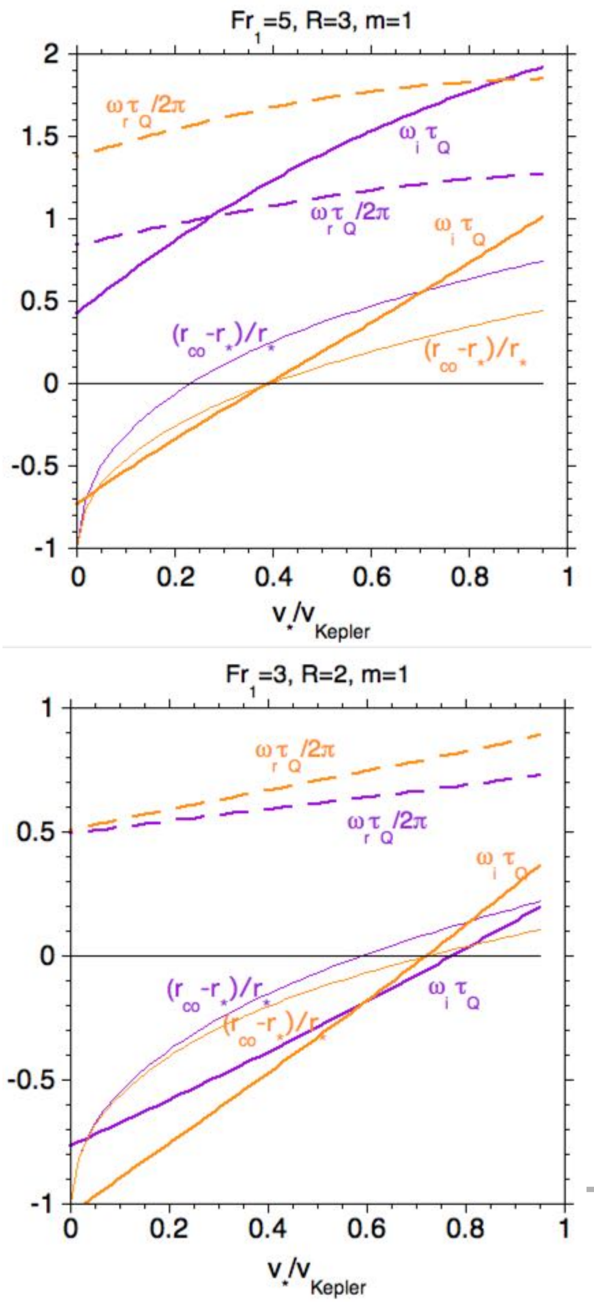
radial advective-acoustic time

$$\tau_Q \equiv \int_*^{sh} \frac{dr}{|v_r|} + \int_*^{sh} \frac{dr}{c - |v_r|}$$



eigenfrequencies ~multiples of $2\pi/\tau_Q$ suggest that the advective-acoustic coupling is dominated by the lower boundary

Comparison of a shocked rotating flow and a trapped acoustic mode



normal shock condition: vorticity + pressure perturbation

- as in Yamasaki & Foglizzo 08, the growth rate of the prograde mode increases with the rotation rate

- a corotation radius can exist for rotation rates as low as 3% v_{Kepler} at r_* (T/W~0.05%)

- a corotation radius is not a sufficient condition for instability (e.g. Fr_1=3, R=2)

- the transition from SASI to an instability with a corotation is very smooth

ad-hoc shock condition: total acoustic reflexion, no vorticity

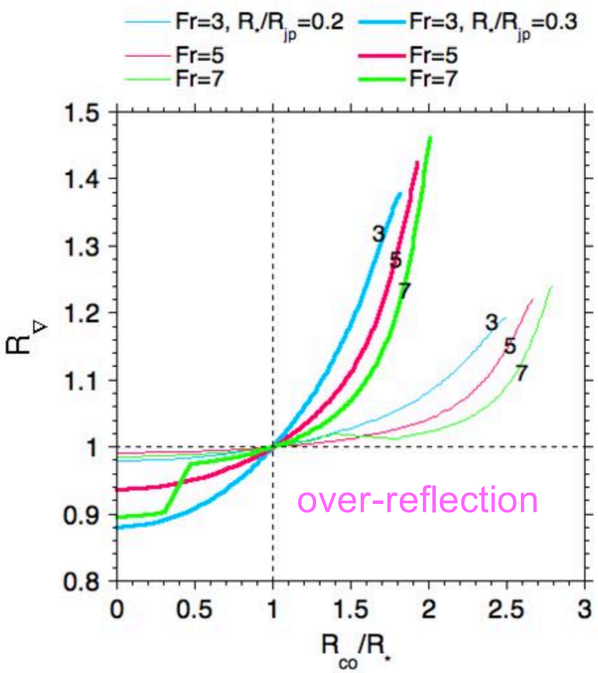
- when acoustic reflexion at the shock is total, the existence of the corotation radius is a sufficient condition for instability: similar to differentialy rotating NS (Watts+05, Passamonti & Andersson 15, Yoshida & Saijo 17)

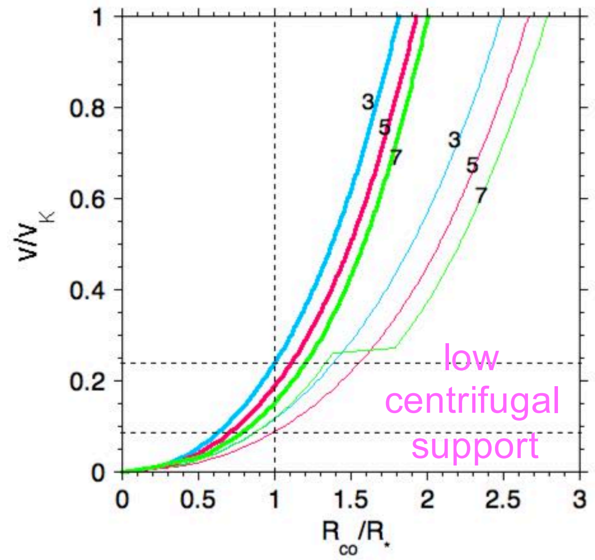
- a corotation radius can exist for rotation rates as low as 6% v_{Kepler} at r_* (T/W~0.02%)

- however, the growth rate of this corotation instability seems loosely correlated with the growth rate of the shocked flow.

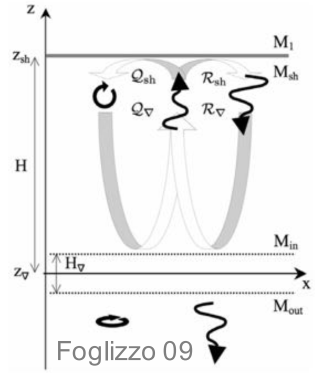
- estimated growth rate: $\frac{\omega_i}{\Omega_{sh}} \propto (2.2 \pm 0.4) \left(\frac{\Omega_{sh}}{\Omega_*}\right)^{\frac{1}{2}} \left[1 - \frac{\Omega_{corot}}{\Omega_*}\right]$

Acoustic over-reflection and corotation





$$\mathcal{Q}_0 e^{i\omega\tau_{\mathcal{Q}}} + \mathcal{R}_0 e^{i\omega\tau_{\mathcal{R}}} = 1$$



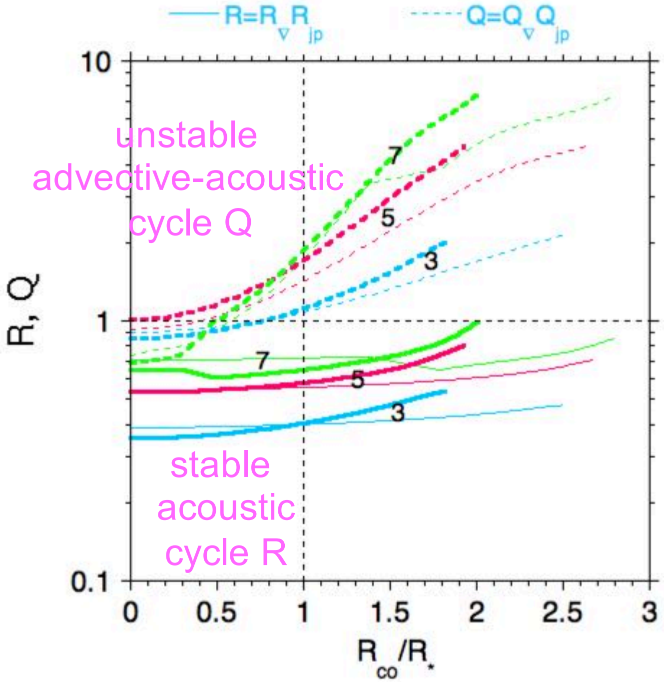
Ingoing acoustic waves are over-refected as soon as a corotation radius stands within the flow boundaries.

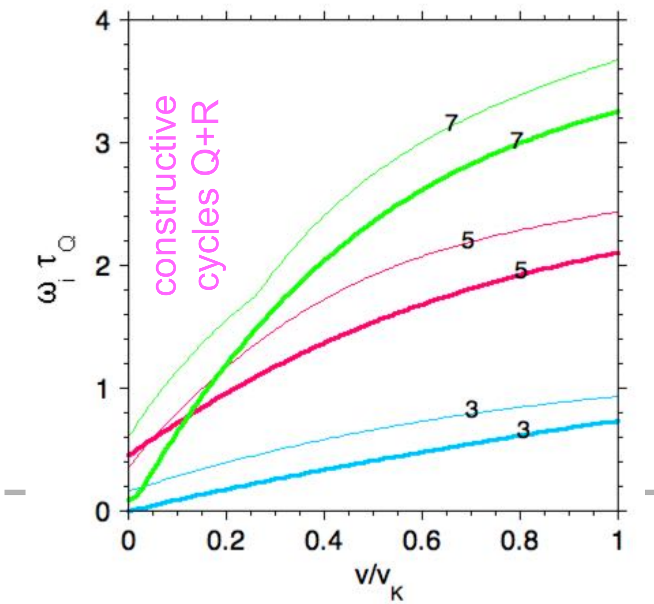
This takes place for moderate centrifugal support (9-25% v_{Kepler})

However, this over-reflection is insufficient to compensate for the acoustic damping at the shock.

The advective-acoustic cycle Q is strongly destabilized by rotation. The acoustic cycle can add a constructive and sometimes decisive contribution.

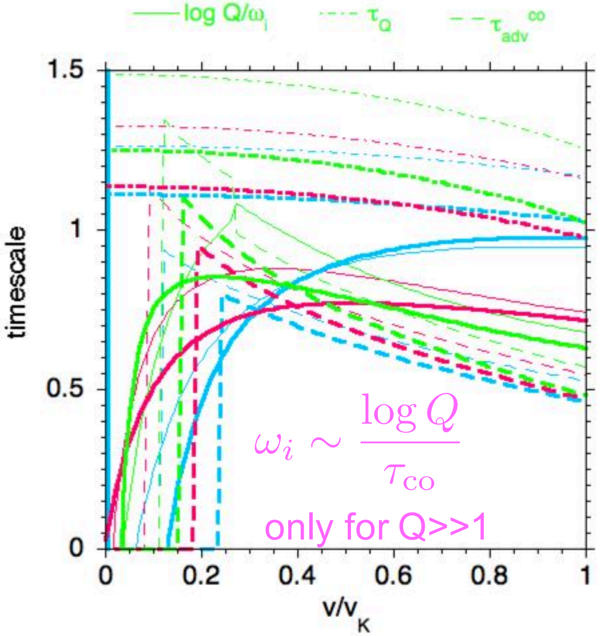
In the astrophysical gas with neutrino cooling, some additional acoustic damping is expected upon reflection in the inner region of the flow (Bruno Pagani)





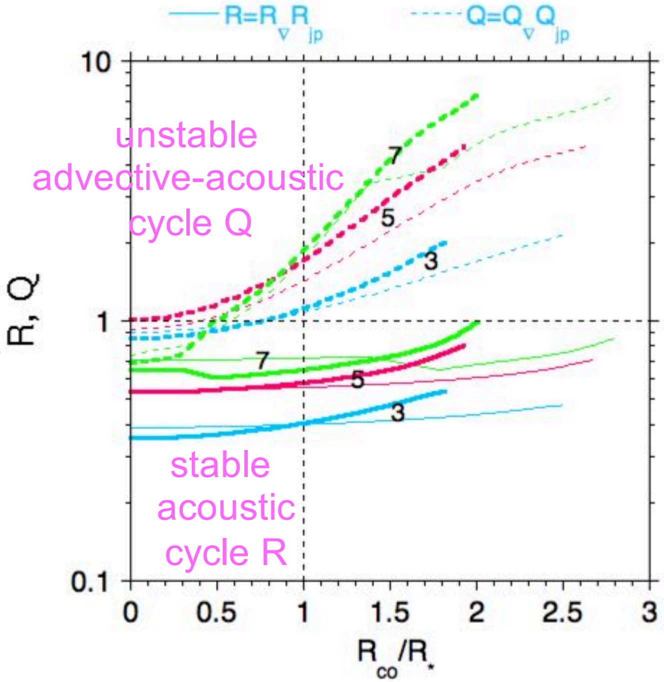
Growth time: hint for the advection time to the corotation radius

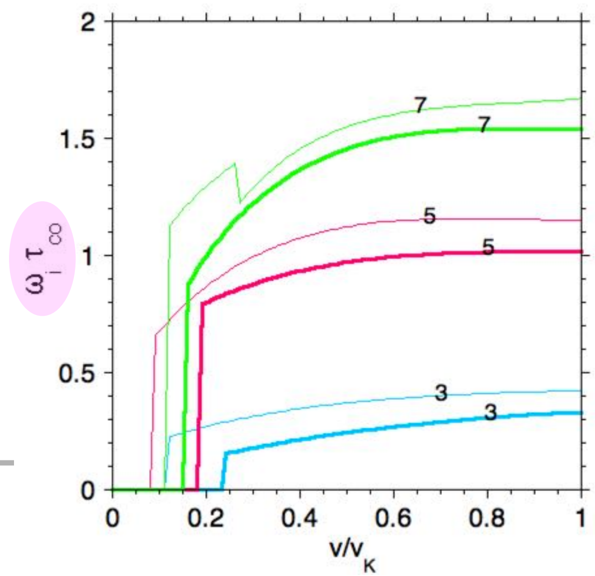
$$\mathcal{Q}_0 e^{i\omega\tau_Q} + \mathcal{R}_0 e^{i\omega\tau_R} = 1$$



As the rotation rate is increased, the growth time becomes significantly shorter than the advection time to the inner boundary τ_Q.

The growth time is better described by the advection time τ_{co} to the corotation radius, as expected by the enhanced advective-acoustic coupling there

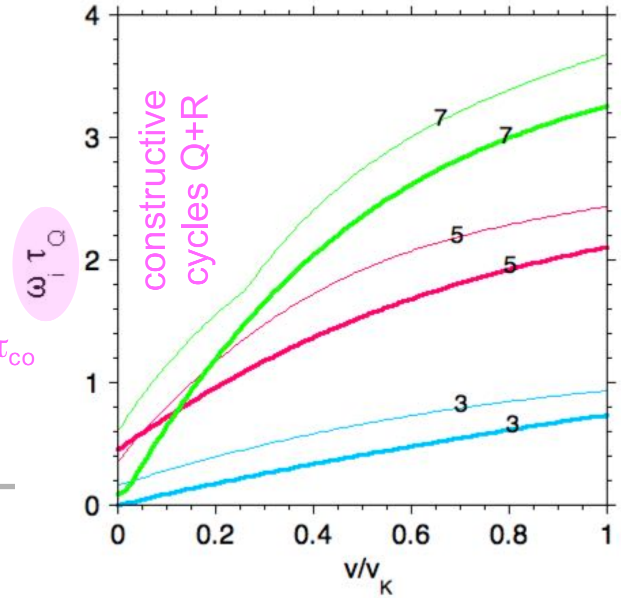




growth rate ω_i in units of the advection time

to the inner radius τ_Q →

← to the corotation radius τ_{co}

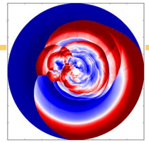


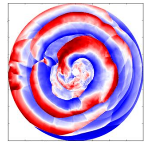
Towards higher Reynolds numbers

with Gilles Durand

Diameter 3m50, Reynolds x 10

Conclusions



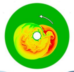


2D Cylindrical gas dynamics (Kazeroni+17) suggests that

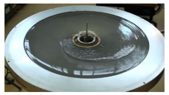
-SASI can account for pulsar rotation periods down to ~50ms

-for rotation rates >100Hz the 'corotation instability' decreases the pulsar spin by <30%

Both instability regimes are captured in the supernova fountain experiment

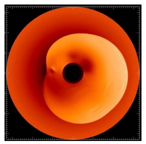


-as the injected angular momentum increases, the prograde spiral mode of SASI seems to connect smoothly to the 'corotation instability'



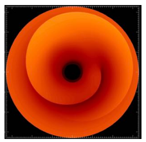
-the offset growth rate in the experiment suggests advection may play a dominant role even when a corotation is present

The shallow water model offers a simple analytical framework to study the interplay of SASI & corotation



-equations are both simple and connected to a real experiment

-the rotational destabilization of the prograde mode of SASI can be explained by its lower doppler shifted frequency which benefits to the advective-acoustic coupling



-a classical corotation instability is recovered as a purely acoustic process, despite radial advection, if the shock is artificially replaced by a total acoustic reflection and no advected vorticity

-the existence of a corotation radius does produce an acoustic over-reflection but this is not a sufficient condition for instability in a shocked flow

-the prograde mode of SASI can be more unstable than an acoustic corotation instability: the stationary phase at the corotation radius favours a strong advective-acoustic coupling

-the growth time scales like the advection time from the shock to the corotation radius

→A sharp transition between SASI and the 'low T/|W|' instability in a shocked flow is not expected

→These results have to be tested in non adiabatic gas dynamics including cooling and the protoneutron star interior which may develop (or not) a classical low T/W instability