Pulsar glitch dynamics in general relativity

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Sourie, Novak, Oertel & Chamel
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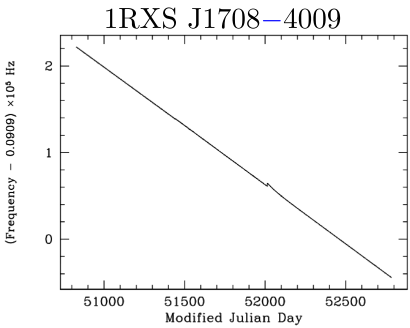
CoCoNuT meeting 2017, Garching, October, 27th

Pulsar glitches



Kaspi & Gavriil, *ApJ*, 2003

- Angular momentum loss through emission of electromagnetic waves

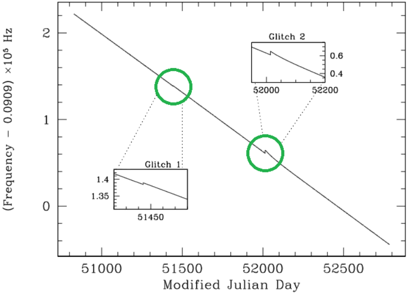
$\Rightarrow$ slowing down of the pulsar with

$$\dot{P} \sim 10^{-21}\text{–}10^{-10}$$

tiny changes in this slowing down = glitches

Pulsar glitches

1RXS J1708−4009



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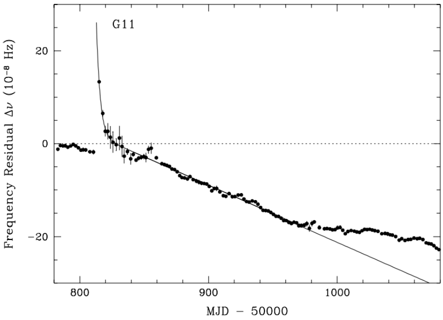
Pulsar glitch observations



Wong, Backer & Lyne, *ApJ*, 2001

- glitch **amplitude** are low:

$$\Delta\Omega/\Omega \sim 10^{-11} - 10^{-5}$$

- **rise time** is quite short :

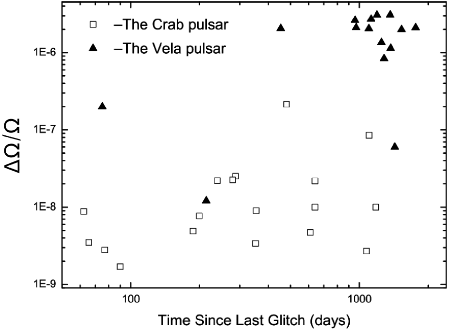
$$\boxed{\tau_{\rm r} < 30 \text{ s}} \quad \leftarrow\text{-- Vela}$$

- exponential **relaxation** during several days, up to months.

$\Rightarrow$ glitches are driven by **internal processes**

Different glitch types

Wang et al., *Ap&SS*, 2012



Giant glitches

- quasi-periodic
- narrow amplitude distribution

Standard glitches

- randomly spaced in time
- various amplitudes

Different glitch models

⇒moment of inertia reduction, with crustquakes

⇒transfer of angular momentum between two components, with superfluidity

Different glitch types

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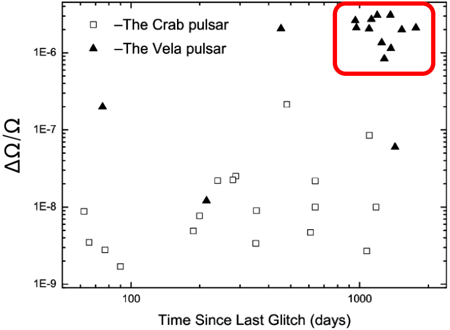

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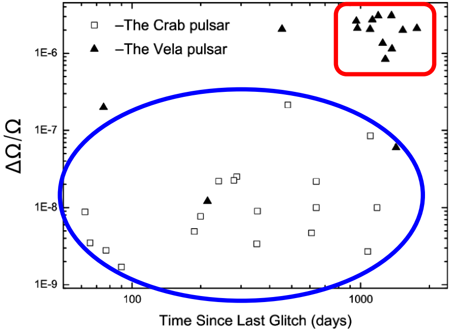

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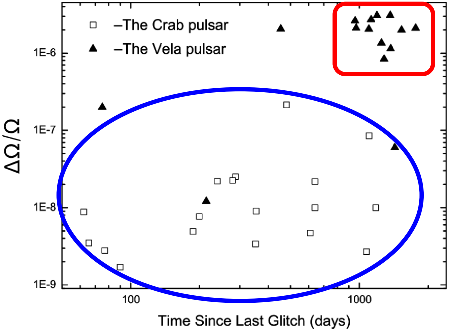

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Hypotheses

- General relativity to describe gravity
- Need to describe rotation $\Rightarrow$ axisymmetry
- Glitch time-scale $\gg$ hydro time-scale $\Rightarrow$ stationarity

$\Rightarrow$Contrary to spherical symmetry no matching to any known vacuum solution is possible (no Birkhoff theorem).
$\Rightarrow$Only boundary condition at $r \to \infty$: flat metric.

Numerical solution obtained using spectral methods (Grandclément & Novak 2009) and the LORENE library (http://lorene.obspm.fr).

Two-fluid approach

Anderson & Itoh 1975

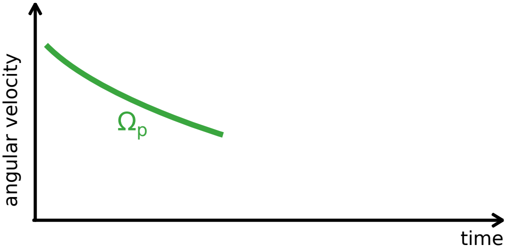
Two-fluid model

- charged particles :

$$\Omega_p = \Omega \leftrightarrow \text{pulsar}$$

- superfluid neutrons :

$$\Omega_n \gtrsim \Omega_p$$



- Superfluid vortices can pin into the crust nuclei
- When a critical threshold is reached in terms of $\delta\Omega = \Omega_n - \Omega_p$, some vortices unpin and can freely move in radial direction

$\Rightarrow$ Transfer of angular momentum between both fluids and glitch

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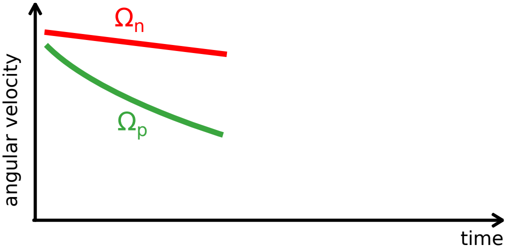

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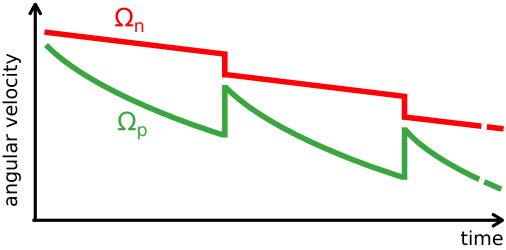

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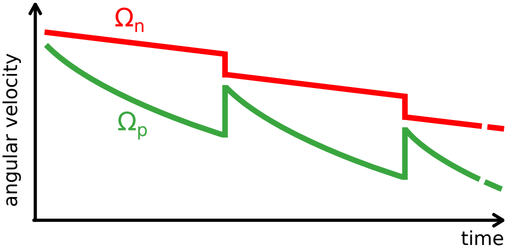

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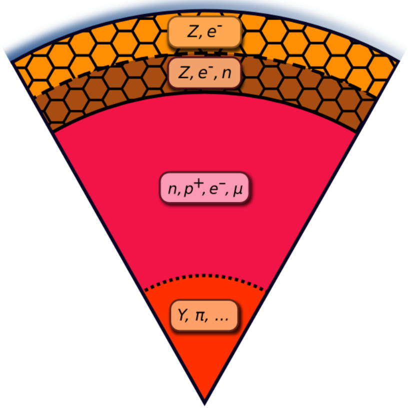
Hypotheses

Prix, Novak & Comer 2005



Equilibrium configurations:

- uniform composition : n, p, e^-
  $\rightsquigarrow$ crust is neglected
- rigid rotation :
  $\rightsquigarrow$ Ω_n and Ω_p = const.
- stationary and axisymmetric spacetime + isolated star.
- $T \ll T_F$, and no magnetic field.
- dissipation effects are neglected.

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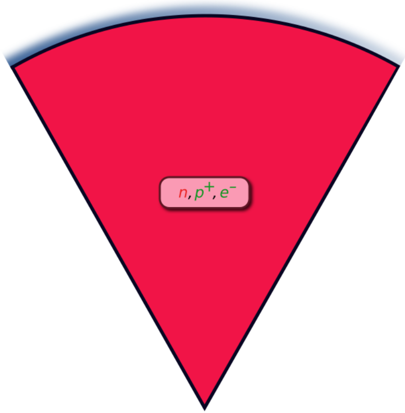

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Two-fluid relativistic hydrodynamics

Carter 1989; Carter & Langlois 1998; Langlois, Sedrakian & Carter 1998

System made of two perfect fluids :

- superfluid neutrons $\rightarrow n_{\rm n}^{\alpha} = n_{\rm n} u_{\rm n}^{\alpha}$,
- protons & electrons $\rightarrow n_{\rm p}^{\alpha} = n_{\rm p} u_{\rm p}^{\alpha}$.

Energy-momentum tensor

1 fluid : $$T_{\alpha\beta} = (\mathcal{E} + P)\, u_\alpha u_\beta + P g_{\alpha\beta}$$

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$$T_{\alpha\beta} = n_{\mathrm{n}\alpha} p^{\mathrm{n}}_{\beta} + n_{\mathrm{p}\alpha} p^{\mathrm{p}}_{\beta} + \Psi g_{\alpha\beta}$$

$\hookrightarrow$ conjugate momenta

$$\begin{cases} p^{\mathrm{n}}_{\alpha} & \propto u^{\mathrm{n}}_{\alpha} \\ p^{\mathrm{p}}_{\alpha} & \propto u^{\mathrm{p}}_{\alpha} \end{cases}$$

without entrainment

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$\hookrightarrow$ conjugate momenta

Entrainment matrix:

$$\left\{ \begin{array}{ll} p^{\rm n}_{\alpha} & \propto u^{\rm n}_{\alpha} \\ p^{\rm p}_{\alpha} & \propto u^{\rm p}_{\alpha} \end{array} \right. \longrightarrow \left\{ \begin{array}{ll} p^{\rm n}_{\alpha} & = \mathcal{K}^{\rm nn} n^{\rm n}_{\alpha} + \mathcal{K}^{\rm np} n^{\rm p}_{\alpha} \\ p^{\rm p}_{\alpha} & = \mathcal{K}^{\rm pn} n^{\rm n}_{\alpha} + \mathcal{K}^{\rm pp} n^{\rm p}_{\alpha} \end{array} \right.$$

without entrainment

$\Rightarrow$ entrainment effect

New, realistic equations of state

$$\mathcal{E}\left(n_{\mathrm{n}}, n_{\mathrm{p}}, \Delta^2\right) \longleftrightarrow \Psi\left(\mu^{\mathrm{n}}, \mu^{\mathrm{p}}, \Delta^2\right)$$

Relativistic mean field model :

nucleon - nucleon interactions $\Leftrightarrow$ effective meson exchange

	DDH Typel & Wolter (1999)	**DDH$\boldsymbol{\delta}$** Avancini et al. (2009)	exp. constraints Oertel et al. (2017)	[units]
n_0	0.153	0.153	0.158 ± 0.005	[fm^{-3}]
B_{sat}	16.3	16.3	15.9 ± 0.3	[MeV]
K	240	240	240 ± 40	[MeV]
J	32.0	25.1	31.7 ± 3.2	[MeV]
L	55	44	58.7 ± 28.1	[MeV]
$M_{\mathrm{G}}^{\max,0}$	2.08	2.16	$\gtrsim 2$	[$\mathrm{M}_\odot$]

The (Komar) angular momentum J_X is such that

$$\boxed{\mathrm{d}J_X = I_{XX}\ \mathrm{d}\Omega_X + I_{XY}\ \mathrm{d}\Omega_Y}$$

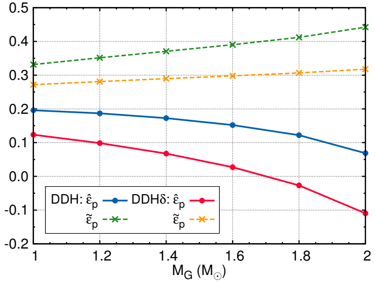
Total coupling coefficient $\boxed{\hat{\varepsilon}_X = I_{XY} / \left(I_{XX} + I_{XY}\right)}$ depends

ENTRAINMENT

- due to strong interaction between nucleons
- measured with the global entrainment coefficient $\tilde{\varepsilon}$ (integration of ε over the star)

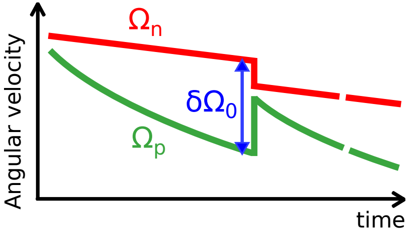
Lense-Thirring effect

- due to GR dragging of inertial frames by each fluid
- measured with the metric term $g_{t\varphi}$



Mutual friction

No external torque $\Rightarrow$exchange of angular momentum between neutrons and protons through mutual friction torque $\Gamma_{\rm mf}$



From Langlois *et al.* (1998), with straight vortices parallel to the rotation axis: interplay between

- Magnus force due to neutron fluid
- drag force caused by charged particles

$$\Gamma_{\rm mf} = -\int \frac{\mathcal{R}}{1+\mathcal{R}^2}\Gamma_{\rm n} n_{\rm n}\varpi_{\rm n}\chi_\perp^2\,{\rm d}\Sigma\times(\Omega_{\rm n}-\Omega_{\rm p}) = -\bar{\mathcal{B}}\times 2\hat{I}_{\rm n}\Omega_{\rm n}\zeta\times\delta\Omega$$

Rise time

Sidery et al. 2010

Evolution equations:

$$\begin{cases} \dot{J}_{\rm n} &= +\,\Gamma_{\rm mf}, \\ \dot{J}_{\rm p} &= -\,\Gamma_{\rm mf}. \end{cases} \dashrightarrow \quad \frac{\delta\dot{\Omega}}{\delta\Omega} = -\frac{\hat{I}\hat{I}_{\rm n}}{I_{\rm nn}I_{\rm pp} - I_{\rm np}^2} \times 2\bar{\mathcal{B}}\zeta\Omega_{\rm n}$$

$\Rightarrow$Analytic approximation:

$$\delta\Omega(t) = \delta\Omega_0 \times \exp\left(-\frac{t}{\tau_{\rm r}}\right)$$

$$\boxed{\tau_{\rm r} = \frac{\hat{I}_{\rm p}}{\hat{I}} \times \frac{1 - \hat{\varepsilon}_{\rm p} - \hat{\varepsilon}_{\rm n}}{2\zeta\bar{\mathcal{B}}\Omega_{\rm n}}}$$

$\Rightarrow$Numerical modeling :

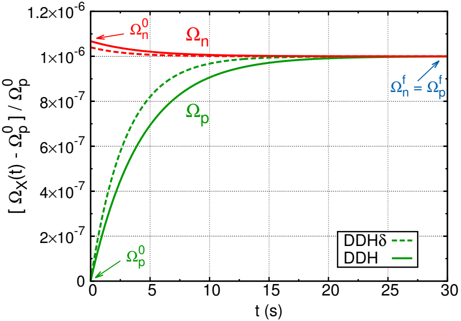
$\Omega_{\rm n}(t)$ and $\Omega_{\rm p}(t)$ are determined by integration from $\Omega_{\rm n,0} > \Omega_{\rm p,0}$

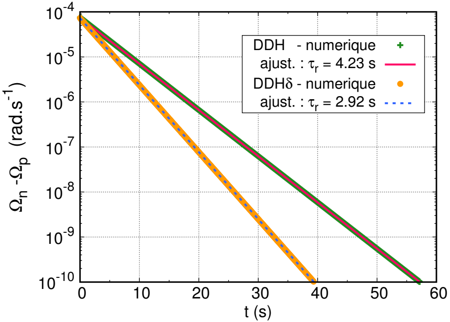
PARAMETERS

$M_G, \Omega, \Delta\Omega/\Omega, \mathrm{EoS}, \bar{\mathcal{B}}$

Time evolution

$$\Delta\Omega/\Omega = 10^{-6},\ \Omega_{\rm n}^f = \Omega_{\rm p}^f = 2\pi \times 11.19\ \text{Hz},$$
$$M_{\rm G} = 1.4\ \text{M}_\odot\ \&\ \bar{\mathcal{B}} = 10^{-4}$$

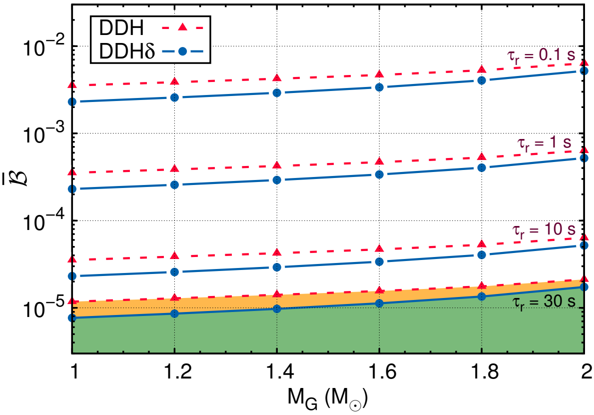




$\dashrightarrow$ Rise times can be estimated with high accuracy without time integration, using only equilibrium models.

Vela pulsar

$\Delta\Omega/\Omega = 10^{-6}$, $\Omega_{\rm n}^f = \Omega_{\rm p}^f = 2\pi \times 11.19$ Hz



- $\bar{\mathcal{B}} \nearrow \Longrightarrow \tau_{\rm r} \searrow$
- Constraints on $\bar{\mathcal{B}}$:

  $\tau_{\rm r} < 30$ s $\Rightarrow \bar{\mathcal{B}} > 10^{-5}$
- $\bar{\mathcal{B}} < 1/2 \rightsquigarrow \tau_{\rm r} > 0.6$ ms

$\hookrightarrow$ a glitch event is a quasi-stationary process, from the hydro viewpoint

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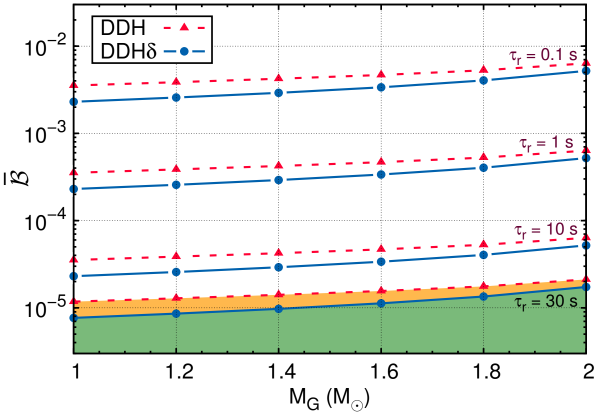

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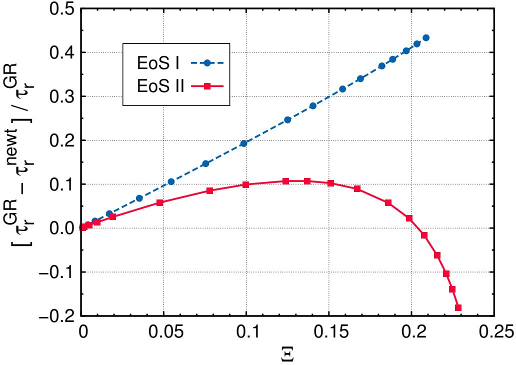
Influence of general relativity

Comparison using two analytic EoSs from Prix *et al.* (2005)



- Compactness defined as

$$\boxed{\Xi = \frac{G\,M_G}{R_{circ}c^2}}$$

  with $\Xi_{NS} \simeq 0.2 - 0.3$
- depends on Ω, too.

$\Rightarrow$impact of general relativity on glitch dynamics can be quite strong!

Conclusions – perspectives

- Precise models of rotating neutron stars in GR
- Realistic EoS for 2 fluids, including entrainment
- Quasi-stationary approach, with analytic formula for rise time
- Additional coupling between fluids due to Lense-Thirring effect
- Strong overall influence of GR on glitch rise time

For the future:

- Looking for accurate data to constrain rise time
- Local modeling of glitch unpinning and movement
- Taking into account crust in global models?

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With entrainment

$$p_X^\alpha = \mathcal{K}^{XX} n_X u_X^\alpha + \mathcal{K}^{XY} n_Y u_Y^\alpha$$

Dynamic effective mass:

$$\boxed{p_X^i = \tilde{m}_X \; u_X^i} \qquad i \in \{1, 2, 3\}$$

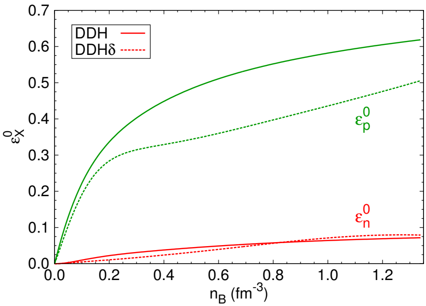
in the fluid-Y rest-frame

In the $u_Y^i = 0$ frame:

relativity

$$\tilde{m}_X = \mu^X \times \left(1 - \varepsilon_X\right)$$

entrainment



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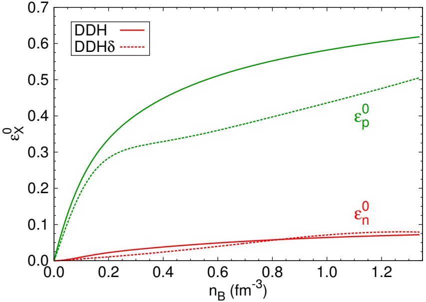

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